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USE OF ANALYTIC REDUNDANCY IN FAULT-TOLERANT SENSOR SYSTEMS

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Abstract: The paper presents a systematic design of fault-tolerant measurement systems for linear time-invariant processes. The discussed methods utilize analytic redundancy of the underlying process. Two multivariable linear model types (PCA for static and ARX for dynamic modeling) are considered. Both model types can be directly identified from the measured process data. Analysis of the primary and structured residuals enables very sensitive detection, isolation, and identification of single and multiple sensor and actuator faults. The fault diagnosis is conducted via analysis of the identified faults and can be followed by appropriate fault accommodation actions. A faulty sensor variable can be reconstructed using fault-free process variables and appropriate models. Faults which degrade sensor performance in a recoverable way (c.f. bias and gain errors) can be compensated using model-based sensor recalibration. Best data source in groups of doubled sensors can be selected through application of the voting scheme (where the third sensor is the soft sensor), thus enabling optimal sensor reconfiguration.

Keywords: fault-tolerant systems, fault accommodation, multivariable models

1 INTRODUCTION

Industrial processes often contain a considerable number of sensors, but even a single sensor fault can spoil the measurement outcome and disturb safe operation of the process. To increase reliability of a sensor system it can be advantageous to diagnose sensor faults by means of a model-based monitoring system. The models applied provide rather accurate consistency relations than the true system description; these relations are multivariable and express analytical redundancy of the process variables.

Sensor faults result in discrepancies between true and measured values of the process variables. Hence, such faults can be modeled as additive disturbances acting at the plant outputs y(k), see

Figure **1**. Similarly, actuator faults, causing discrepancies between intended control signals and their realization by actuators, can be modeled as additive disturbances acting at the plant inputs u(k). For this reason sensor and actuator faults are often called additive faults; their diagnosis is a feasible task [1].

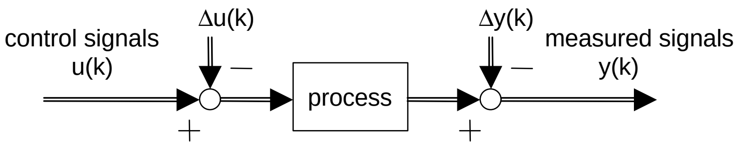


Figure 1. Sensor and actuator faults

Additionally, the fault diagnosis can be followed by various model-based fault accommodation actions (e.g., an optimal reconstruction of the missing sensor data or sensor re-calibration).

2 MODELS

In the paper we consider two basic linear discrete-time model types which can be obtained using experimental modeling: PCA (static models) and ARX (dynamic models, state-space equivalent).

2.1 Principal Component Analysis

A PCA model describes multivariable linear static relations between the measurable process variables. Let the process variables (zero-mean unit variance) be stored in the matrix $X \in \Re^{K \times N}, K >> N,$ such that its columns correspond to *N* process variables (*nu* actuators and *ny* sensors, *N* = *ny* + *nu*) and rows $x(k) \in \Re^N$ to *K* time samples

$$X := [x(0) \quad x(1) \quad \cdots \quad x(K-1)]^T . \tag{1}$$

In no-noise case the samples *x(k)* are assumed to be located on an *L* dimensional model hyperspace. (Note that *L<N.)* The model hyperspace is spanned by a set of *L* orthonormal vectors p_i, constituting an orthogonal matrix *P*. The model hyperspace is orthogonal to a set of *N-L* orthonormal vectors q_i, constituting an orthogonal matrix Θ, such that $\Theta^T \Theta = I, \Theta^T P = 0$. For any vector *x*(k) from the hyperspace spanned by the column vectors of *P* (representing a fault-free behavior) it holds

$$\Theta^T x(k) = \underline{0} . \tag{2}$$

This set of *N-L* linear equations defines the model. Using it, the original noisy data (1) can be expressed as a sum of two components $X = \hat{X} + \tilde{X}$ where $\hat{X} = PP^T X$ represents the data orthogonally projected on the model hyperspace, and $\tilde{X} = \Theta\Theta^T X$ the residuals. PCA is particularly well suited for process monitoring [7], there exist also extensions for dynamic [4] and nonlinear systems [6].

