XVI IMEKO World Congress
Measurement - Supports Science - Improves Technology - Protects Environment ... and Provides Employment - Now and in the Future
Vienna, AUSTRIA, 2000, September 25-28

NOISE INFLUENCE ON THE MULTIPLE MEASURING METHOD

T. Wada and T. Ono
Department of Mechanical Systems Engineering
Osaka Prefecture University
1-1 Gakuen-cho, Sakai, Osaka 599-8531, Japan

Abstract: The multiple measuring method, that is, the double measuring method or its improved version, provides a precise calibration procedure of linear sensors as well as a linear system identification method, whose results are independent of dynamics of measuring devices. These methods are established on the assumption that measurement noise can be ignored. In practice, however, measurement noise exists and it cannot be ignored. In this paper, the influence of measurement noise is investigated on the result obtained by applying the multiple measuring method. Consequently, the influence is appeared as the signal-to-noise ratio at an observed output and it is removed by the principle of the multiple measuring method.
Keywords: Multiple measuring method, Measurement noise, Signal-to-noise ratio

1 INTRODUCTION

The differences in dynamics among measuring devices affect the measured results when plural devices are simultaneously used for measurement of physical quantities. The multiple measuring method, the “double measuring method” and the “improved double measuring method”, was developed to remove such differences in dynamics from the resulting values [1]. In another word, they provide the methods of relative compensation of the differences in dynamics. That is, the differences in dynamics among measuring devices are identified as mathematical models, the identified models are realized as filters, the filters are implemented to the measurement system, and then the differences in dynamics among the plural measuring devices are relatively compensated. The effectiveness of the relative compensation was proved in an experimental measurement system for sound transmission loss [2].

These methods are available on the assumption that measurement noise can be ignored. In practice, however, measurement noise cannot be ignored, and analysis of its influence on the resulting values has been desired.

In this paper, the influence of measurement noise is investigated on the results obtained by applying the multiple measuring method. Consequently, the influence is appeared as the signal-to-noise ratio at an observed output and can be removed by the principle of the multiple measuring method. The analytical investigation as well as a numerical experiment is shown when the improved double measuring method is applied to the linear system identification.

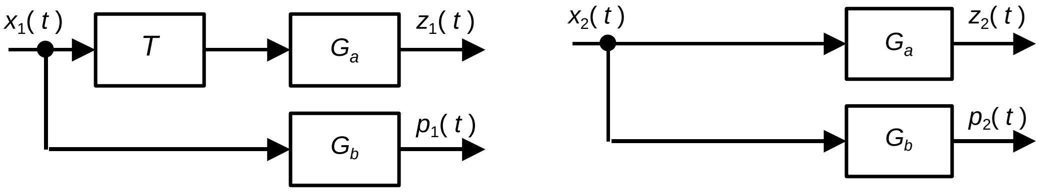
2 IDENTIFICATION BY THE IMPROVED DOUBLE MEASURING METHOD

Let us consider the identification problem of a linear system T by using input and output signal under noise free conditions. As shown in (a) of Figure 1, the input signal and output one of the system T are simultaneously observed as $p_1(t)$ and $z_1(t)$ through the measuring devices G_b and G_a, respectively. If $p_1(t)$ and $z_1(t)$ are adopted as the input and output signal, respectively, then the identified result is not equal to T since each of the signals are affected by the dynamics of the devices G_b or G_a.

On the other hand, the identification by the improved double measuring method can remove the dynamics of these devices from the identified results [1]. This method requires another measurement (b) of Figure 1, and signals $p_2(t)$ and $z_2(t)$ are obtained. Then, the system T is identified by

$$T(s) = \frac{Z_1(s)}{P_1(s)} \frac{P_2(s)}{Z_2(s)}, \qquad (1)$$

where the capital letter $Z_1(s)$ indicates the Laplace transformation of the signal $z_1(t)$, and so on. The identified result (1) does not include the dynamics of the devices G_a and G_b.



(a) Measurement 1 (b) Measurement 2

Figure 1. Improved double measuring method.

