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METROLOGICAL CHARACTERISATION OF TIME-FREQUENCY AND TIME-SCALE ANALYSERS

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Abstract: The coupled time-frequency and time–scale analysis are more and more applied in the most different fields, and they are becoming a standard component of the measurement chain. To properly design a measurement chain, the metrological characteristics of the processing has to be known as for all other components such as sensors and conditioners. On these premises a systematic investigation of the metrological properties of the processing techniques considered is carried out, by applying the analysers to a set of properly designed synthetic test signals and by evaluating the result through subjective judgement by a jury. Results of the investigation are presented, giving guidelines for application.
Keywords: time-frequency analysis, wavelet analysis, metrological properties

1 INTRODUCTION

In the last years, the time-scale analysis has found more and more applications in the measurement field. This kind of analysis is seen as a valid option in addition to the time-frequency analysis, in order to improve the time and frequency resolutions. Looking at some applications [2][4] it is clear that nor one or the other technique is the unique solution to all measurement problems: a careful processing design is needed according to the particular application. Moreover the processing step is a fundamental block in the measurement chain and often the result of a measurement is presented as output for final interpretation of the singled out observation feature.

On these premises a metrological characterisation of the processing is required as for all other blocks in the measurement chain such as sensors, conditioners and so on. The metrological properties of the time-frequency analysis are known [2], but the time-scale is far less investigated from this point of view. There is a lot of work in the theoretical field and some publications regarding particular applications but it is difficult to find out an engineering characterisation [1][3][5][6].

This paper investigates the metrological properties of the time-scale analysis in direct comparison with the time-frequency analysis, by applying them on a set of synthetic signals which present interesting properties from the measurement point of view. The metrological properties are the starting point for the definition of a rough application field for each 'analysing window' considered, in such a way that the processing design for worthwhile applications will move from a small set of opportunities, instead of evaluating the whole set.

Starting from typical measurement results, a set of significative sample signals is generated. The first step deals with simple stationary multicomponent signals, then modulation is inserted. Pulse signals of different shapes are studied alone and in presence of a main periodic signal. The Signal to Noise Ratio (SNR) can be adjusted in order to evaluate its effect over the processing result. The signals are processed using the two techniques with different configurations. Results are compared to identify the performance scores of each processing configurations and then the most appropriate technique is sought for each case.

The restitution of the result is also investigated, considering its ergonomic aspects. There is a fundamental difficulty in the time-scale analysis: the result is never directly related to a frequency. There is a common feeling for the frequency concept: it is well placed in every technician brain. The presentation of a result in terms of scale gives a sort of uneasiness to the observer, which reduces the readability of the result. Besides that, often the usual colour map presentation is not able to enhance the principal characteristics of the signal.

2 ANALYSERS UNDER INVESTIGATION

The investigation has considered the analysers described in the following, in view of their a priori complementary features and of their application interest.

2.1 Short Time Fourier Transform, STFT

This analysis technique aims at estimating local spectra, by means of a time running window . It is defined as:

$$\tilde{x}(f,t) = \int_{-\infty}^{+\infty} w(t-\tau)\cdot x(\tau)\cdot e^{-j2\pi f\tau} d\tau \qquad (2.1)$$

which, for each time t, results in a spectrum of the signal x, as viewed through the *observation window* w(.) of duration T_W, centred at time t.

It allows for a constant time and frequency resolution to be optimised for each application by a proper choice of duration T_W, once the shape of the running window has been fixed.

2.2 Evolutionary Power Spectral Density, EPSD

This method is based on an average of the square modulus of time adjacent spectra as defined before. It results in the averaged power spectral density evolving with time. It may be defined as:

$$\tilde{\Psi}(f,t) = \int_{-\infty}^{+\infty} v(t-\tau)\cdot \left|\tilde{x}(f,\tau)\right|^2 d\tau \qquad (2.2)$$

where v(.) is an *averaging window* of duration $T_V > T_W$.
Now time averaging occurs, enabling a variance reduction for stochastic processes. Now a trade off between frequency resolution, time resolution and variance reduction is in order, by a proper choice of both T_W and T_V.