2.2 The multivariable ARX model

The multivariable ARX model is capable of describing linear dynamic relations between the system inputs $u(k) \in \Re^{nu}$ and outputs $y(k) \in \Re^{ny}$ $(nu + ny = N)$. The model is defined as

$$\begin{aligned} &A(q)y(k) = B(q)u(k) + e(k), \quad \text{where} \\ &B(q) = B_1 q^{-1} + \cdots + B_{nb} q^{-nb}, \quad A(q) = I + A_1 q^{-1} + \cdots + A_{na} q^{-na} \end{aligned} \tag{3}$$

Here *A(q)* and *B(q)* are polynomial matrices of *ny x ny* and *ny x nu* dimensions and e(k) represents modeling imperfections. The above equation can also be written in a form similar to (2) as

$$\Theta(q)^T x(k) = e(k) \tag{4}$$

where $\Theta(q)^T = [A(q) \quad -B(q)]$, $x(k) = [y(k)^T \quad u(k)^T]^T$ (5),(6)

3 FAULT DETECTION AND ISOLATION

Figure 2 shows the principle of model-based process monitoring. The residual generator utilizes an implicit process model and produces residuals, which are the measure of the data inconsistency.

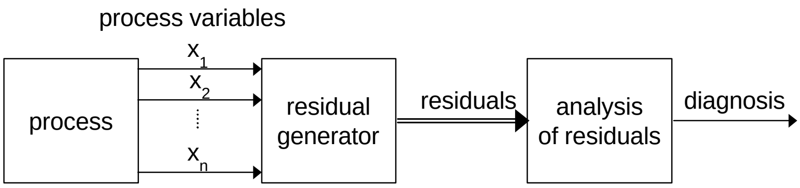


Figure 2. Model-based process diagnosis using a residual generator

Analysis of the residuals enables fault diagnosis, and comprises the following diagnostic tasks: (i) fault detection: indication of a fault disturbing the process, (ii) fault isolation: determination of the fault location, (iii) fault identification: determination of the fault magnitude, (iv) actual fault diagnosis: determination of the physical cause of the fault. In the sequel we assume that process faults do not appear and that the system under consideration is influenced at most by a single sensor or actuator fault, i.e., a fault influencing single entries in the data vector *x*(*k*), (1), (6). An extension for multiple additive faults is straightforward, and discussed at the end of the Section.

3.1 Fault detection

Fault detection means *indication of a fault disturbing the process.* It is performed by testing the residuals against appropriately selected thresholds or by applying statistical tests [1].

Fault detection for the PCA model

According to (2), the (primary) PC residual is computed from the observation x(k) as $e(k) = \Theta^T x(k)$. In the ideal case (no-noise, perfect modeling) $e(k) = \Theta^T \Delta x(k)$, i.e., the residual depends only on the faults *Dx(k).* Fault detection may then rely on testing the magnitude of residuals, i.e.,

$$r(k) = \sqrt{x(k)^T \Theta\Theta^T x(k)} = \| \Theta^T x(k) \|_2 \qquad (7)$$

Fault detection for the ARX model

The (primary) residuals of the ARX model can be written in computational form as

$$e(k) = A(q)y(k) - B(q)u(k) = \Theta(q)^T x(k)\ . \qquad (8)$$

The residual is a dynamic function of the data. Assuming perfect modeling and a noise-free situation, the residuals can also be expressed (in the so called internal form, containing the faults) entirely in terms of sensor faults $\Delta y(k)$ and actuator faults $\Delta u(k)$ (see Figure 1) as

$$e(k) = A(q)\Delta y(k) - B(q)\Delta u(k) = \Theta(q)^T \Delta x(k) \qquad (9)$$

If the spectrum of the disturbance $\Delta x(k)$ is known, filtering of the original residuals may help to make them sensitive with respect to the faults and insensitive with respect to the disturbances.

3.2 Fault isolation

Fault isolation follows fault detection. Its goal is the *determination of the fault location* (particular sensor, actuator, or one of internal process components). Fault isolation is performed in the following steps: (i) computation of the structured residuals, and (ii) evaluation against the fault signatures. The structured residuals are designed in such a way that each residual is sensitive to a subset of faults. These residuals are obtained from the primary residuals in an algebraic way by means of a linear transformation, satisfying the so-called *decoupling conditions* (11). The transformation procedure eliminates given variables from the original equation set. Each fault causes a characteristic response of the structured residuals, called fault signature. For single sensor and actuator faults it is then natural to make the *i*-th structured residual (*i=1..N*) dependent on all but *i*-th process variable. Table 1 shows signatures of faults in single variables (one column is the signature of the corresponding fault) for this choice of the structured residuals.

