XVI IMEKO World Congress
Measurement - Supports Science - Improves Technology - Protects Environment ... and Provides Employment - Now and in the Future
Vienna, AUSTRIA, 2000, September 25-28

SELECTION OF PFM SIGNAL PERIOD FOR T/D CONVERSION

D. Swisulski
Technical University of Gdañsk, Electrical and Control Engineering Department
ul. Narutowicza 11/12, 80-952 Gdañsk, Poland

Abstract: This paper presents principles of selecting the period of pulse-frequency modulated (PFM) signals used as intermediate representation of measured signals. It is assumed that the PFM signal is to be digitized using time-to-digital (T/D) conversion, and that the measured signal can be modeled as a dc component plus a sinusoidal fluctuation of known frequency. The measurement error, mainly composed of dynamic error and counting error, can significantly be reduced by properly selecting the period of the PFM signal with respect to other measurement conditions. Included are considerations pertaining to both on-line and off-line postprocessing of the T/D conversion readouts. In the case of speed measurement by means of incremental encoder, the proposed method of period selection indicates the most favorable number of encoder pulses per shaft revolution.
Keywords: PFM signals, period counting, dynamic error

1 INTRODUCTION

Modern measurement systems usually rely on conversion of the measured physical quantity to a voltage signal, the latter being subsequently converted to a digital representation for processing, storage, and displaying purposes. An alternative approach is to convert the measurand to frequency (period) of a pulse train, and then use pulse counting to obtain digital representation of the measurand. This kind of conversion is conveniently applied with parametric sensors, where a measurand-dependent capacitance or inductance can be used to establish the frequency of a pulse-generating circuit. The use of frequency or period as intermediate representations of measured variables is also an obvious solution in those cases where the measurand itself has temporal or frequency nature (time interval, frequency, rotational speed, etc.) [1].

A variety of methods are available for digitizing PFM signals [2]. The most widespread methods include counting PFM pulses over a constant gating interval, and counting reference pulses over a gating interval equal to a PFM pulse-to-pulse interval. The former method yields measurement results at constant time intervals. A drawback of this method is a long gating time required to achieve the required measurement accuracy. The latter method permits radical shortening of the measurement time.

The conversion of pulse-to-pulse intervals of the encoder output signal can be effected by counting clock pulses from a reference generator that fit between two consecutive encoder pulses. Such conversion exhibits the following errors [3]:

- counting error due to the finite resolution of pulse counting,
- reference frequency error [4],
- triggering error caused by the noise contained in the measured signal.

When the frequency changes during the measurement, there occur additional dynamic errors [5].

2 ERRORS IN MEASUREMENTS BASED ON T/D CONVERSION

2.1 Dynamic error

Conversion of period of pulse-frequency modulated (PFM) signals to a digital format is usually implemented by counting pulses from a reference oscillator between successive PFM pulses. This process, referred to as the time-to-digital (T/D) conversion, is characterized by the following errors: the counting errors (due to the granularity of reference pulse times), the frequency error of the reference oscillator, and the triggering errors. If the T/D conversion is used in a measurement of a PFM-encoded signal (e.g. motor speed), another error comes into play; it is called the dynamic error, because it only appears if the measured signal varies in time.

The conversion result corresponding to the average value of interval T_m (between t_i and t_{i+1}) is determined with delay is T_c after t_{i+1} (Fig. 1), where T_c represents the time required for computation [6].

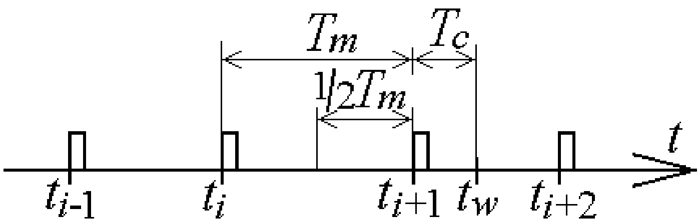

Figure 1. Illustration of the dynamic error

Therefore the dynamic error is represented by the following equation:

$$\Delta x_d = \frac{1}{T_m}\int_{t_i}^{t_i+T_m} f(t)dt - f(t_i + T_m + T_c) \quad (1)$$

The dynamic error Δx_d is made up of three components. The first Δx_{d1}, results from the fact that each particular readout is a time average over the gating interval (i.e., the period of the PFM signal). Assuming that the readout is assigned to the midpoint of the gating interval, this error is the difference between the readout and the actual value of the measured signal in this instant of time. The second error component Δx_{d2}, is due to the fact that the counting concludes only at the end of the gating interval; this error is the difference between the actual values of the measured signal in two different time instants: at the midpoint and at the end of the gating interval. The third error component Δx_{d3}, is caused by additional time delay between the conclusion of counting and the conclusion of final computations. This error is the difference between the actual values of the measured signal corresponding to the end of the gating interval and the end of computations.

