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OPTIMIZED LOOK-UP TABLE FOR NON-LINEAR RECONSTRUCTION OF MEASURAND

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Abstract: A new procedure for determining the size and the values of a look-up table for reconstructing the measurand values in a measurement system employing a non-linear sensor is presented. This procedure uses the A/D converter quantization error to calculate the table entries. The proposed methodology is illustrated applying it to a thermistor based thermometer.
Keywords: measurement theory, measurand reconstruction, look-up table, signal processing, linear interpolation, piecewise linear approximation

1 INTRODUCTION

In many measurement systems, sensors with non-linear transfer characteristics are employed. Considering a digital measurement, the sensor output signal must be processed by an analog conditioning circuit and converted to a digital format using an A/D converter. Next, it must be digitally reconstructed to take into account the non-linear transfer characteristic of the sensor.

The measurand reconstruction function must be an inverse or an approximate inverse of the sensor transfer function. Although, for embedded system, employing microcontrollers or ASIC's, the implementation of such functions can be cumbersome considering that one might use fixed-point arithmetic with relatively slow calculation speed. The use of look-up tables (LUT) with linear interpolation (also called polygonal interpolation) is an attractive approach for this problem compared to other types of interpolation like splines and polynomial interpolation [1], which require more computational power.

The use of LUT's for non-linear function approximation is well known and well referenced in the literature, although, generally, only the problem of approximation error minimization is resolved without addressing the problem of minimizing the number of table entries, as in [2]. The canonical piecewise linear (CPWL) method can be used to calculate the coefficients of a piecewise linear function that approximates the reconstruction function, minimizing the mean squared approximation error, when the breakpoints are given [3]. The values of the coefficients and breakpoints can be used to calculate the number of LUT cells and their values. The breakpoint locations can also be optimized in order to minimize the approximation error, using an optimization method just like the Newton-Gauss Algorithm [4]. However, a method for calculating the optimal number of points in order to achieve a desired reconstruction resolution (or precision) does not exists.

In the following, a method to calculate the values and number of entries of a LUT necessary for reconstructing the measurand from a non-linear sensor using linear interpolation in order to achieve a specified measurement resolution. The procedure uses the system effective resolution due to the A/D conversion and non-linear sensor output signal [5], to calculate the LUT. The necessary number of bits for storing the LTU values is also analyzed. Finally, a temperature measurement system using a thermistor is analyzed to illustrate the proposed procedure.

2 PROBLEM DEFINITION

Let us consider a measurement system, shown in Figure 1, using a sensor with non-linear transfer characteristics. The conditioning circuit has the main function of adjusting the sensor output to lies within the A/D converter input range, over the full measurement range. The non-linear reconstruction function can be approximated by a piecewise linear function.

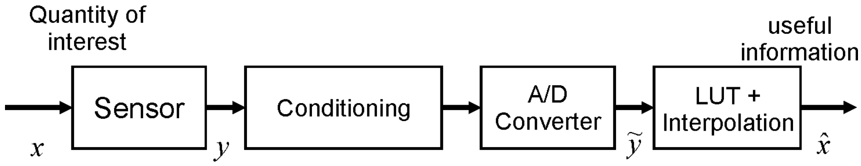

Figure 1. Measurement system using LUT with interpolation

In Figure 1, the sensor output signal y as function of the quantity of interest x is $y = f(x)$, f: $S \rightarrow \Re$, $S \subset \Re$ corresponds to the full measurement range. For an N-bit A/D converter, the digital signal can be represented as: $\tilde{y} = y + \varepsilon_{AD}$, where ε_{AD} is the quantization error defined as: $-q/2 \leq \varepsilon_{AD} \leq q/2$, with q equivalent to the value of 1 LSB for an ideal A/D converter.

The reconstruction function must be an inverse or approximate inverse of the sensor's transfer function and can be defined as: $g(y) = \arg[y = f(x)]$ where the measurand can be estimated as: $\hat{x} = g(\tilde{y})$. One might define then $\tilde{g}(.) \approx g(.)$ as a piecewise linear function that approximates the reconstruction function. The approximate function breakpoints $(Py_i, Px_i), i = 1, \ldots, M$ must be stored in a LUT (M breakpoints) and the function can be generated for each measured value using the immediately upper and lower break points in relation to the measured value. We also define $\hat{\varepsilon}_x(\tilde{y})$ as the maximum acceptable measurement error for each quantized value of the variable y that must be an association of the quantization error and a weighting factor.