3 MEASUREMENT NOISE INFLUENCE ON THE IDENTIFICATION

The identification by the improved double measuring method succeeds under the noise free condition. However, in the case when the measurement noise $n_i(t)$ and $v_i(t)$ (i=1,2) cannot be ignored in Figure 2, only the signals $u_i(t)$, $y_i(t)$ (i=1,2) affected by the noise are observed, where it is assumed that the noise $n_i(t)$ and $v_i(t)$ (i=1,2) are stationary time series with mean 0 and they are uncorrelated to each other. If the signals $u_i(t)$, $y_i(t)$ (i=1,2) are used instead of $p_i(t)$, $z_i(t)$, respectively, for the identification of the improved double measuring method, then the result

$$\tilde{T}(s) = \frac{Y_1(s)U_2(s)}{U_1(s)Y_2(s)} = \frac{Z_1(s)+V_1(s)}{P_1(s)+N_1(s)}\frac{P_2(s)+N_2(s)}{Z_2(s)+V_2(s)} \tag{2}$$

is not expected to coincide with $T(s)$.

Since the noise $n_i(t)$ and $v_i(t)$ (i=1,2) are uncorrelated to each other, the Fourier transformation of the cross-covariance for each of the signals and $u_i(t)$ in measurement i (i=1,2) yields

$$S_{y_iu_i}(w) = S_{z_ip_i}(w)\ ,\quad S_{u_iu_i}(w) = S_{p_ip_i}(w) + S_{n_in_i}(w),\quad i = 1, 2, \tag{3}$$

where $S_{y_iu_i}(w)$ is the cross power spectral density with y_i and u_i, and $S_{u_iu_i}(w)$ is the power spectral density of u_i. On the other hand, the cross spectral densities, the spectral densities and the frequency transfer functions satisfy

$$\frac{S_{z1p1}(w)}{S_{p1p1}(w)} = \frac{G_a(jw)T(jw)}{G_b(jw)},\quad \frac{S_{z2p2}(w)}{S_{p2p2}(w)} = \frac{G_a(jw)}{G_b(jw)}\ . \tag{4}$$

From (3) and (4), the system T is expressed by the cross spectral and spectral densities:

$$T(jw) = \frac{S_{z1p1}(w)}{S_{p1p1}(w)}\frac{S_{p2p2}(w)}{S_{z2p2}(w)} = \frac{S_{y1u1}(w)}{S_{u1u1}(w)}\frac{S_{u2u2}(w)}{S_{y2u2}(w)}\frac{1+\{S_{p1p1}(w)/S_{n1n1}(w)\}^{-1}}{1+\{S_{p2p2}(w)/S_{n2n2}(w)\}^{-1}} \tag{5}$$

where $S_{p_ip_i}(w)/S_{n_in_i}(w)$ denotes the signal-to-noise (SN) ratio at the observed output from the device G_b. Therefore, the system T can be identified from the observable signals u_i, y_i and the SN ratios. That is, the measurement noise influence on the identified result appears in the last multiplied term

$$\frac{1+\{S_{p1p1}(w)/S_{n1n1}(w)\}^{-1}}{1+\{S_{p2p2}(w)/S_{n2n2}(w)\}^{-1}}.$$

Since the SN ratios $S_{p_ip_i}(w)/S_{n_in_i}(w)$ (i=1,2) are determined by the output channel of the device G_b, they are approximately regarded as equivalent. That is, the system T can approximately be expressed by

$$T \cong \frac{S_{y1u1}(w)}{S_{u1u1}(w)}\frac{S_{u2u2}(w)}{S_{y2u2}(w)}\ . \tag{6}$$