2.3 Time Scale Analysis, TS

The wavelet technique aims to identify any local feature of signal by breaking it into a series of local basis. Continuous and discrete wavelet transform may be performed according to the following formulas, respectively:

$$\text{CWT: } \hat{x}(a,t) = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} w\left(\frac{t-\tau}{a}\right)\cdot x(\tau)\cdot d\tau \qquad (2.3)$$

$$\text{DWT: } \hat{x}(a,t) = \frac{1}{\sqrt{a}} \sum_{m,n\in Z} w\left(\frac{t-t}{a}\right)\cdot x(t), \text{ with } a = 2^m \text{ and } \tau = n\cdot 2^m \qquad (2.4)$$

where $w\left(\frac{t-\tau}{a}\right)$ is the analysing wavelet, suitably scaled and shifted in time. Considering w(t) as the observation window, in this case its width its variable and given by $\mathbf{a}T_W$, where **a** is the scale parameter and T_W is the window width (wavelet length) at unit scale.
A mixed mode implementation has also been considered: CWT step 2^n, a continuous wavelet transform in which the scale parameter assumes only discrete values a_k (as in the case of the DWT), but time varies continuously (as in CWT).
Two analysing reference wavelets have been used: a binary wavelet (Haar) and a signal-like wavelet (Daubecihies10). Considering the implementations and the different weavelets, the total number of analysers evaluated is 8.

3 METROLOGICAL PROPERTIES

Time and frequency resolution as already mentioned are the most relevant and conflicting requirement for time frequency and time scale analysis. The behaviour of the analysers in noisy condition is also of outmost importance and not fully investigated by now, despite the immense bibliography available on this kind of analysis.
Possible qualitative definitions are as follows:

- *frequency resolution*: ability of distinguishing close spectral lines or rapid variations in the spectrum;
- *time resolution*: ability of detecting time events or rapid variation of spectrum along time.

If we now turn to convert these qualitative definitions into quantitative ones, we have some measure of both frequency an time resolution, referred to the analysing window, such as:

$$\Delta f_W = \sqrt{\int_{-\infty}^{+\infty} f^2 \cdot \left|\tilde{w}(f)\right|^2 df \Big/ \int_{-\infty}^{+\infty} \left|\tilde{w}(f)\right|^2 df} \qquad (3.1)$$

$$\Delta t_W = \sqrt{\int_{-\infty}^{+\infty} t^2 \cdot \left|w(t)\right|^2 dt \Big/ \int_{-\infty}^{+\infty} \left|w(t)\right|^2 dt} \qquad (3.2)$$

where $\tilde{w}(f) = \int_{-\infty}^{+\infty} w(t) \cdot e^{-j2\pi f\tau} d\tau$, is the Fourier transform of the window.

However such definition only refer to the performance of the window, not to the final result of the analysis, which is highly dependent on the structure of the signal, including interaction between the required time and frequency resolution and the influence of noise.

So a different approach is here followed, consisting in applying the analysers to a properly designed training set of test signals and in evaluating the resulting performance by subjective judgement.

4 TEST SIGNALS AND PROCEDURE

In order to test the performance of the analysers with respect to frequency and time resolution, under different noise conditions, a set of test signal of the following kind has been used:

$$x(t) = x_1(t) + x_2(t) + x_3(t) \qquad (4.1)$$

where:

- $x_1(t)$ is a fixed sinusoid with frequency f_1;
- $x_2(t)$ is an amplitude modulated sinusoid, with variable carrier frequency f_2;
- $x_3(t)$ is a white noise, with variable variance ó.

Thereafter the study will, both, considered: the frequency resolution, defined as the possibility of correctly distinguishing the fixed frequency f_1 from the carrier frequency f_2; and the time resolution, defined as the possibility of distinguishing the amplitude modulation along the time axis, in contrast with the reference sinusoid, whose amplitude should be perceived as constant along time.

A number of trials was made, with different frequency separation, Äf/ f_1=(f_2- f_1)/ f_1, and noise, ó, conditions; 6 different values for both Äf/ f_1 and ó were considered, for a total number of 36 signals.

