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USING INTERMEDIATE FFT RESULTS IN MEASUREMENT SIGNAL ANALYSIS

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Abstract: A possible use of information contained in the well known Fast Fourier Transform, calculated for some subsets of samples belonging to an analyzed sequence is proposed. The above mentioned information will be particularly important if the sequence to be identified takes nonzero values only within a certain subinterval of the analyzed period. The described method refers also to some properties of nonharmonic orthogonal sets of functions and makes use of them. Also some remarks concerning signals buried in noise are given. Typical signals coming from electromagnetic transducers are analyzed; the results illustrate the proposed method.

Keywords: digital signal analysis, fast algorithms, nonharmonic orthogonal functions

1 INTRODUCTION

The discrete Fourier transform and the corresponding fast algorithms are a very well known tool for representing a sequence of N signal samples in the frequency domain. Generally N/2 different spectral components are then to be used to reconstruct or transmit the original signal, even if it consists of segments of elementary functions, but takes nonzero values <u>only</u> within a certain previously unknown subinterval (example see Fig. 1).

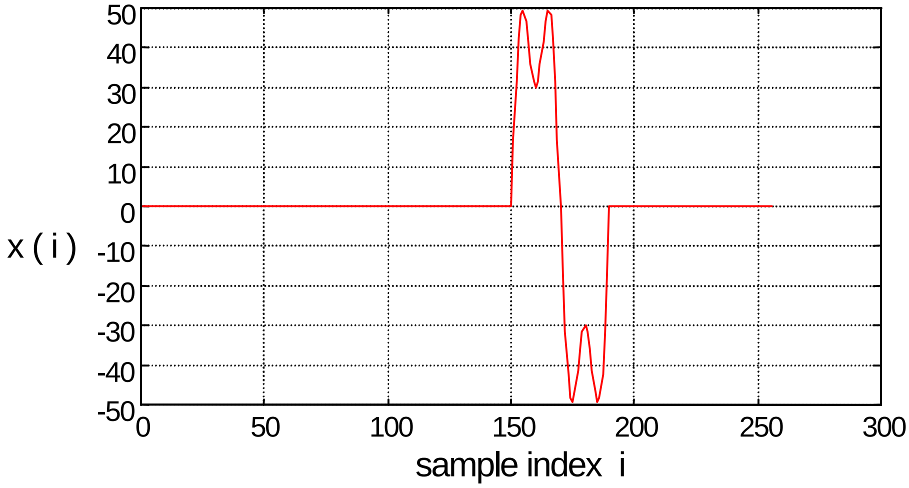

Figure 1. Example of analyzed signal

It seems to be possible to propose a method of spectral identification and representation of the above mentioned type of signals, using certain intermediate results obtained from "inside the FFT calculation procedure". It would be also interesting to find out a simple mean to detect the subinterval of nonzero signal values, taking into consideration the presence of additive noise. The main goal here is to reduce the number of data characterizing the signal.

2 OUTLINE OF THE METHOD

As it was mentioned in [1], the idea of comprehensive spectral representation of signals taking nonzero values only within a subinterval of the whole analyzed period, can be already observed if we are looking through old mathematical papers, where some specific orthogonal systems of functions were defined, for instance texts written by Alfred Haar. Already at the beginning of our century he has proposed a system of "rectangular" functions, having interesting properties [2]. Each of Haar functions stretches over $1/2^k$-th part of the basic period (*k* - integer) and does not exceed the positive or negative half of the next lower order function (for lower *k)*. A sufficient condition of orthogonality is thus fulfilled. Beside orthogonality the Haar system is complete and the corresponding Fourier series converges uniformly for every continuous function. This very interesting idea of "local" analysis cannot be however extended to the harmonic system. The sections of sine wave, cut out in a manner resulting from the Haar function shape are not orthogonal for different frequencies, even for frequencies taking values related to each other like integer powers of two. Generally, it has been shown that only some properties of the Haar and Fourier transforms are similar and exactly of this fact it will be made use below.

