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TOPOLOGICAL PROPERTIES OF ELECTROMAGNETIC MEASUREMENT

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Abstract: Values of physical quantities are real numbers. These values do not contain information on the physical nature of measured quantity, this information is given by the physical unit. But the physical unit is only a name while the information on the nature of measured quantity is given by physical laws and design of measurement devices (sensors). We propose a topological theory of measurement, which mathematically describes both values of physical quantities and their nature. The model of measurement is fibre bundle space with base space corresponding to topological properties of measured quantity and fibres corresponding to values. This model combined with the field theory gives us a new concept of spherical ammeter and additionally a new field of structure generalising of Maxwell electromagnetic field.

Keywords: theory of measurement, measurement theory, topological theory of measurement, spherical ammeter

1 INTRODUCTION

Measurement is the basic source of knowledge in natural science. Together with the development of new physical ideas, the viewpoint on measurement was rapidly changing. The theory of relativity showed the relativity of time and length measurement and originated quite a new geometrical description of the field theory. Quantum mechanics gave entirely new viewpoint on the measurement on microscopic level that was developed by physicists in the range of quantum theory measurements. Independently of this quantum theory, the representation measurement theory [1] as well as the dimensional analysis were developed. The representational theory is still a recognised theoretical base of metrology [2,3], and measurement theory in metrology remains classic though quantum mechanics has been practically used for metrology: standards of basic values are of quantum origin. We have not found any literature combining the quantum theory of measurement with metrology. This way one can observe extremely specific circumstance: the quantum theory of measurement and metrology have been independently developed for one hundred years.

This situation has its reason in unsolved fundamental problems of measurement. We assume that the basic unsolved problem is the mathematical relation between a measurement device structure and the physical theory experimentally verified using that measurement device.

Therefore a new formula of measurement theory unifying both the classical and quantum theory must account for the following problems:

1. Physical nature of measured quantity is determined by structure of measurement device, which is designed on the basis of physical laws describing measured quantity. In the previous paper [4] we came up with the idea that physical quantities are distinguished by the topological structure of measurement device. Thus measurement theory ought to describe topological properties of both researched reality and measurement device. Topological properties are important factors that characterise the fundamental physical interactions (fields) and allow us to classify them [field theory]. The real numbers which represent the values of physical quantities do not include information of researched phenomenon topology (all results of measurement have topology of ***R*** - real numbers).
2. The most fundamental theory in physics is the field theory. The major research aim of a physicist is to search for the unified theory of all fields. In our opinion it is not possible to complete unification of the field theory before compatibility of the field theory with the measurement theory has been established. Elaboration of measurement theory combined with field theory allows us to define consequently the physical quantities associated both with the structure of measurement device and properties of physical fields.

3. The results of measurement are relative (in epistemological meaning) in the following way: exploring the word we measure the quantity defined by a physical theory, using devices designed basing upon this theory. Simultaneously, these theories are experimentally verified by applying devices designed under this theory (physical theories are "frozen" in measurement devices). We want to obtain a mathematical expression for the relative character of measurement as the source of cognition, using a formal mathematical structure identical to those applied in the physical field theory, namely the concept of fibre bundle spaces.

2 TOPOLOGICAL THEORY OF MEASUREMENT

2.1 Measurement idea

Most often measurement is defined as the process of empirical mapping of object properties (or processes) under study into numbers (or other mathematical values in such way that enables description of these objects and processes [2]. A mathematical model of measurement is a function:

$$M : Q \rightarrow R, \qquad (1)$$

While ***Q*** the set of properties, ***R*** relational numerous system (usually real numbers).

We describe values of all physical quantities as real numbers; quantities differ only in physical units. Such classical measurement theory does not respect specific physical properties of different measurement devices, and the role of measurement device is reduced to comparison of the measured value with its standard value belonging to the object, being the standard for a certain physical value. Hence, we suggest a new topological measurement theory that takes into account geometrical structure of measurement device. Because geometrical properties of modern measurement systems are hidden in electronics, and physical nature of measured quantity is determined by the physical properties of measuring sensor, we will discuss the primary (physically defined) classical measurement devices according to original classical definitions.