Each analyser processed all the signals and the 288 results were judged by a jury of researchers with different background, which assigned them a score for both time and spectral resolutions in a range from 0 to 10. Results are shown in the next paragraph.

5 RESULTS OF THE INVESTIGATION

The output from the different implementations of the wavelet analysis is presented in figure 1 for a signal without noise and Äf/ f_1=0.4, with the db10 reference wavelet. In the DWT, the discrete and varying time step is evident, while in the CWT the continuous variation of both time and scale are evident. In particular the CWT result is interesting because, in correspondence of the fixed reference frequency, the large bandwidth of the wavelet analyser clearly appears. The row width, which is about 8 levels, is reduced to a single 2^n step when the CWT 2^n result is considered.

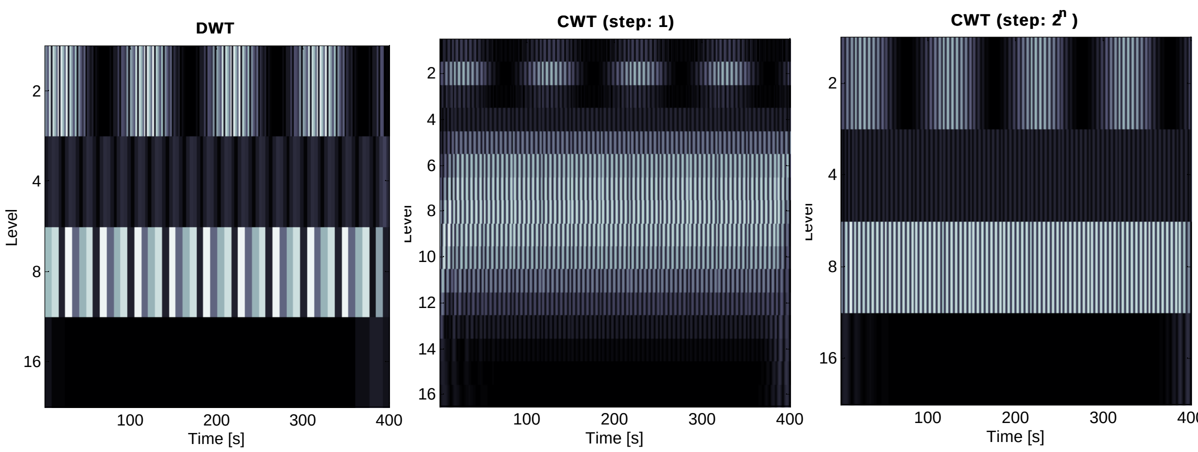
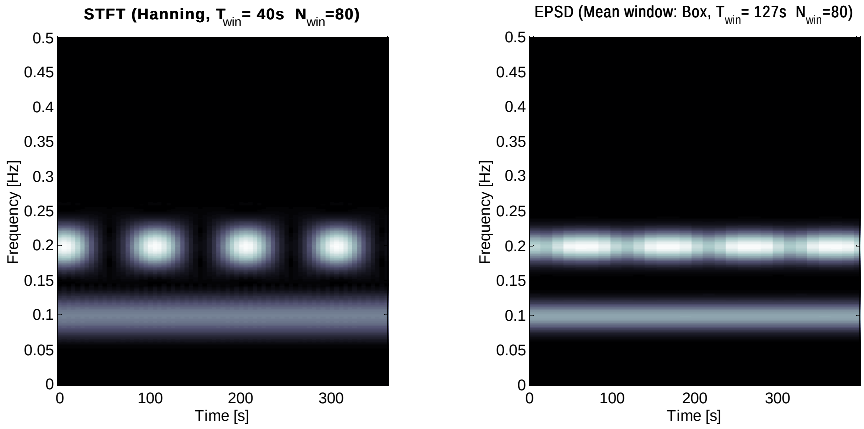


Figure 1. comparison of the outputs from the 3 algorithms for wavelet analysis.