Table 1. Signatures of faults in single components of *x*(*k*)

	$\Delta x_1(k)$	$\Delta x_2(k)$	$\cdots$	$\Delta x_N(k)$
$r_1(k)$	0	$\neq 0$	$\cdots$	$\neq 0$
$r_2(k)$	$\neq 0$	0	$\cdots$	$\neq 0$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
$r_N(k)$	$\neq 0$	$\neq 0$	$\cdots$	0

Fault isolation for the PCA model

The general form of the structured residuals for PCA model can be obtained by pre-multiplying the primary residuals (2) by a vector V_i in $\Re^{N-L}$ [2], such that

$$e_i(k) := V_i \Theta^T x(k)\ . \qquad (10)$$

The transformation vector V_i must satisfy the following decoupling condition

$$V_i (\Theta^T)_{i,\bullet} = \underline{0} \qquad (11)$$

where $(\Theta^T)_{i,\bullet}$ denotes *i*-th column of Θ^T. The condition ensures that the residuals do not depend on the *i*-th process variable composing the data vector *x(k)*. A particularly simple choice of vector V_i is equivalent to define the structured residuals as in [3]:

$$e_i(k) := [I - P(P^T \Pi_i P)^{-1} P^T]\Pi_i x(k) \qquad (12)$$

where Π_i is a matrix of dimension *N´N* which differs from the identity matrix only in having zero instead of one at the *i*-th diagonal element.

Fault isolation for the ARX model

Isolation of single sensor faults is analogous as in the PCA case. The primary residuals $e(k) = \Theta(q)^T x(k)$ must be transformed via linear filter (vector of polynomials in q operator) $V_i(q)$

$$e_i(k) := V_i(q)\Theta(q)^T x(k) \tag{13}$$

to become insensitive with respect to single entries of vector $x(k)$; the transforming filter $V_i(q)$ must satisfy decoupling condition (14). An algorithm to determine $V_i(q)$ can be found in [1].

$$V_i(q)(\Theta(q)^T)_{i,\bullet} = 0 \tag{14}$$

Isolation of multiple faults

Multiple additive faults (faults in m variables $i_1..i_m$,) can be isolated by means of structured residuals which are not sensitive to these variables, e.g., in the PCA method, the condition

$$V_i\left((\Theta^T)_{i_1,\bullet} \cdots (\Theta^T)_{i_m,\bullet}\right) = [\underline{0} \quad \cdots \quad \underline{0}] \tag{15}$$

ensures that the variables $i_1..i_m$ are eliminated in (10). Formula (12) can be also adapted for the isolation of multiple faults: Π_i must differ from the identity matrix only in having zeros instead of ones at the i_1-i_m-th diagonal elements. Similarly, the decoupling condition for the ARX model becomes

$$V_i(q)\left((\Theta(q)^T)_{i_1,\bullet} \cdots (\Theta(q)^T)_{i_m,\bullet}\right) = \underline{0}\,. \tag{16}$$

4 FAULT RECONSTRUCTION, DIAGNOSIS AND ACCOMODATION

Sensor fault diagnosis and supervision are possible *first* after detection and isolation of the fault.

4.1 Fault reconstruction

Fault reconstruction means *determination of the fault magnitude* by means of the following steps: (i) reconstruction of the faulty variable using the fault-free variables and partial models, such models can be computed from one joint model (as in this paper) or directly estimated from the fault-free data (see [1] and [4]), and (ii) reconstruction of additive faults by subtracting the reconstructed variable values from the known (faulty) variable values.

Reconstruction of a single sensor or actuator fault for the PCA model

Fault identification can be performed via data reconstruction, the least-squares solution is

$$\hat{x}_i = -(\Theta_i\Theta_i^T)^{-1}\Theta_i\Theta_{-i}^T \cdot x_{-i}\,. \tag{17}$$

$\hat{x}_i$ is a vector containing the reconstructed variable(s). x_{-i} is a vector containing all measured variables except the variable(s) i ($i_1..i_m$) to be reconstructed. Θ_i contains only the column(s) i ($i_1..i_m$) of Θ and Θ_{-i} contains the remaining columns. The reconstructed additive fault becomes

$$\Delta x_i = x_i - \hat{x}_i = x_i + (\Theta_i\Theta_i^T)^{-1}\Theta_i\Theta_{-i}^T \cdot x_{-i} \tag{18}$$

where x_i is (are) the faulty variable(s).