The components of dynamic error are given by equations (2), (3) and (4).

$$\Delta x_{d1} = \frac{1}{T_m}\int_{t_i}^{t_i+T_m} f(t)dt - f\left(t_i + \frac{1}{2}T_m\right) \quad (2)$$

$$\Delta x_{d2} = f\left(t_i + \frac{1}{2}T_m\right) - f(t_i + T_m) \quad (3)$$

$$\Delta x_{d3} = f(t_i + T_m) - f(t_i + T_m + T_c) \quad (4)$$

Assume that the measured signal varies as a sinusoid (equation (5)) of amplitude X_m, with dc offset X_0, and frequency $f_p=1/T_p$, and define, for the sake of convenience, the following normalized quantities: $k_i = t_i/T_p$, $q = T_m/T_p$, $c = T_c/T_p$, and $z = X_m/X_0$, where t_i denotes the pulse time of the PFM signal that begins the i-th gating interval, T_m is the gating interval (PFM period), and T_c represents the final computational delay. The above normalized quantities have the following interpretations: k_i is the dimensionless start time of the i-th measurement, q is the interval between successive pulses of the PFM signal, c is the final computational delay, and z is the ratio of the ac and dc components of the measured signal. Using the adopted conventions, the relative dynamic error (related to the actual value at the end of conversion) can be evaluated from (6), while the components of this error are given by (7), (8) and (9).

$$f(t) = X_0 + X_m \sin(2pf_p t) \quad (5)$$

$$dx_d = \frac{\sin(p\,(2 \cdot k_i + q)) \cdot \sin(pq) - pq \sin(2p\,(k_i + q + c))}{\frac{pq}{z} + pq \sin(2p\,(k_i + q + c))} \tag{6}$$

$$dx_{d1} = \frac{\sin(p\,(2 \cdot k_i + q)) \cdot \sin(pq) - pq \sin(p\,(2k_i + q))}{\frac{pq}{z} + pq \sin(2p\,(k_i + q + c))} \tag{7}$$

$$dx_{d2} = \frac{\sin(p\,(2 \cdot k_i + q)) - \sin(2p\,(k_i + q))}{\frac{1}{z} + \sin(2p\,(k_i + q + c))} \tag{8}$$

$$dx_{d3} = \frac{\sin(2p\,(k_i + q)) - \sin(2p\,(k_i + q + c))}{\frac{1}{z} + \sin(2p\,(k_i + q + c))} \tag{9}$$

Figure 2 presents the plots of these errors as a function of k_i for q = 0,1, c = 0,01 and z = 0,1.

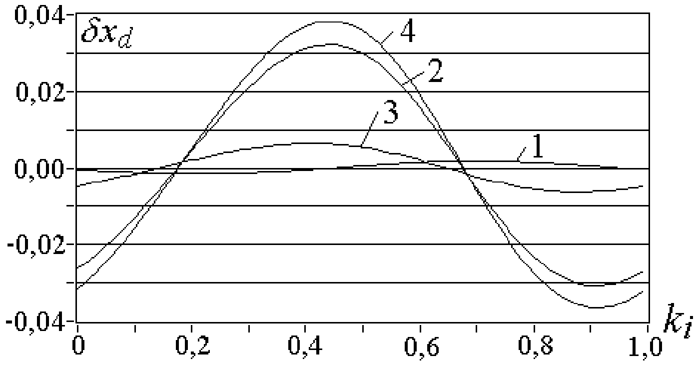

Figure 2. The dynamic error and its components as a function of k_i for q = 0,1, c = 0.01 and z = 0,1:
1 - δx_{d1}, 2 - δx_{d2}, 3 - δx_{d3}, 4 - $\delta x_d = \delta x_{d1} + \delta x_{d2} + \delta x_{d3}$

As can be seen in the figure 2, the main influence upon the dynamic error has the component due to the delay of the completion of conversion.