It is a well known fact that the so called "frequency division algorithm" is one of the basic variants of FFT [3]. The algorithm enables us to determine complex amplitudes of particular signal components as the following sums

$$\sum_{n=0}^{N-1} x(n) \exp\left(- j2pnm / N\right) \tag{1}$$

separately for even and odd values of *m,* that is for *m = 2r* and for *m = 2r+1* where *r* Î [0, N/2-1]. After the first decomposition following transforms having reduced dimensions are obtained

$$\begin{aligned} X(2r) &= \sum_{n=0}^{N/2-1} \left(x(n) + x(n+N/2)\right) \exp\left(- j4pnr / N\right) \\ X(2r+1) &= \sum_{n=0}^{N/2-1} \left(x(n) - x(n+N/2)\right) \exp\left(- j2pn / N\right) \exp\left(- j4pnr / N\right) \end{aligned} \tag{2}$$

It results from Eqs. (2) that the discrete Fourier transform of order *N* has been reduced to two transforms having dimensions equal *N/2.* Repeating this procedure $log_2 N$ times leads to the above mentioned frequency division algorithm. If however at an arbitrary stage the following partial sums denoted as *X'(.)* are calculated separately

$$\begin{aligned} X'(2r) &= \sum_{n=0}^{N/2-1} x(n) \exp\left(- j4pnr / N\right) \\ X(2r) &= \sum_{n=0}^{N/2-1} x(n+N/2) \exp\left(- j4pnr / N\right) \end{aligned}$$

and respectively (3)

$$\begin{aligned} X(2r+1) &= \sum_{n=0}^{N/2-1} x(n) \exp\left(- j2pn / N\right) \exp\left(- j4pnr / N\right) \\ X(2r+1) &= -\sum_{n=0}^{N/2-1} x(n+N/2) \exp\left(- j2pn / N\right) \exp\left(- j4pnr / N\right) \end{aligned}$$

Fourier transforms (spectra) for particular subintervals corrsponding to $N/2^k$ samples will be obtained. These spectra, if calculated for subintervals with nonzero signal values, have a much more "concentrated' form than spectra calculated for all N samples.

As it was assumed above, the analyzed signals take nonzero values only within a certain subinterval. Different situations resulting from this fact are discussed in [1]. In all cases a quantitative index should be found which locates the signal with respect to the sequence of N samples. For signals being symmetrical with respect to a point it seems to be convenient to use a certain transform coefficient as criterion. The first non-diappearing Haar coefficient may be applied here, because of the specific form of Haar functions. Such a coefficient does not depend on any specific form of the signal (if a zero shift with respect to the subinterval is assumed; if this is not the case, a complementary character of the Haar coefficients for two neighbouring subintervals is to be expected).

It is a well known fact that the structures of fast Fourier and Haar algorithms are similar to some extent. A substantial difference is the fact of multiplication by certain powers of a complex factor in Fourier case, hence the complex character of all intermediate results. If however at all stages, additionally (together with typical Fourier "butterflies") simple sums and differences are calculated, a parallel computation of Haar coefficients will occur. Obviously in practical implementations only the general frame of the program can be used for calculating both sets of coefficients.

3 RANDOM ERROR ANALYSIS

In case of additive noise the analyzed signal will be corrupted (Fig. 2) and certain random errors of the above mentioned coefficients are to be expected.