Reconstruction of a single sensor fault for the ARX model

The ARX model can be used to reconstruct the full output vector $y(k)$ from the input vector $u(k)$

$$\hat{y}(k) = A(q)^{-1}B(q)u(k)\,. \tag{19}$$

However, it can be easily shown that reconstruction of a single output variable $y(k)_i$ from the input vector $u(k)$ and the remaining other output variables can be performed in a more accurate way as

$$\hat{y}_i(k) = -\left([\mathrm{diag}(A(q))]^{-1}[A(q) - \mathrm{diag}(A(q))]y(k) + [\mathrm{diag}(A(q))]^{-1}B(q)u(k)\right)_{\bullet,i} \tag{20}$$

where $(\cdot)_{\bullet,i}$ denotes i-th row of a matrix and diag($A(q)$) the polynomial matrix with the main diagonal identical to the diagonal of $A(q)$ and zeros outside. The reconstructed fault becomes

$$\Delta y_i(k) = y_i(k) - \hat{y}_i(k)\,. \tag{21}$$

4.2 Sensor fault diagnosis

Sensor fault diagnosis means determination of physical sensor fault by analyzing the reconstructed additive fault signal (denoted Δy_i for ARX and Δx_i for PCA model), as summarized in Table 2.

Table 2. Fault diagnosis depends on the reconstructed additive fault signal

reconstructed sensor fault Dy_i	**diagnosis**
is close to zero	sensor is OK
has a non-zero mean	offset error
has an increased variance	additive noise error
is highly correlated with $\hat{y}_i$	gain error
is opposite to $\hat{y}_i$	zero gain error
its sign depends on signal derivative	Hysteresis error

4.3 Accommodation of sensor faults

Accommodation of sensor faults may incorporate soft-sensing and sensor re-configuration (after an isolation of the sensor fault), as well as sensor re-calibration (after a diagnosis of the sensor fault).

Soft sensing

By soft sensing we mean model-based reconstruction of a sensor variable using the other (fault-free) process variables after an *isolated* severe sensor failure, like the zero gain error. Under the assumption that all the other variables are fault-free, such a reconstruction is given for the PCA model by (17) and for the ARX model by (20).

Sensor re-configuration

By sensor re-configuration we mean selection of the best data source in the situation when some sensors are doubled. If one of the sensors fails the voting scheme can be utilized to isolate the faulty sensor since the additional third sensor is the soft sensor (i.e., the model-based reconstruction of the measured value using the other, fault-free process variables).

Sensor re-calibration

Sensor re-calibration means soft compensation of some sensor faults, which deteriorate the sensor performance in recoverable way. The correction means, summarized in Table 3, are possible for the diagnosed *offset error, gain error, and additive noise error*.

Table 3. Sensor re-calibration depending on the fault diagnosis

reconstructed sensor fault Dy_i	**correction means**
has a non-zero mean (offset error)	the estimated sensor offset is the mean value (22) of the additive fault Δy_i, the compensated sensor value $\breve{y}_i(k)$ is given by (23)
has an increased variance (additive noise error)	the compensated sensor value $\breve{y}_i(k)$ is appropriately filtered measured value $y_i(k)$
is highly correlated with $\hat{y}_i$ (gain error)	the estimated sensor gain can be computed using the LS method (24); the compensated sensor value $\breve{y}_i(k)$ is given by (25)

$$\Delta\bar{y}_i = \frac{1}{K}\sum_{k=0}^{K-1}\Delta y_i(k), \qquad \breve{y}_i(k) = y_i(k) - \Delta\bar{y}_i \qquad (22),(23)$$

$$\hat{G}_i = \sum_{k=0}^{K-1}\hat{y}_i(k)y_i(k) \Big/ \sum_{k=0}^{K-1}\hat{y}_i(k)^2, \qquad \breve{y}_i(k) = y_i(k)\big/\hat{G}_i \qquad (24),(25)$$

5 EXAMPLE

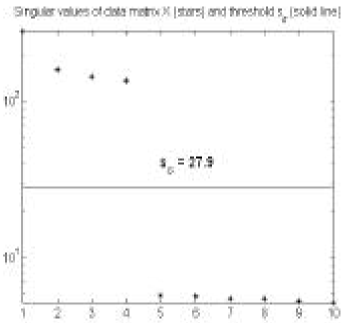
The following example describes an application of the PCA method for diagnosis of a single sensor fault in a simulated static linear multivariable process with L=4 degrees of freedom.