In the case of on-line postprocessing of the T/D conversion results, the individual readouts represent the measured signal at the respective time instants. Therefore, all components of the dynamic error should be taken into account. When the conversion results are processed off-line, and assigned to the midpoints of gating intervals, the dynamic error due to averaging (the first component δx_{d1}) is of secondary, if any, importance.

Therefore the measuring error in the case of off-line processing, is smaller than the error in the case of on-line processing. For the example in Fig. 2, maximal value of the error due to a difference between the average value of the gating interval and the instantaneous value corresponding to the midpoint of this interval is about 20 times smaller than the error due to delay of the completion of conversion.

The maximum value of dynamic error, as well as its time of occurrence k_i, depends on q, c and z.

Figure 3a shows the plots of the maximal value of the total dynamic error $\delta x_{d\max}$ as a function of q for different value of c and Figure 3b for different value of z. As can be seen this error is approximately proportional to q.

The expression for the maximum dynamic error $\delta x_{d\max}$ is fairly complicated, and thus an approximate formula has been found, as given in (10).

$$dx_{d\,\max} = 3 \cdot z \cdot q + 6.5 \cdot z \cdot c \tag{10}$$

Formula (10) was obtained by linear regression after analysis of different functions. and gives satisfactory results for parameters q, z and c, which occur in practice.

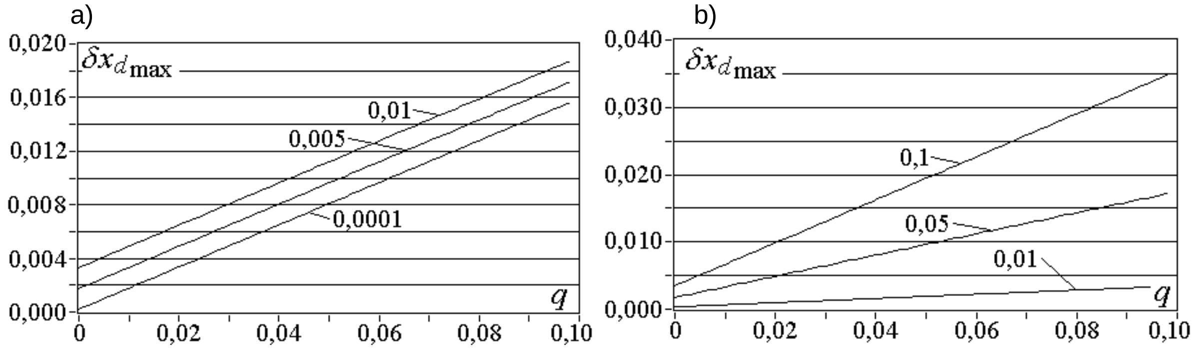

Figure 3. The maximal value of the total dynamic error as a function of *q*:
a) for *z* = 0,05 and *c* = 0,0001, 0,005, 0,01, b) for *c* = 0,005 and *z* = 0,01, 0,05, 0,1

Figure 4 shows the plots of the maximal value of the component dx_{d1} versus *q* for different value of *z*. As can be seen the increase of this error is larger for larger value of *q*. Parameter *c* has a small influence upon the error $dx_{d1\max}$.

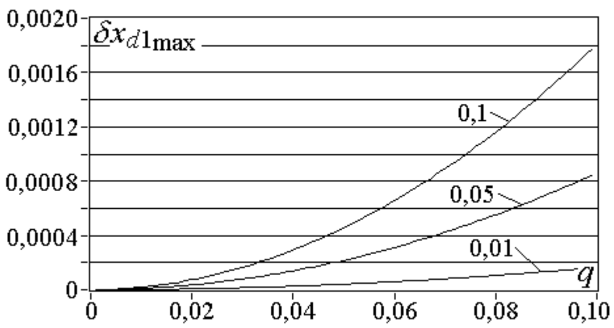

Figure 4. The maximal value of the component of the dynamic error $\delta x_{d1\max}$ as a function of *q* for *z* = 0,01, 0,05, 0,1 and *c* = 0,005

An approximate expression for $\delta x_{d1\max}$ (i.e., the maximum value of the δx_{d1} component) is given by (11).

$$dx_{d1\max} = z \cdot \left(1 - \frac{\sin(pq)}{pq}\right) \tag{11}$$

2.2 Counting error

The counting error, dx_b, does not exceed the reciprocal of the pulse count.

$$dx_b = \frac{1}{N} = \frac{T_g}{T_m} = \frac{1}{f_g \cdot T_m} \tag{12}$$

Let *g* denote the ratio of the reference oscillator period T_g and the PFM signal period T_p, that is, $g = T_g/T_p$. Then, the counting error can be estimated by formula (13).

$$dx_b = \frac{g}{q} \tag{13}$$

Figure 5 shows the plots of the maximum dynamic error (formula (10)), the component of the dynamic error (formula (11)) and the counting error (formula (13)) as a function of *q*.