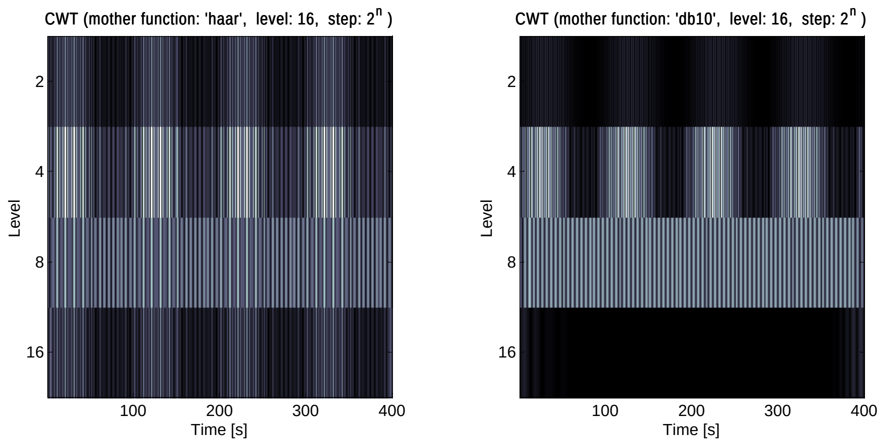
In the next figures and in the score presentation, only the CWT step 2^n wavelet analysis will be considered because, in general, it was the best implementation.

A typical example of the output available from the analysers for judgement is presented in figures 2-5 for signals with the same frequencies but different NSR. The judgement is quite sensitive to the output format: while a colour scale with huge variations is recommended for STFT and EPSD results, a more smooth colour scale is more suitable for all the wavelet analysis. For printing reasons all results are here presented in a grey scale. Before judgement, a short training is necessary in order to be able to read time scale plots.



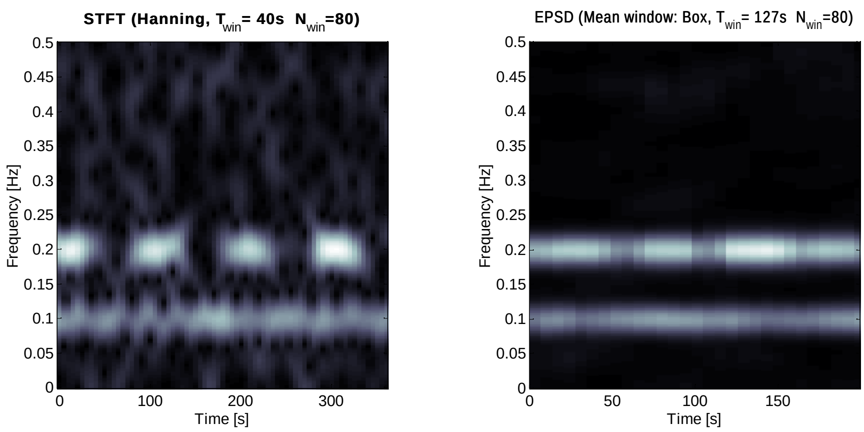
A B

Figure 2. STFT (A) and EPSD (B) analysis of signal Δf=0.15, without noise



A B

Figure 3. CWT analysis with Haar (A) and Daubechies10 (B) of signal Δf=0.15, without noise



A B

Figure 4. STFT (A) and EPSD (B) analysis of signal Δf=0.15, NSR=3.5

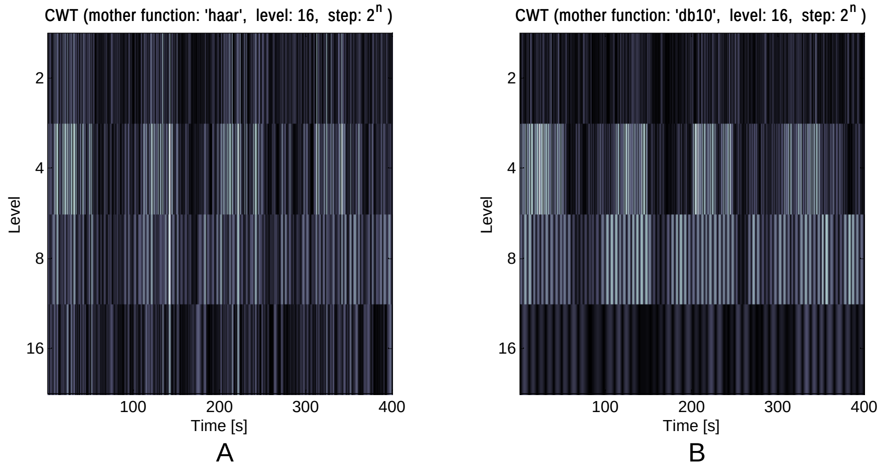


Figure 5. CWT analysis with Haar (A) and Daubechies10 (B) of signal Δf=0.15, NSR=3.5