Therefore suggestion is based upon the following assumptions:

1) every physical fundamental quantity measured directly (as time, length, mass, electrical current and temperature) is defined by geometrical properties of a measurement device;

2) measurement device is characterised by permissible measurement states which may be observed as geometrical changes;

3) measurement device states are numbered in such a way that each state corresponds to the element of mathematical space which structure is dependent on the topology of measurement device.

Measurement consists in mapping the states of a researched object (properties of object or process) with the measurement device at its actual physical state.

$$f : Q \rightarrow Z \qquad (2)$$

Where: ***Q*** is the set of the researched object properties, and ***Z*** - the space (topological space) of the states of measurement device (physical states of sensor).

In calibration process of the device we label the states of measurement device with numerous values by mapping ***s***:

$$s : Z \rightarrow R \qquad (3)$$

where: ***R*** – numerous relational system (body of real numbers or other geometrical structures as circle or sphere and so on).

Let us consider two examples: measurement of length, and of electric current intensity.

2.2 Length measurement

Measurement device is a ruler with two marked points on it as a certain length (standard). Topological structure of length measurement device is the set of two points called S^0 (zero dimensional circle). The permissible states of measurement device are the possible distances between two points. If we enumerated all possible lengths in the body of real numbers, then ***R*** would be obtained.

In this case there is homomorphism between the measurement device states and the set of measurement results (values of scale)

2.3 Electric current intensity measurement

The measurement device comprises any closed circuit that changes its geometry due to electric current flow. In moving coil permanent magnet type ammeter its coil is moving in magnetic field.

In moving coil dynamical type ammeter one coil moves due to interaction with other coil. Both of them are the elements of the same circuit (row connection). If we want to determine geometrical structure of electric current intensity measurement, it is necessary to design a system based on the following features:

1) An ammeter is a conductor (usually a metallic one) connected in series of the researched circuit in such way that it remains closed. Each closed circuit is topologically equivalent to one-dimensional circuit $\boldsymbol{S^1}$.

2) The elements of the circuit influence each other changing the circuit geometry due to electric current flow. All possible geometrical states of such circuit are the permissible states of an ammeter. A standard ammeter is designed in such a way that one of the circuit elements – usually coil shaped – is able to move around one axis of its rotation.

Mathematical expression of this idea may be written using the language of space which is called fibre bundles. A fibre bundle above certain space $\boldsymbol{X}$ together with the bundle $\boldsymbol{R}$ is a Cartesian product of generalisation $\boldsymbol{X} \times \boldsymbol{R}$ [5].

2.4 Mathematical structure of measuring space and time

Let us start with the following examples describing the association between some activity of measurement and geometric objects hidden in the nature of this activity. Let us consider the simplest one, measurement of distance in metric structured space (X,ρ). Any metric determines the section of a one dimensional trivial vector bundle over the space $X \times X$ (Cartesian product). This section

$$s : X \times X \to X \times X \times R \tag{4}$$

is defined by the formula:

$$s(x_1, x_2) = ((x_1, x_2), \rho(x_1, x_2))$$

It is clear that the act of distance measurement is equivalent to the following two steps. The first one is based on the choice of a map:

$$f : S^0 = \{-1,1\} \to X \times X,$$

where $f(1)=(x_1,x_1)$ is the starting point and $f(-1)=(x_1,x_2)$ is the ending point of measured distance.

The second step includes the pull-back section [5] in reading the value f^*s at the ending point **2**

$$\begin{array}{ccc} S^0 \times R & \rightarrow & X \times X \times R \\ p_1 \uparrow \uparrow f^*s & & p_{12} \downarrow \uparrow s \\ S^0 & \rightarrow & X \times X \end{array} \tag{5}$$

where π_1 is projection on the first coordinate.