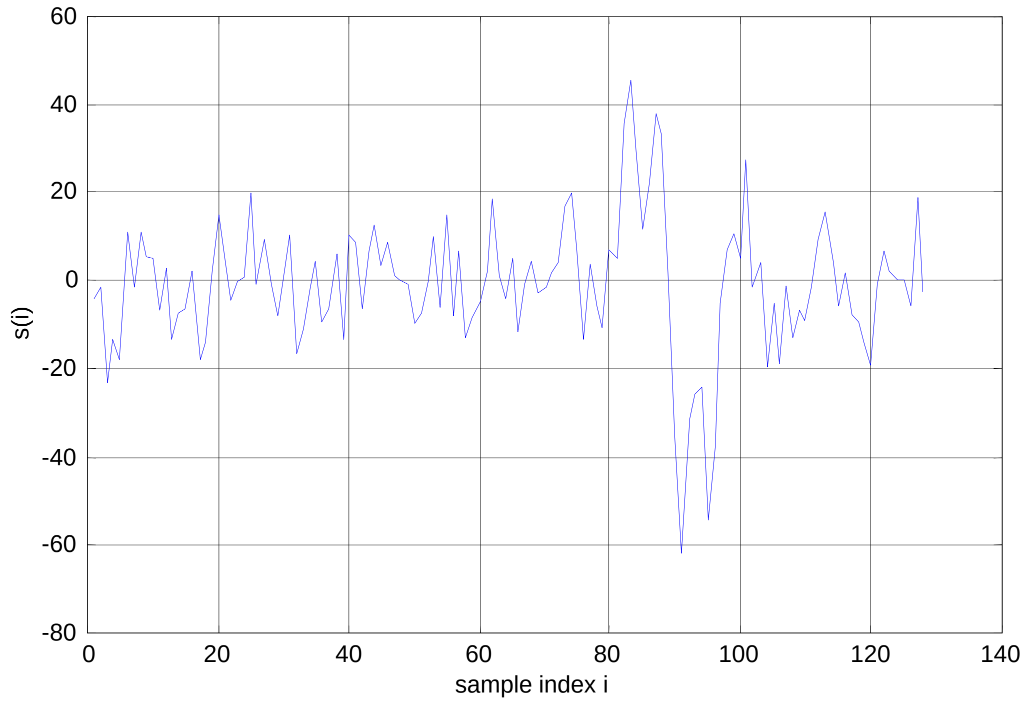

Figure 2. Example of disturbed signal s(i)

The signal s(i) shown in Fig. 2 is assumed to be a sum

$$s(i) = x(i) + n(i) \tag{4}$$

of analyzed signal *x(i)* and random gaussian noise n(i). Coefficients H_i calculated with respect to the i-th Haar function haar(i,k) result from the formula

$$H_i = \sum_{k=N_1}^{N_2-1} S(i) haar(i,k) \tag{5}$$

where $[N_1, N_2]$ denote the interval of nonzero Haar function values (depending on index i). H_i takes random values and can be characterized by two parameters: expected value and expected squared value. The first one is defined as

$$E[H_i] = E\left[\sum_{k=N_1}^{N_2-1} S(i)haar(i,k)\right] = \left[\sum_{k=N_1}^{N_2-1} x(i)haar(i,k)\right] \quad (6)$$

and takes the greatest absolute value if the signal x(i) has nonzero values exactly within the interval [N_1, N_2]. The second one is given by the formula

$$E[H_i^2] = E\left[\left(\sum_{k=N_1}^{N_2-1} S(i)haar(i,k)\right)^2\right] \quad (7)$$

In particular case, if the signal x(i) is symmetric with respect to the point $(N_1+N_2)/2$, taking into consideration the "whiteness" of the noise we obtain

$$E[H_i^2] = \left(\sum_{i=N_1}^{N_2} x(i)haar(i,k)\right)^2 + (N_2 - N_1)s^{\,2} \quad (8)$$

where σ^2 denotes dispersion of the white noise.

A H_i^2 vs. Haar function group index curve is given in Fig. 3.