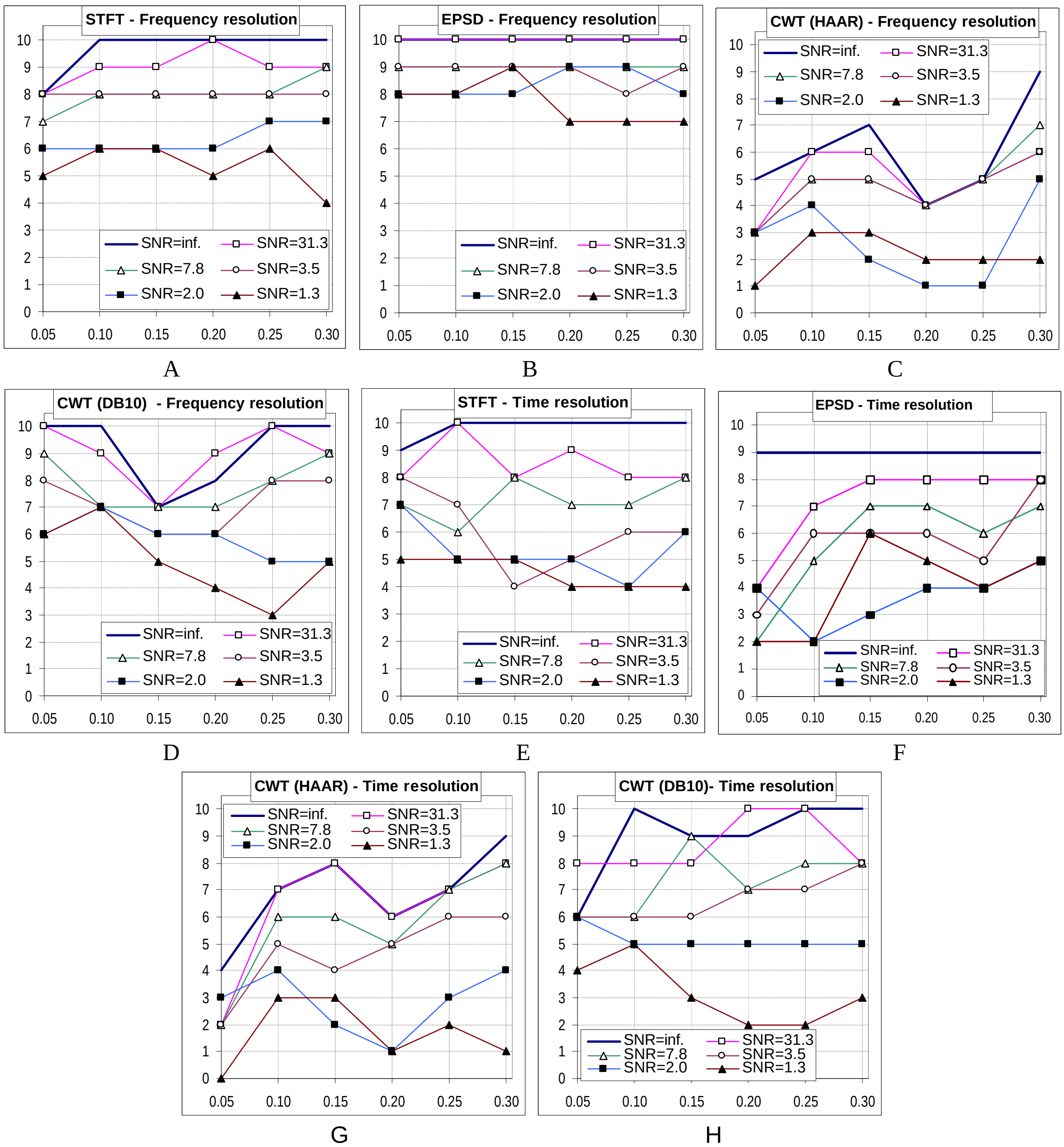


Figure 6. Example of scores results for frequency and time resolutions.

