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VECTOR MEASUREMENTS BY A DIGITAL OSCILLOSCOPE

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Abstract: Vector measurement, that is, the determination of the magnitude and phase of a complex quantity, is usually associated with rather sophisticated measurement equipment. The paper shows that a straightforward and cost-effective implementation can be based on a two-channel digital oscilloscope, which acquires data for a simple signal processing algorithm. An analysis of measurement uncertainty indicates that the main limit of the approach is given by the signal-to-noise ratio in the two measurement channels. Experimental results from an application in the telecommunications field are presented.
Keywords: vector measurement, digital oscilloscope, signal processing

1 INTRODUCTION

A vector measurement allows the determination of the two components of a complex quantity, usually given in terms of magnitude and phase. Several quantities of practical interest are defined in terms of a complex variable: in the electronics and telecommunications fields, impedances, reflection coefficients and frequency responses are among the most common examples. To perform a vector measurement an instrument must obviously possess two input channels, whose set-ups are co-ordinated to optimise the measurement task. Specialised instruments have been produced for this purpose, e.g., impedance meters and vector voltmeters, but the principle is also applicable using general purpose instruments, such as oscilloscopes. Digitising oscilloscopes are particularly well-suited, since the acquisition of one record of signal samples for each input channel can be followed by digital signal processing algorithms, which directly provide the required vector quantity. This paper analyses the performances of digital oscilloscopes in vector measurements, presents the appropriate signal processing algorithms and discusses the experimental results obtained in two application cases.

2 VECTOR MEASUREMENT

Vector measurements are usually related to frequency-dependent quantities, whose magnitude and phase are determined at some given frequency. It can therefore be assumed that sinusoidal waveforms are involved, and that a two-channel digital oscilloscope acquires the data sequences $x_1(nT)$ and $x_2(nT)$, having finite length NT and defined as:

$$x_1(nT) = A_1 sin(2pf_0 nT + j_1) + e_1(nT); \quad x_2(nT) = A_2 sin(2pf_0 nT + j_2) + e_2(nT) \qquad (1)$$

where T is the sampling interval and n = 0, …, N-1. The two quantities e_1 and e_2 represent additional contributions due to quantisation and noise, and can be considered as mutually incorrelated white noise stationary random processes, having finite variances s_1^2 and s_2^2, respectively.

The aim of a vector measurement is the determination of the two quantities $A = A_2/A_1$ and $F = j_2 - j_1$ at the frequency f_0. The signal processing algorithm proposed in this paper is very straightforward, requiring just the calculation of the Discrete Fourier Transforms (DFTs) of the two data sequences. The Fourier coefficient having the maximum magnitude is taken from each transform, the complex ratio of the two providing the desired vector quantity. The main features of this algorithm will now be analysed, and in so doing it will be made clear why such a simple processing algorithm is usually also the most appropriate for this kind of measurement.

2.1 Signal processing algorithm

Whereas in the frequency domain a sinusoidal voltage corresponds to a single frequency component with its image, it is well known that the DFT coefficients of the two finite sequences (1) may all have non-zero values, an effect called spectral leakage [1]. The two transforms can be written as follows:

$$X_1(kF) = \frac{A_1}{2} \cdot e^{jj_1} \cdot W_R(kF - f_0) + \frac{A_1}{2} \cdot e^{-jj_1} \cdot W_R(kF + f_0) + E_1(kF) \qquad (2)$$

$$X_2(kF) = \frac{A_2}{2} \cdot e^{jj_2} \cdot W_R(kF - f_0) + \frac{A_2}{2} \cdot e^{-jj_2} \cdot W_R(kF + f_0) + E_2(kF) \qquad (3)$$

where k = 0, …, N-1 and F = 1/NT. Equations (2) and (3) are both written as sums of three terms: the first two are associated with the sinusoid frequency and its image; $E_1(kF)$ and, respectively, $E_2(kF)$ are the DFTs of e_1 and e_2. The complex function $W_R(.)$, which is non-zero for any value of the index k, unless f_0NT is an integer, is called the Dirichlet kernel and accounts for leakage. Spectral leakage makes the estimate of a single sinusoid inaccurate for two reasons: the interference between the contribution of the signal component with frequency f_0 and that of its image, and the fact that a single DFT coefficient cannot provide a direct estimate of the waveform parameters, because the multiplication by $W_R(.)$ alters both amplitude (scalloping loss) and phase [1].