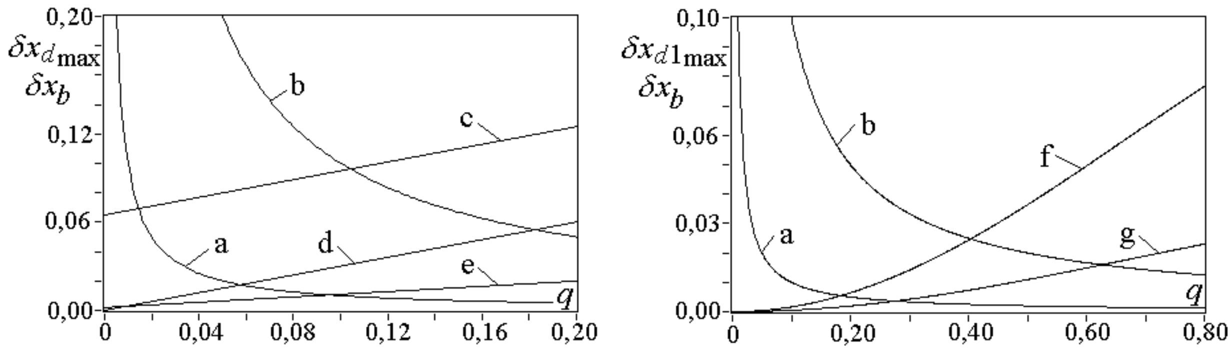

Figure 5. The counting error by formula (13) for: a) g = 0.001, b) g = 0.01; the dynamic error by formula (10) for: c) z = 0 and c = 0,1, d) z = 0,1 and c = 0,0001, e) z = 0,03 and c = 0.01; the component of the dynamic error by formula (11) for: f) z = 0,1, g) z = 0,03 versus q

Assuming that the frequency error of the reference oscillator is negligible, and so are the triggering errors, the total measurement error amounts to the sum of dynamic error and counting error. For low values of q, the counting error is the dominant error component. For higher values of q, however, the dynamic error can become the major error component. The value of q that minimizes the total error dx_c, denoted q_0, can be evaluated from (15). We obtain this formula from (14). For the case of off-line postprocessing, the minimum will be located at $q = q_{01}$, the latter being given by (17). Formula (17) is obtained by conversion $\sin(pq_{01})$ and $\cos(pq_{01})$ (formula (16)) into power series and taking into account only two addends.

$$\frac{\P dx_t}{\P q} = 3 \cdot z - \frac{g}{q_0{}^2} = 0 \qquad (14)$$

$$q_0 = \sqrt{\frac{g}{3 \cdot z}} \qquad (15)$$

$$\frac{\P dx_{t1}}{\P q} = \frac{z \cdot \sin(pq_{01})}{q_{01}^2} - \frac{zp \cdot \cos(pq_{01})}{q_{01}} - \frac{g}{q_{01}^2} = 0 \qquad (16)$$

$$q_{01} = \frac{1}{p}\sqrt[3]{\frac{3 \cdot g}{z}} \qquad (17)$$

The value of c has an effect on the error but have not an effect on the value of q_0.

3 EXAMPLE SELECTION OF PFM SIGNAL PERIOD

Consider the motor speed measurement by means of an incremental encoder. Assume that the maximum fluctuations of the measured speed correspond to k_{max} = 0.01, and their frequency is 50 Hz (the speed fluctuations can be caused by pulsating voltage waveforms feeding the motor). The assumed reference oscillator frequency is 1 MHz, and thus g = 50 Hz / 1 MHz = $5 \cdot 10^{-5}$. The time required for computations is T_c = 100 μs, and thus c = 100 μs / 20 ms = 0,005.

If the speed variation has sinusoidal form, parameter z is equal the maximal value of the irregularity of rotational speed $k_{\max}$ [7]:

$$k_{\max} = \frac{\left|n - n_{avg}\right|_{\max}}{n_{avg}} \qquad (18)$$

where n is an instantaneous and n_{avg} an average rotational speed.