The measurand reconstruction problem consists of the following steps: a) Calculate the values and the number of the LUT entries; b) Define the size (in number of bits) of the LUT cells; in order to guarantee the measurement error to be always less than or equal to maximum acceptable measurement error, or: $\varepsilon_x(\tilde{y}) \leq \hat{\varepsilon}_x(\tilde{y})$.

3 CALCULATION OF THE LOOK-UP TABLE

In order to calculate the number of entries in the table one needs to calculate the initial measurement error due to quantization, for the values of $\tilde{y}$. This error is inherent in the measurement system for the best case of reconstruction. Using a resolution of reconstruction much higher than needed does not ameliorate the final resolution of the measurand. The initial measurement error, ε_{x0}, can be calculated as the largest measurement error using the quantized values of the variable y in the ideal reconstruction function, considering the maximum quantization error:

$$\varepsilon_{x0}(\tilde{y}) = \max\left[\left|g(\tilde{y}) - g(\tilde{y} + \varepsilon_y)\right|, \left|g(\tilde{y}) - g(\tilde{y} - \varepsilon_y)\right|\right]. \quad (1)$$

where ε_y is the worst case quantization error, equal to $q/2$.

The maximum acceptable measurement error must be higher than or equal to the initial measurement error: $\hat{\varepsilon}_x(\tilde{y}) \geq \varepsilon_{x0}(\tilde{y})$, and it can be arbitrarily attributed or defined as a function like:

$$\hat{\varepsilon}_x(\tilde{y}) = w(\tilde{y}).\varepsilon_{x0}(\tilde{y}),\ w(\tilde{y}) \geq 1, \forall \tilde{y} \in S\ , \quad (2)$$

where w is arbitrary and can be considered as a function of loss of precision (resolution) in the reconstruction. For values of w = 1, one has no loss of resolution in the reconstruction. Further, the loss of resolution in number of bits, due to the reconstruction can be calculated as function of the quantized variable as:

$$L_R(\tilde{y}) = -\log_2 w(\tilde{y}). \quad (3)$$

From the maximum acceptable measurement error one can define the upper and lower acceptable limits of x respectively as: $x_s(\tilde{y}) = g(\tilde{y}) + \hat{\varepsilon}_x(\tilde{y})$ and $x_i(\tilde{y}) = g(\tilde{y}) - \hat{\varepsilon}_x(\tilde{y})$. Then, one can implement a search algorithm to find the break points in such a way that the straight line defined by two consecutive points lies always in the band between these two curves.

3.1 Search algorithm

The search algorithm can be implemented in many different ways. For simplification and for maximizing the measurement resolution the search algorithm can be implemented considering a function having only positive values, i.e.: $\min(x)$ = 0 and $\min(y)$ = 0, to avoid the use of sign in the reconstruction block. This can be performed by applying a level shift to the function if necessary, that can be subtracted after the LUT block, to guarantee always positive values. The break points are defined by (Px, Py) where Px stands for the quantized values of the variable x that correspond to the values of Py, and Py are integer numbers that can assume values from 0 to 2^N-1, equivalent to the A/D converter output considering the analog signal from the sensor to be conditioned to lie within the A/D input range over the full measurement range. The number of bits required for the LUT cells is addressed in the next section. The search algorithm must perform at least the following steps according to (4):

1. Index $i = 0$
2. Define the initial break point as $Py_0 = 0$, $Px_0 = g(0)$;
3. While $Py_i < 2^N - 1$ do 4 and 5
4. $i = i + 1$;
5. Search the next break point $Px_i = g(Py_i)$, for the greatest value of Py_i where eq. (4) holds;
6. Define the last break point as $Py_i = 2^N - 1$ and $Px_i = g(2^N - 1)$.

$$\left(r(l) < x_s(l)\right) \wedge \left(r(l) > x_i(l)\right), \forall Py_{i-1} \le l \le Py_i, \quad r(l) = \frac{Px_i - Px_{i-1}}{Py_i - Py_{i-1}}\left(l - Py_{i-1}\right) + Px_{i-1}, \tag{4}$$

where ^ is the AND operator.

An example of the algorithm application is shown in Figure 2, for a reconstruction function defined as: $g(\tilde{y}) = \tilde{y}^2/15, \tilde{y} = 0 \ldots 15$. which is calculated using double precision considering a constant measurement error of: $\hat{\varepsilon}_x(\tilde{y}) = 0.6$.