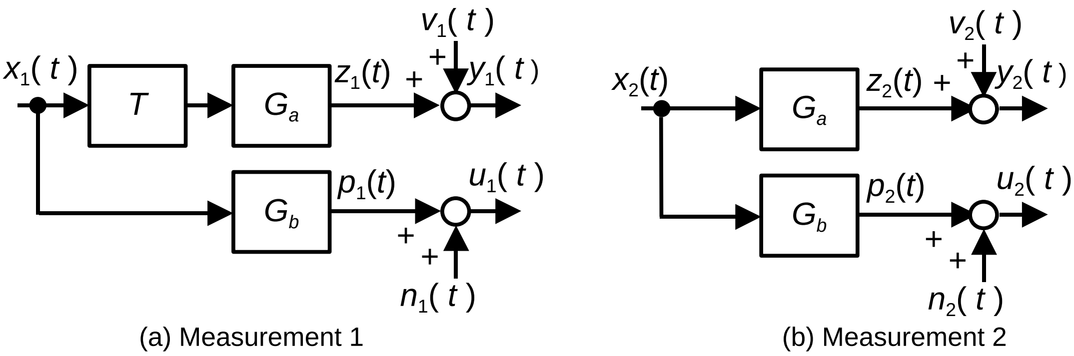


Figure 2. Improved double measuring method with measurement noise.

Moreover, $S_{yiui}(w)/S_{uiui}(w)$ in equation (6) is considered as the frequency transfer function of the system whose input is u_i and whose output is y_i. Thus, the system T is approximately equal to $\tilde{T}$ of (2), which is the identified result by the improved double measuring method. This means that the noise influence on the result identified by the improved double measuring method can be removed by the principle of the method that two measurements 1 and 2 are required for the same measurement devices.

Remark: In measurements 1 and 2, the input signals $x_1(t)$, $x_2(t)$ are not necessarily the same. Especially, when these signals $x_1(t)$, $x_2(t)$ are generated from the same stochastic source or the statistical properties of these signals are the same, the SN ratios $S_{p1p1}(w)/S_{n1n1}(w)$, $S_{p2p2}(w)/S_{n2n2}(w)$ coincide with each other and the equality of equation (6) holds. This means that the noise influence can be completely removed.

4 A NUMERICAL EXAMPLE

The measurement noise influence on the identified result is examined by applying the improved double measuring method to an identification problem in a numerical example. The example shows for discrete time systems since the above discussion is also valid for discrete time systems although the previous section dealt with continuous time systems. In the following example, the systems and measuring devices are same as those in reference [1], and only difference from [1] is that measurement noise exists. The sampling interval for each of the pulse transfer functions is 0.02.

The pulse transfer function of the system T in Figure 2 is given by

$$T(z) = \frac{0.31433 + 0.012327z^{-1} - 0.302003z^{-2}}{1 - 1.913713z^{-1} + 0.938367z^{-2}}, \tag{7}$$

and the measuring devices G_a, G_b are given by

$$G_a(z) = \frac{0.909091 + 0.909091z^{-1}}{1 - 0.818182z^{-1}}, \tag{8}$$

$$G_b(z) = \frac{1.7 + 1.7z^{-1}}{1 - 0.6667z^{-1}}, \tag{9}$$

respectively. In Figure 2, input signals $x_1(t)$, $x_2(t)$ are generated as white Gaussian time series with mean 0, variance 1, and measurement noise $n_i(t)$ and $v_i(t)$ (i=1,2) are generated as white Gaussian time series with mean 0, variances 0.03 and 0.01, respectively.

For these systems, numerical simulations of measurement 1, 2 are achieved and the system T is identified. That is, in measurement i (i =1,2), the frequency response function $S_{yiui}(w)/S_{uiui}(w)$ is identified as an ARX model whose input and output are u_i and y_i, respectively. The identified results are substituted for equation (6), and that yields the model identified by the improved double measuring method. The frequency response of the identified model is plotted in Figure 3 as the solid line. The dashed line indicates that of the true system T, and the dotted line indicates that of the model identified by the conventional method. Here, the conventional method means the identification method by which the influence of the dynamics of measuring devices is ignored. That is, the model identified only by using data u_1 and y_1 of measurement 1 is regarded as the model identified by the conventional method. Since the data are affected by the measuring devices G_a, G_b as well as noise $n_1(t)$, $v_1(t)$, the identified result by the conventional method also includes the difference in dynamics between these devices and the influence of the noise. This fact can be seen in Figure 3, where the dotted line is apart from the others. While Figure 3 shows that the response of the model identified by the improved double measuring method is close to the true one. This implies that the identification method by the improved double measuring method removes the influence of the difference in dynamics between two measuring devices as well as the measurement noise. Therefore, the improved double measuring method is useful even if there exists measurement noise.