Since the magnitude of $W_R(.)$ decreases with frequency, spectral interference becomes negligible if the frequency separation between the two components is high enough. For a single sinusoid, this condition is satisfied when f_0 > K/NT, with K ≅ 20, which simply requires the adjustment of the digital oscilloscope timebase setting so that the acquisition interval contains at least K signal periods. The coefficients that correspond to the peak magnitude of the two DFTs can then be expressed in a simplified form. Let k_0 be the peak index (the same index is obtained for both transforms, since the signal frequency is the same); one has:

$$X_1(k_0F) = \frac{A_1}{2} \cdot e^{jj_1} \cdot W_R(k_0F - f_0) + E_1(k_0F) \qquad (4)$$

$$X_2(k_0F) = \frac{A_2}{2} \cdot e^{jj_2} \cdot W_R(k_0F - f_0) + E_2(k_0F) \qquad (5)$$

The amplitude and phase of the two sinusoids can be estimated by the expressions:

$$\hat{A} = \frac{2\left|X_i(k_0F)\right|}{W_R(0)} \qquad \hat{j}_i = \arg\left[X_i(k_0F)\right] \qquad i = 1,2 \qquad (6)$$

Taken singly, such estimates are not very accurate, and could be enhanced by introducing further processing steps at this stage; this is unnecessary for a vector measurement, since scalloping loss and phase effects due to leakage are cancelled out when the required complex ratio is computed.

The most important factor affecting measurement accuracy is broad-band noise; in this respect, the best performance is obtained when samples in the data record from which the DFT is computed are weighted uniformly. In fact, the terms $E_1(k_0F)$ and $E_2(k_0F)$ are proportional to the white noise variances s_1^2 and s_2^2; weighting by any of the window types normally employed for spectral analysis would increase their values by a factor equal to the equivalent noise bandwidth [1] of the window, which turns out to be always greater than 1. Therefore, the most appropriate processing algorithm is the straightforward calculation of the DFT coefficients. The vector measurement is obtained simply as:

$$\hat{A} = \frac{\hat{A}_2}{\hat{A}_1} = \frac{\left|X_2(k_0F)\right|}{\left|X_1(k_0F)\right|} \qquad (7)$$

$$\hat{j} = \hat{j}_2 - \hat{j}_1 = arg\left[X_2(k_0F)\right] - arg\left[X_1(k_0F)\right] \qquad (8)$$

2.2 Statistical analysis

Performances in this approach are essentially determined by the signal-to-noise ratios in the two channels, SNR_1 = $X_1^2/2s_1^2$, SNR_2 = $X_2^2/2s_2^2$. Even assuming no electrical noise is present at all, the effect of quantisation is to introduce an upper bound which, in the common case of an 8-bit analogueto-digital converter, is 49 dB for a full-range sinewave. This limits the range of values that can be measured with an acceptable degree of accuracy: it is useful, therefore, to assess performances by a statistical analysis. It should be remembered, however, that the analysis based on a noise model cannot account for such factors as possible mismatches between the two oscilloscope channels, whose effects should be removed by a suitable calibration.

Cramér-Rao lower bounds of the amplitude and phase estimators in equations (6) can be given in terms of standard deviations as follows [2]:

$$\frac{StdDev\left[\hat{A}_i\right]}{A_i} = \frac{1}{\sqrt{N \cdot SNR_i}} \qquad StdDev\left[\hat{\varphi}_i\right] = \frac{2}{\sqrt{N \cdot SNR_i}} \qquad i = 1,2 \tag{9}$$

where the quantity for amplitude is expressed in relative form. If all potential causes for uncertainty can be modelled by the additive white noise processes introduced in equation (1), these expressions provide an indication of the best achievable uncertainty.