A – STFT analysis versus $f-f_{rif}$. Frequency resolution; B - EPSD analysis versus $f-f_{rif}$. Frequency resolution; C - Haar CWT step 2^n analysis versus $f-f_{rif}$. Frequency resolution; D - db10 CWT step 2^n analysis versus $f-f_{rif}$. Frequency resolution; E - STFT analysis versus $f-f_{rif}$. Time resolution; F - EPSD analysis versus $f-f_{rif}$. Time resolution; G - Haar CWT step 2^n analysis versus $f-f_{rif}$. Time resolution; H - db10 CWT step 2^n analysis versus $f-f_{rif}$. Time resolution

Score results are presented in figure 3. Considering the spectral resolution it is evident that the classical EPSD analyser performs in a better way than all the others, while wavelet analysers are a bit in difficulty, decreasing their scores with increasing noise.

When considering the time resolution things are different for the wavelet analysers: the db10 reference wavelet performs rather well also in presence of noise, while EPSD presents a rapid degradation of the time resolution performance while the two frequencies are getting nearer.

It emerges that wavelet analysers have some interpretation difficulties, due to the concept of scale instead of frequency. Their behaviour as proportional band pass filters affects the spectral resolution above all in low NSR conditions, due to a poor filtering, but the variable bandwidth improves their time resolution above all when the frequencies to be distinguished are very close each other.

6 CONCLUSIONS

The judgement procedure seems to be a promising way in order to evaluate actual interpretation of time scale analysers. A lot of work has to be done by using different sets of signals with different time evolutions. At this stage of the work the followings emerge.

The result interpretation is quite sensitive to the restitution; the proper choice of the colour scale can improve dramatically the result interpretation. New approaches in this sense has to be considered in order to overcome the dualism between frequency and scale and to identify a proper presentation for the wavelet analysis results, which could not be connected to the usual presentations of time frequency analysis.

The EPSD analyser performs very well as spectral resolution, but it needs careful choice of the time resolution for which long time sequence of signals are requested.

The reference wavelet has to be chosen according to the signal shape and to the event to be detected. The Haar wavelet has a very good computational efficiency, but in our case db10 performs much better.

The wavelet analyser implementation is quite relevant: CWT takes lot of computation time, above all for long signals, and not always such a level of detail is useful for interpretation; CWT2n, improves the computation efficiency and avoids the display of the filter bandwidth; DWT is the most efficient, but it provides such a low level of details which implies difficult interpretation.

REFERENCES

[1] G. Strang, T. Nguyen, *Wavelets and filter banks*, Wellesley-Cambridge Press, 1996.

[2] D'Alessio, R.C. Michelini, F. Porfiri, G.B. Rossi, Comparison of spectral analysis methods for mechanical measurements (in Italian), *Proceedings II Congresso Nazionale di Misure Meccaniche e Termiche* (Bressanone,19-21 June 1995), Bressanone, Italy, 1995, p. 267-277.

[3] K.C. Chui, Wavelets: A Tutorial In Theory And Applications, Academic Press, 1992.

[4] S. Conforto, T. D'Alessio, Spectral analysis for non-stationary signals from mechanical measurements: a parametric approach, *Mechanical systems and signal processing*, **13** (3) (1999) 395-411.

[5] W.J. Wang, P.D. McFadden, Application of wavelets to gearbox vibration signal for fault detection, *Journal of Sound and Vibration*, **192** (5) (1993) 927-939.

[6] B.A. Paya, I.I. Esat, Artificial neural network based fault diagnostics of rotating machinery using wavelet transforms as a preprocessor, *Mechanical systems and signal processing*, **11** (5) (1997) 751-765.

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