It follows from (15) that $q_0 \approx 0.04$, whereby $T_m = q_0 \cdot T_p$ = 0.8 ms. From (17), in turn, there is $q_{01} \approx 0.08$, and thus $T_{m1} = q_{01} \cdot T_p$ = 1.6 ms. For the assumed example data, hence, the desirable PFM signal period corresponding to on-line postprocessing is twice the period corresponding to off-line postprocessing.

The desirable period of PFM signal immediately translates into the desirable number of encoder pulses per shaft revolution. Assuming the motor speed of 50 rev/s and on-line postprocessing, the optimum number of pulses per revolution is 0.02 s / $8 \cdot 10^{-4}$ s = 25. However, when the motor speed is 10 rev/s, the optimum number of pulses per revolution becomes 0.1 s / $8 \cdot 10^{-4}$ s = 125.

In the case of off-line postprocessing, the optimum number of pulses per revolution is twice times greater than in the case of on-line postprocessing.

The total measurement error dx_t consists of the dynamic error dx_d (formula (10)) and the counting error dx_b (formula (13)). The value of this error is $dx_t = \sqrt{dx_d^2 + dx_b^2} = 0.2\%$. In the case of off-line postprocessing the total measurement error dx_{t1} consists of the component of the dynamic error dx_{d1} (formula (11)) and the counting error dx_b (formula (13)). In this case the value of the error is $dx_{t1} = \sqrt{dx_{d1}^2 + dx_b^2} = 0.06\%$ and it is smaller than the error in the case of on-line postprocessing. In assessing the accuracy of speed measurement, encoder errors should also be taken into account [8].

The counting error influences on the total error for encoder with greater number of pulses per revolution, the dynamic error influences on the total error for encoder with smaller number of pulses per revolution.

4 CONCLUSION

The total measurement error of the period, mainly composed of dynamic error and counting error, can significantly be reduced when the period of the PFM signal is properly selected.

We can use the presented method for selecting the period of pulse-frequency modulated signals as the PFM signal can be modeled with the help of a dc component and a sinusoidal fluctuation of known frequency.

The method can be use to both on-line and off-line postprocessing of the T/D conversion readouts.

We can use the method if we have a possibility of changing the period of the PFM signal. E.g. in the case of speed measurement by means of incremental encoder, the proposed method indicates the most favorable number of encoder pulses per shaft revolution. In the case of measurement with parametric sensors, the method indicates the most favorable capacitance or inductance of sensor.

REFERENCES

[1] D. Œwisulski, Elimination of errors due to measurement-time variation in continuous measurements of time-encoded signals; *Proceedings of 9th International Fair and Congress for Sensors, Transducers & Systems SENSOR’99*, (Nürnberg, May 18-20, 1999), vol. II, p. 463-468.
[2] M. Prokin, Dynamic Response of Frequency Measuring System; *IEEE Transactions on Instrumentation and Measurement*, vol. 41, nr 3, June 1992, 390-396.
[3] Hewlett Packard, *Fundamentals of Time Interval Measurements*; Application Note 200-3, 1997, 67 p.
[4] A. Härting, Considering Clock Errors in Numerical Simulations; *IEEE Transactions on Instrumentation and Measurement*, vol. 45, nr 3, June 1996, 715-720.
[5] G. Schommartz, Zur Meßdynamik digitaler Frequenzmesser; *Messen Steuern Regeln*, Heft 5, Mai 1979, 261-267.
[6] D. Œwisulski, Wp³yw b³ê du dynamicznego na dobór okresu sygna³u impulsowego przy przetwarzaniu T/C, *Proceedings of Conference Metrologia Wspomagana Komputerowo MWK’99*, (Rynia, June 7-10, 1999), vol. II, p. 43-48.
[7] L. Referowski, L. Swê drowski, The chosen problems of the measurements of the irregularity of rotational speed; *Summary of papers EMISCON’89*, Dum techniky CSVTS (Praha 1989), p. 198-200.
[8] D. Œwisulski, Effect of the incremental encoder’s errors while measuring the angular velocity; *Proceedings of XIV IMEKO World Congress* (Tampere, June 1997), Vol. VIII, p. 222-227.

AUTHOR: Dr. Ing. Dariusz ŒWISULSKI, Technical University of Gdañsk, Electrical and Control Engineering Department, ul. Narutowicza 11/12, 80-952 Gdañsk, Poland, Phone Int. +48 58 3471397, Fax Int. +48 58 3471726, E-mail: dswis@ely.pg.gda.pl