Assuming the digital oscilloscope employed for measurement has a record lenght N = 500, and $SNR_1 = SNR_2 = 40$ dB, from (9) one has approximately 0.05% relative standard deviation for the amplitude estimate and 0.05 degrees for the phase estimate. Recommended methods of analysis [3] can be applied to express the uncertainty of the quantities given in equations (7) and (8): these require the calculation of the combined standard deviations of the measured quantities, and the multiplication by a suitable coverage factor to obtain an uncertainty interval. Taking three times the combined standard deviation, it follows that the amplitude of the vector quantity could be measured with a ±0.2% uncertainty, while phase uncertainty would be ±0.2 degrees.

These values are best-case results for several reasons. Vector measurements are often concerned with attenuation or amplification, in which cases it cannot be assumed that the same *SNR* value applies to both oscilloscope channels. Furthermore, the two channels could be operating with different vertical gain settings, which may correspond to different noise levels; while 40-45 dB is a common range of values for signal-to-noise ratio in a digital oscilloscope, high-sensitivity settings may contribute a higher amount of noise. If SNR_1 differs from SNR_2, the worse of the two obviously has greater weight in determining uncertainty.

3 IMPLEMENTATION AND EXPERIMENTAL RESULTS

The vector measurement approach has been tested in a number of different applications, the simpler being frequency response measurements. The results presented in this paper refer to an application in the telecommunications field: the conformance test of terminal equipment for ISDN primary rate access [4]. The complete test suite comprises over twenty different tests and is implemented by a complex automatic test system. The experimental analysis refers to the determination, over a given frequency range, of the return loss and the impedance towards ground of the interface of the item under test (IUT).

The vector measurement procedure described in this paper has been implemented by a Hewlett-Packard HP 3325B signal generator and a HP 54510B digital oscilloscope, already present in the test system; the proposed algorithm, written in HP-VEE, is executed on the workstation that controls system instruments via an IEEE-488 bus. The approach represents an alternative to the standard procedure, where a Hewlett-Packard HP 3588A scanning spectrum analyser with a tracking generator, also part of the test system, is employed to obtain selective measurements over a broad frequency range. It should be noticed that the proposed algorithm also guarantees frequency selectivity, since a single Fourier coefficient is picked from each signal transform to compute the complex ratio.

3.1 Return loss measurement

The purpose of this test is to verify the adaptation of the IUT receiving input to the line impedance, according to the concept measurement scheme given in Fig. 1. Ideally, return loss is determined as the ratio of the reflected voltage, measured by the voltmeter, to the incident voltage provided by the generator. In practice, to account for non-idealities of the measurement circuit, it is determined instead from two measurements of reflected voltage, one with the IUT connected to the measurement system, the other with no terminal, in which case the resulting open circuit causes total reflection. Provided the generator output remains stable, the ratio of the two measurements yields the return loss.

In the test implementation the reflected voltage was measured by one of the input channels of the digital oscilloscope; the second channel was employed to measure the generator output voltage, so that its amplitude could be monitored. Test specifications [4] indicate that the IUT should achieve a return loss of at least 18 dB between 100 kHz and 2048 kHz, 12 dB below 100 kHz, and 14 dB above 2048 kHz. It is important, therefore, to determine measurement accuracy in these conditions. For this reason reference devices having return losses of 12 dB, 14 dB and 18 dB (± 0.03 dB) are included in the automatic test system "calibration kit", being routinely employed to check its calibration. To assess the performances of the proposed algorithm, these devices were connected in place of the IUT and measurements of their return loss over the relevant frequency ranges were taken. Measurements were then repeated using the automatic test system standard procedure, to obtain data for a comparison.

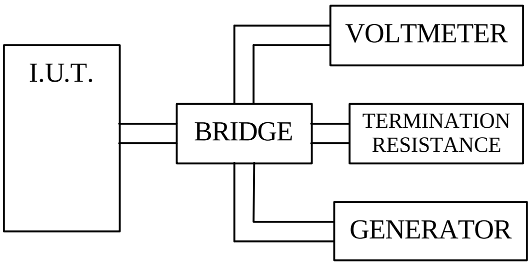

Figure 1. Block diagram of the return loss measurement [4].