The distance $\rho(x_1,x_2)$ is the value of $f^*s(2)$. Note that such structure gives us an opportunity to determine a distance between points in any "locally" metric space as the greatest lower bound of the lengths of all broken lines that connect two points. The bundle characteristics for the act (understanding as the map f) of length measurement is denoted by x_ρ:

$$x_p = \begin{pmatrix} S^0 \times R \\ p_1 \downarrow \quad \uparrow f^* \rho \\ S^0 \end{pmatrix}, \tag{6}$$

A very similar structure may be accomplished for the act of time measurement. The only difference is that the nature of time determining the sense makes it possible to define the function $\tau : T \times T \to R$, which is antisymmetric, i.e. $\tau(t_1, t_2) = -\tau(t_2, t_1)$. This implies that orientation of fibres is an additional structure of the characteristic bundle x_τ:

$$x_t = \begin{pmatrix} S^0 \times R \\ p_1 \downarrow \quad \uparrow f^* t \\ S^0 \end{pmatrix} \tag{7}$$

Now let us consider the example of a theory based on the concept of time-space. In such theory any act of measurement should be decomposed to the pair ($f^*\tau$, f^*s), and moreover, there should exist a continuous deformation f^*s_l joining f^*s and $f^*\tau$ (l - parameter of deformation). The above postulates are satisfied when we consider a well known in algebraic topology (compare with Milnor construction) so called join $S^0 * S^0 = S^1$ of bundle bases $B(x_\rho)$ and $B(x_\tau)$. In general, the space $X_1 * X_2$ is defined as a Cartesian product of factors and unit interval I divided by the relation **~** :

$$(X_1 \times X_2 \times I) / \sim$$

the relation **~** identify points of the third coordinate $l=1$ and points of the third X_1 coordinate : $l=0$ with X_2:

$(x_1, t_1, 1) \sim (x_1, t_2, 1)$ and $(x_1, t_1, 0) \sim (x_2, t_1, 1)$

For any two maps f_1 and f_2 the join:

$$f_1 * f_2 : X_1 * X_2 \to Y_1 * Y_2$$

is well defined by a natural relation:

$$[(f_1 * f_2)(x_1, x_2, t)] = [f_1(x_1), f_2(x_2), t]$$

The total space of the bundle over the join $X_1 * X_2$ we define as the space $(X_1 \times R \times X_2 \times R \times I)/\sim_x$

where the relation $\sim_x$ is given by the formula:

$$(x_1, n_1, w_1, l_1) \sim_x (x_2, n_2, w_2, l_2) \cong$$
$$\cong ((x_1, t_1, l_1) \sim (x_2, t_2, l_2) \text{ and } (n_1 l_1, w_1(1 - l_1)) = (n_2 l_2, w_2(1 - l_2)))$$

The above gives a locally trivial subbundle of the two-dimensional trivial bundle over $X_1 * X_2$

The following diagram describes the result section of the measurement action:

$$\begin{array}{ccc} (S^0 * S^0) \times R & \longrightarrow & (X * T) \times R^2 \\ p_1 \uparrow \ \uparrow f^* s & & p_1 \downarrow \ \uparrow s \\ S^0 * S^0 & \xrightarrow{f_1 * f_2} & X * T \end{array} \tag{7}$$

2.5 Mathematical structure of electric current intensity measurement

As it was mentioned above, an ammeter is built of the closed circuit so its geometry is topologically equivalent to one dimensional circle S^1. The space of all possible measurement results has the structure of fibre bundle over a circle. The result of given measurement is a section of this bundle. According to theorems of algebraic topology [5], a bundle over a circle splits into direct sum of trivial bundles ($R \times S^1$) and a Möbius bundle. Viewing on the field theory, the measurement result obtained using ammeter gives us information on components of electric field. In our theory an electrical component is a section of a trivial bundle component, and magnetic field is a section of Möbius component. Such topological analysis of measurement process allows us to understand why the field researched by electric measurement devices has its magnetic and electric components.

3 CONCLUSIONS

A standard ammeter comprises a coil placed on its rotation axis. Topological analysis of the circuit freedom degrees shows that it is possible to design a spherical ammeter with two degrees of freedom fitting the above mentioned expectations (the patent is being under way). Field researched by such a device has its structure richer than Maxwell electromagnetic field.

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