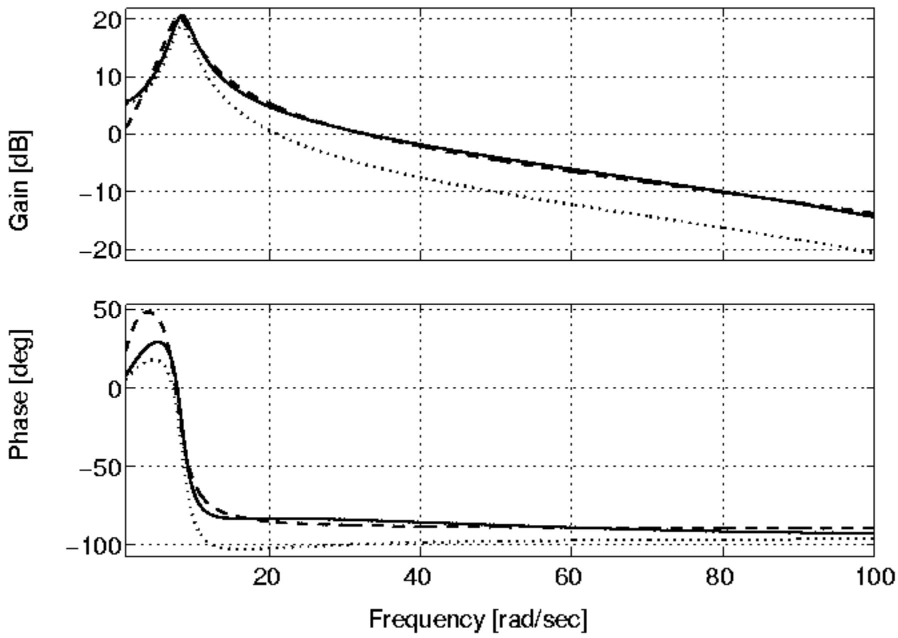


Figure 3. Frequency responses.

5 CONCLUSION

The relative compensation of differences in dynamics among plural measuring devices are significant for precise measurement. The multiple measuring method has been developed and applied for the relative compensation by the authors and their research group[1][2]. In the application of this method to practical systems, the analysis of the measurement noise influence must be made and it has been left as a problem. In this paper, the influence of the measurement noise is analyzed when the method is applied to an identification problem. As a result, the noise influence is appeared as the SN ratios at measuring devices and it can be removed from the compensated results because the method requires two measurements for the same measuring devices. Here, we analyzed only the case when the improved double measuring method is applied to the system identification problem, but similar discussion is available for the application of the double measuring method or the improved double measuring method to calibration problems and control problems.

REFERENCES

[1] T. Ono and J. Sun, On measuring methods independent of measuring device dynamics, *Basic Metrology and Applications*, Editrice Levrotto & Bella, Torino, 1994, pp. 12-17.

[2] T. Wada, A. Ishizaki, T. Ono and Y. Kurobe, An application of relative compensation methods in dynamics of measuring devices: Influence of microphone dynamics on sound transmission loss, *Proceedings of the XIV IMEKO World Congress* (Tampere, 1-6. June 1997), vol. V, Tampere, Finland, 1997, pp. 7-11.

AUTHORS: Assistant Prof. Dr. Teruyo WADA and Prof. Dr. Toshiro ONO, Department of Mechanical Systems Engineering, College of Engineering, Osaka Prefecture University, 1-1 Gakuen-cho, Sakai, Osaka 599-8531, Japan, Phone Int. +81 722 54 9218, Fax Int. +81 722 58 6885, E-mail: wada@ mecha.osakafu-u.ac.jp and ono@ctrlab.mecha.osakafu-u.ac.jp