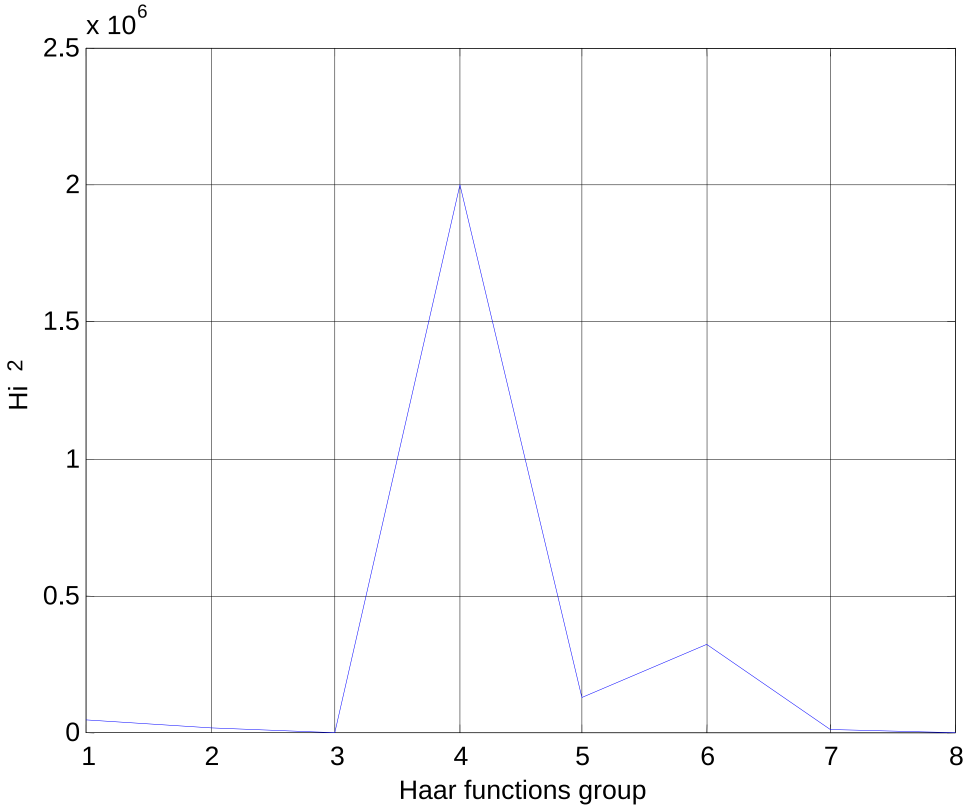

Figure 3. Squared Haar coefficient vs. Haar functions group index curve for signal given in Fig. 2

The following conclusion should be drawn from this figure: A certain threshold depending on the noise level can be determined; if the coefficient is exceeding it for a certain function index, the signal is positioned on the time axis.

4 EXAMPLE

A signal induced in a coil moving across magnetic field (e.g. a transducer reading binary data from a magnetic carrier – see [4]) is shown in Fig. 4. The same signal buried in noise is depicted in Fig. 5. It was localized by the use of the above presented method and the corresponding curve, similar as in Fig. 3, is shown in Fig. 6.