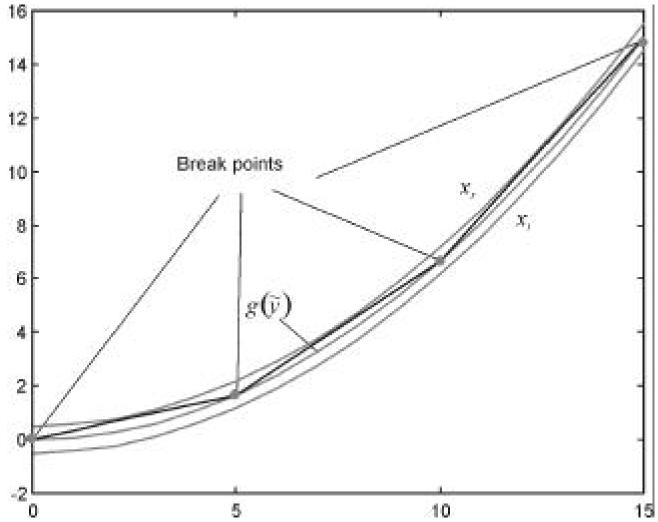

Figure 2. Application of the algorithm for piecewise linear approximation of a square function.

3.2 Resolution of the table entries

The breakpoint values in the variable x must be stored in the LUT with a specific resolution, using fixed point arithmetic. For maximizing the measurement resolution these values must be stored considering the lower end of the measurement range as zero. Thus, the variable x can be redefined as function of the initial value of the measurand as:

$$x = x' - min(x'), \tag{5}$$

where x' is the initial value of the measurand.

The number of bits for the entries in the LUT must guarantee that no break point lies outside of the acceptable limits of the variable x. Thus, this resolution, defined as N_T, can be found as the integer greater than or equal to:

$$N_T = \log_2\left(\frac{\max(x) - \min(x)}{2\min(\hat{\varepsilon}_x)}\right), \tag{6}$$

4 APPLICATION EXAMPLE

In order to illustrate the application of the proposed procedure, we consider a thermistor based thermometer, in the current biased mode. In this example, for a specific resolution of the table entries, we want to find the values and the number of break points and the loss of resolution in the reconstruction. For the current biased thermistor, one has the output voltage as function of the temperature given by:

$$v = I.r_0.\exp\left(\frac{b}{x+273}\right) \tag{7}$$

where, I is the bias current, x is the temperature in °C and the thermistor parameters are: β = 4000 K and $r_0 = 3.288\text{x}10^{-3}\Omega$, as given in [6].

For the present system, we desire to measure temperature from 0 to 50°C using a 10-bit A/D converter. It is assumed that the sensor output signal is conditioned in such a way that it lies exactly within the A/D input range for the full measurement range. The design problem consists in finding the number or entries and values of a LUT using 10 bits of resolution, that approximates the reconstruction function of the sensor.

The ideal reconstruction function must map the quantized values to the real temperature values: $\tilde{y} \to \hat{x}$, $\tilde{y} = 0 \ldots 1023$, $\hat{x} \subset [0;50]$. Thus, one can easily get the following reconstruction function:

$$\hat{x} = \frac{\beta}{\ln\left(\frac{k.\tilde{y} + \min(v)}{I.r_0}\right)} - 273,\ k = \frac{\max(v) - \min(v)}{2^N - 1}. \tag{8}$$

In order to use a 10-bit resolution LUT, the maximum acceptable measurement error, can be defined as:

$$\hat{\varepsilon}_x(\tilde{y}) = \begin{cases} \varepsilon_{x0}(\tilde{y}), & \varepsilon_{x0}(\tilde{y}) \le \frac{\max(x) - \min(x)}{2^{N+1}} \\ \frac{\max(x) - \min(x)}{2^{N+1}}, & \varepsilon_{x0}(\tilde{y}) > \frac{\max(x) - \min(x)}{2^{N+1}} \end{cases}, \tag{9}$$

where ε_{x0} is given by (1), for $\varepsilon_y = 0.5$.

The curve of the initial effective resolution N_0, calculated using the initial measurement error, the curve of the final effective measurement resolution N_E, with LUT based reconstruction, and the curve of loss of resolution L_R, calculated from (2), are shown in Figure 3. We can observe that if we want to uses a LUT with 10 bits of resolution, the loss of resolution cannot be zero over the whole range. For the values of the quantized variable less than 300, the final resolution is less than 10 bits, due to the non-linear sensor transfer characteristic that affects the initial measurement effective resolution.