Experimental results were obtained as part of an interlaboratory comparison organised within the research project “Generic ATE Uncertainty Characterisation, Calibration and Harmonisation Initiative” (GAUCCHI), sponsored by the European Union [5]. During the round robin, tests were carried out in three different laboratories which make use of the same type of automatic test system. The plots in Fig. 2 present the measurements of the 12 dB, 18 dB and 14 dB reference loads obtained in one of the three laboratories; dotted lines refer to the spectrum analyser measurements, while continuous lines refer to the oscilloscope measurements.

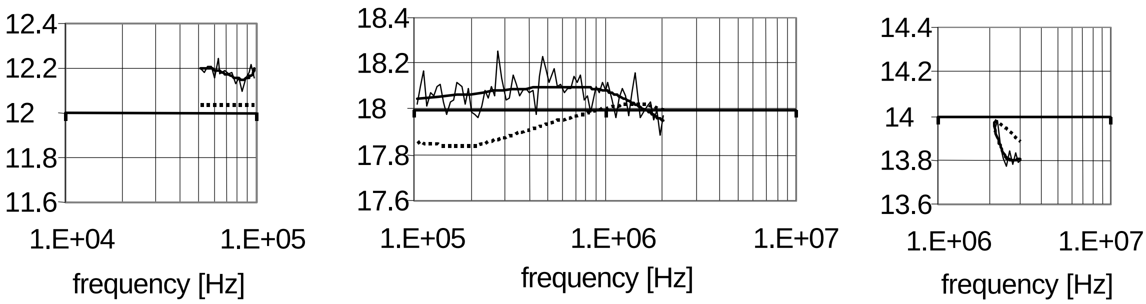

Figure 2. Measurements of reference loads with nominal 12, 18 and 14 dB return losses.

As noted in Section 2, signal-to-noise ratio is the factor that most affects measurement accuracy; however, it must also be considered that return loss is not constant with frequency, but varies slightly within the given frequency range. It is important, therefore, to be able to analyse the two effects separately. For this purpose, polynomial fits of the measured data were calculated, and are indicated by thicker continuous lines in Fig. 2; statistical analysis of the residuals provided the experimental standard deviation of return loss estimates. Its value, expressed in relative terms, is always smaller than 1%, which agrees well with the theoretical analysis.

Uncertainty due to the effect of noise has been calculated from the estimated standard deviation using a 3-σ confidence level: the resulting values, converted into a logarithmic scale indication, show that, for a measured return loss of 18 dB, the contribution to the total uncertainty is comprised between ±0.15 dB and ±0.25 dB, depending on the laboratory. These slight differences can be justified by the fact that measured signal-to-noise ratios were found to vary by a few dB among the digital oscilloscopes of the three automatic test systems. The same analysis showed that measurements performed on the 12 dB and 14 dB references (i.e., below 100 kHz and above 2048 kHz) are within about ±0.1 dB of the mean value. The smaller variability is a direct consequence of the fact that a lower return loss implies stronger reflection and, consequently, a better signal-to-noise ratio in the oscilloscope channel measuring the reflected signal.

The polynomial fits agree with the corresponding spectrum analyser measurements taken in the same laboratory within 0.2 dB, with just one exception; differences may be related to the slightly different measurement set-up employed. The much better dynamic range and lower noise of the spectrum analyser, however, produce measurements with a much reduced statistical variability, as can be seen in Fig. 2. As far as reproducibility of results is concerned, for a measured return loss of 18 dB the polynomial fits obtained in the three laboratories agree to within 0.5 dB or better, while at 12 dB and 14 dB differences do not exceed 0.15 dB. By comparison, spectrum analyser measurements always agree within 0.2 dB or better. In the authors' opinion, however, the fact that reproducibility of results changes between the two methods should be related to the calibration state of some instruments, rather than to any particular feature of the methods themselves.

3.2 Impedance to ground measurement

The set-up for this measurement comprises a generator, that applies a sinusoidal common-mode voltage of 2 Vrms to the IUT port under test, and an instrument that measures the resulting current. Measurements have to be taken at several frequencies between 10 Hz and 1 MHz and the impedance to ground of the IUT is required to be at least 1 kΩ at all frequencies ([4]; this requirement has been modified in the subsequent ETSI document, TBR 4, where a -20 dB/dec roll-off, starting at 500 kHz, is allowed).