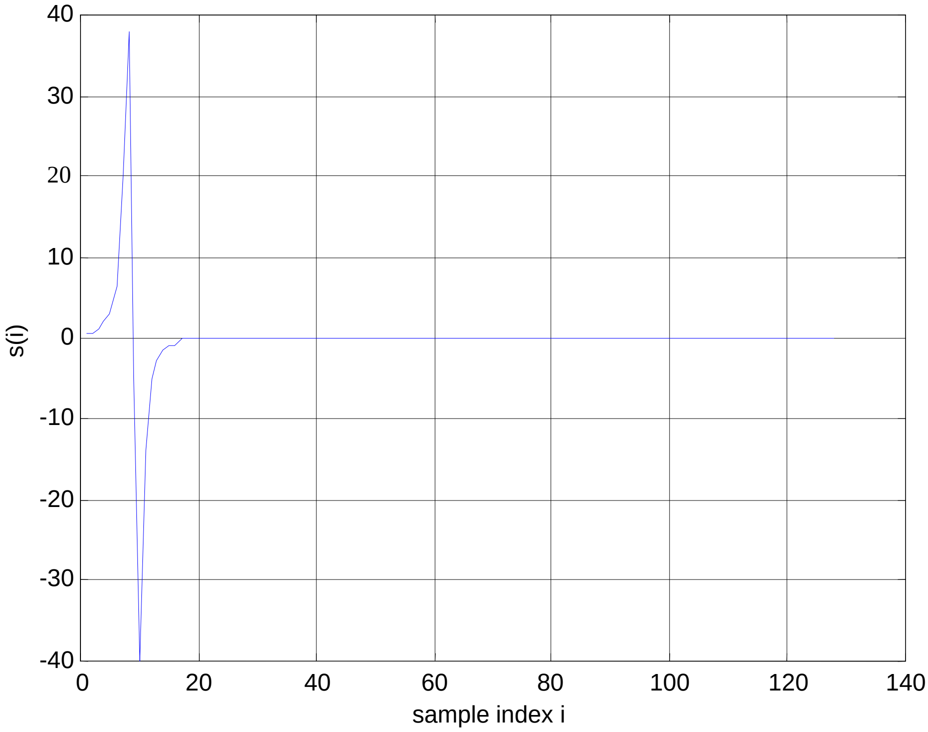

Figure 4. Transducer output signal

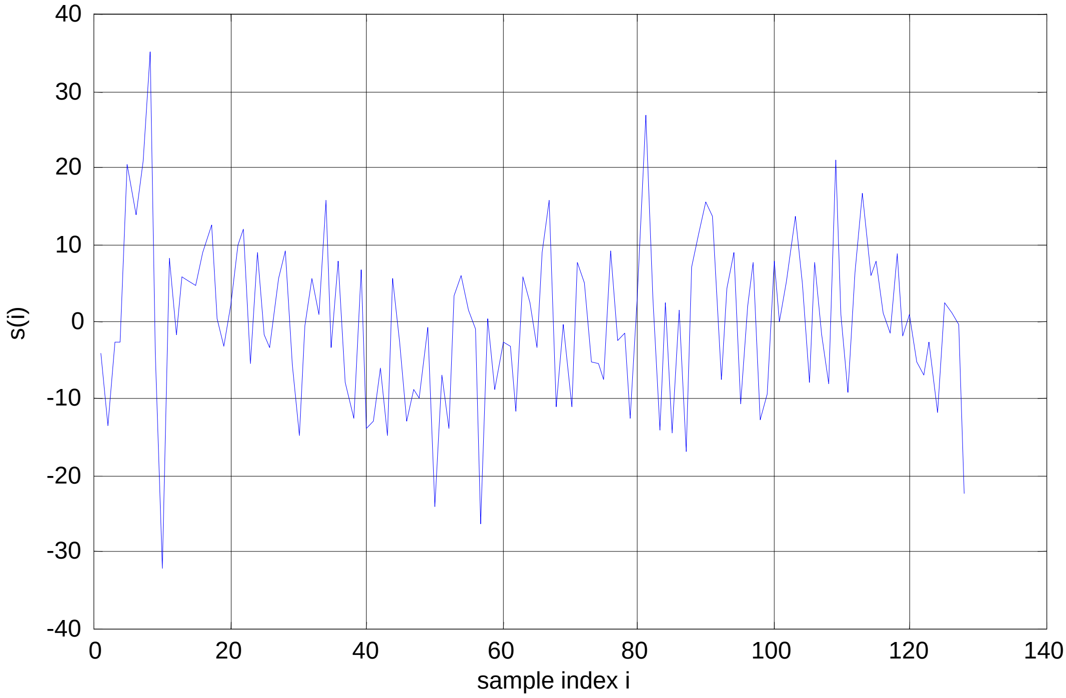

Figure 5.

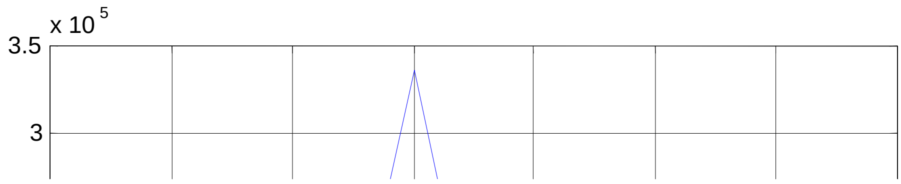

Figure 6. Squared Haar coefficient vs. Haar functions group index curve for signal given in Fig. 5

5 CONCLUSIONS

A combined method of spectral analysis, using traditional FFT and some specific orthogonal transform (Haar transform) was proposed in the contribution. It can be used if the analyzed signal takes nonzero values only within a certain subinterval of the analyzed period. If the signal position does not correspond exactly to particular Haar functions, the information about signal shift should be derived from two neighbouring Haar coefficients. This problem should be probably analyzed separately.

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