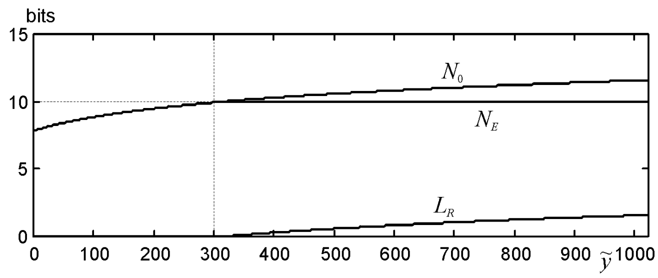

Figure 3. Initial and final effective resolution and loss of resolution in reconstruction.

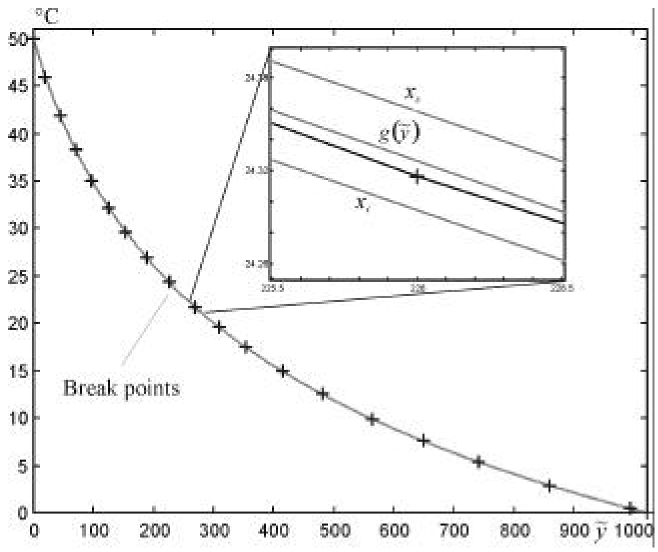

Figure 4. Reconstruction function curve and LUT break points.

The reconstruction function curve and the LUT break points are shown in Figure 4. For the present case the LUT is required to store 20 break points. The effect of the quantization in the LUT entries is shown in detail in the upper right hand corner of the figure 4.

The maximum acceptable measurement error envelope and the reconstruction function approximation error, calculated as the difference between the reconstruction function (8) and the piecewise linear function are shown in Figure 5. One can observe that the approximation error lies always within the limits defined by the maximum acceptable error.

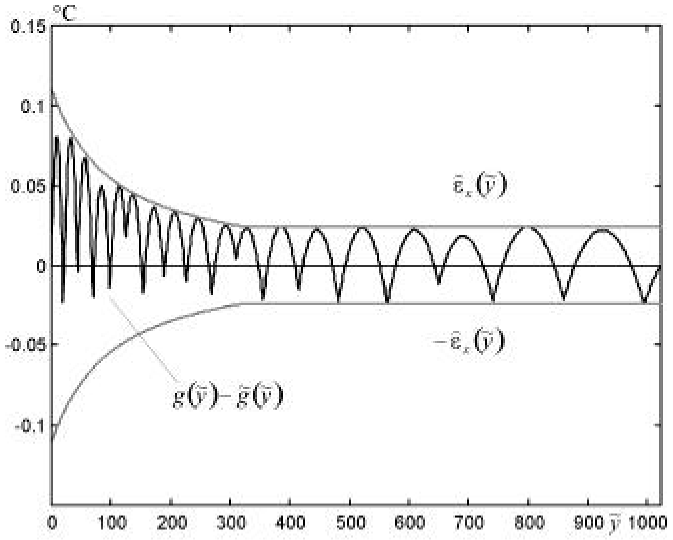

Figure 5. Maximum acceptable measurement error envelope and approximation error.

5 CONCLUSIONS

A new procedure for dimensioning a look-up table for the reconstructing of measurand, in a measurement system using a sensor with non-linear transfer characteristics was presented. The proposed procedure tends to minimize the number of cells that guarantees the measurement error to lie within a specified limit. This procedure can also be easily extended for the cases where sensor has a linear transfer function but is non-linearly dependent on an interfering quantity. In this case the LUT will store the correction values of the main measurand as function of this interfering quantity.

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