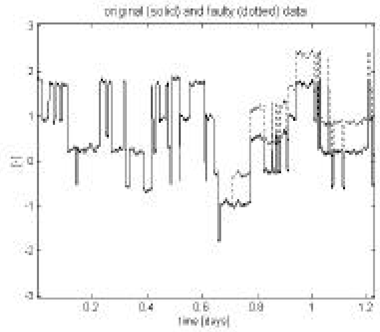
Process data and model building

The simulated process variables $x_1..x_{10}$ represent measured physical quantities and compose the process data vector x(k). Around 14500 process samples are used to build the data matrix X. Singular values of this matrix are shown in Figure 3. The process model contains N-L=6 equations.

Simulated sensor fault

Figure 4 depicts the original process variable x_5 (solid line) and the same variable disturbed by a simulated additive fault (sensor offset error) for t > 0.72. The difference between the original and corrupted data is not large and may not be properly detected by testing x_5 against fixed thresholds.





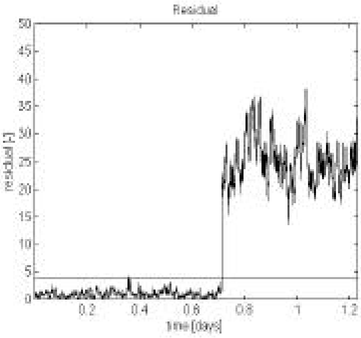
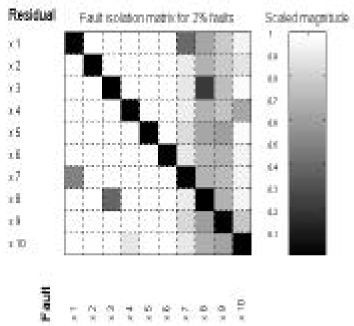
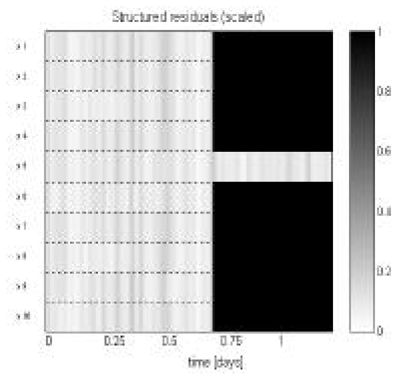


Figure 3. Singular values of the data matrix X.
Figure 4. The original (solid) and disturbed (dotted) variable x_5.
Figure 5. Fault detection via testing magnitude of the residuals (7).

Fault detection, isolation, diagnosis, and compensation

The additive fault can be clearly detected by testing the residuals (7), as shown in Figure 5. Since the fault isolation matrix, shown in Figure 6, has almost a diagonal structure, isolation of the fault is feasible (cf. Table 1). The isolation is conducted by testing the magnitude of the structured residuals (12), shown in Figure 7. The fault is clearly related to the variable x_5 since magnitudes of all structured residuals are high and only the one without the variable x_5 remains low. The additive fault influencing the variable x_5 can be reconstructed by applying the least-squares estimate (18). The reconstruction is shown in Figure 8. The reconstructed fault is close to the original simulated "true" fault. Such a fault would be correctly diagnosed as an offset error, as stated in Table 2. According to Table 3, the fault for t > 0.72 could be compensated via (23) by subtracting its mean value (22) from the measured value.





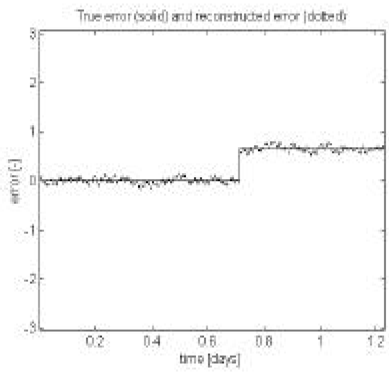


Figure 6. Fault isolation matrix has almost a diagonal structure–the fault isolation is feasible.
Figure 7. Structured residuals (12) enable isolation of the fault related to the variable x_5.
Figure 8. Original (solid) and identified (dotted) additive fault, influencing the variable x_5.

6 SUMMARY

The paper presents design of fault-tolerant sensor systems. The most promising methods, described here in detail, are the model-based methods. These methods make use of the so-called analytical redundancy of the underlying process, which is represented by a model. Two basic model types are considered: PCA for static systems and ARX for dynamic systems. These models can be obtained directly from measured data by means of experimental modeling. The so-called residuals, quantifying discrepancy between the data and the model are then utilized as a consistency measure of the measured data. Analysis of the residuals enables very sensitive detection, isolation and identification of sensor and actuator faults. The fault diagnosis follows then from analysis of the reconstructed faults. The fault diagnosis and process model are necessary for the fault accommodation: soft sensing, sensor recalibration, and reconfiguration.

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