The automatic test system employed in this work makes use of a Tektronix P6021 current probe to convert the measured current into a voltage that, in the standard procedure, is measured by the HP 3588A spectrum analyser. To implement the proposed algorithm, the test system has been modified to allow the acquisition of the same signal by the HP 54510B digital oscilloscope, while the second input channel of the oscilloscope measures the generator output voltage. Regardless of the approach, the accuracy of the system at frequencies below 120 Hz is limited by the bandwidth of the current probe, that degrades significantly the overall performances. For this reason, the analysis that follows is limited to frequencies from 200 Hz upwards.

The automatic test systems employed in the round robin were first calibrated by using the reference load included in each system own calibration kit, which provides 1 kΩ impedance to ground. Then, all systems measured the same set of three references, having the values 980 Ω $\pm$ 0.1%, 1000 Ω $\pm$ 0.1%, and 1020 Ω $\pm$ 0.1%, which were purposely realised during the GAUCCHI project.

Measurements of the 1000 Ω impedance, obtained in three different laboratories, are presented in Fig. 3; thicker lines on the impedance plots represent the polynomial fits of results, that were necessary also in this case to help separate the statistical variability of the impedance estimate from frequency-dependent variations. Accuracy is significantly affected by the low level of the signal provided by the current probe, whose nominal factor is 10 mV/mA. In fact, signal-to-noise ratio in the oscilloscope channel that measures current gives the prevailing contribution to uncertainty.

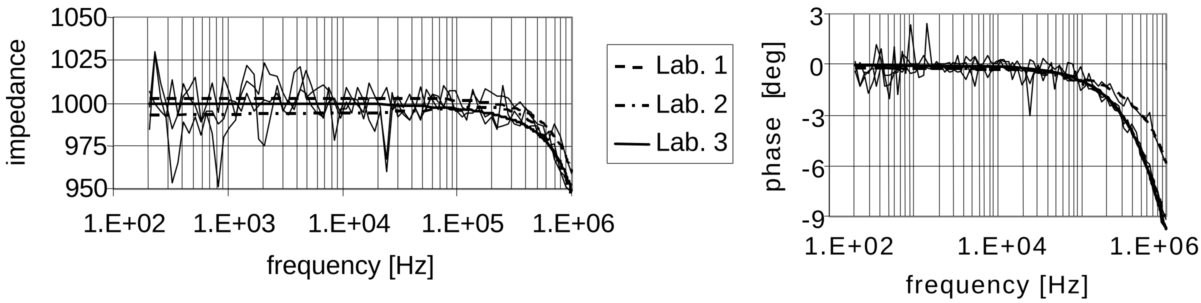

Figure 3. Vector measurements of impedance towards ground for a nominal value of 1000 Ω.

Experimental results are summarised in Table 1, where nominal and measured values of the impedance towards ground are reported for each of the three laboratories taking part in the round robin. Experimental standard deviations, obtained by statistical analysis of the residuals of the polynomial fits, are also given in the table. Standard deviation of the impedance is between 0.5% and 2% of the measured value; for the phase estimate, standard deviation is between 0.3 and 0.7 degrees. These values are, again, in very good agreement with the theoretical analysis presented in Section 2; variations in the standard deviations obtained by the three laboratories are explained, as in Section 3.1, by the different values of signal-to-noise ratio associated with the three digital oscilloscopes.

One way to further reduce variability is the use of waveform averaging during signal acquisition. In an automatic test system the advantages of this technique have to be carefully balanced against the increase in measurement time caused by repeated acquisitions. In fact, the increase in measurement time is approximately proportional to the number of averages, while standard deviation is reduced by the square root of that number. To find an acceptable compromise, repeated measurements were made at a single frequency, their mean and standard deviation being plotted as functions of the number of repetitions. The resulting graphs evidenced that a satisfactory reduction of the standard deviation had been achieved after averaging 10 waveforms. As shown in Table 1 for the 1000 Ω impedance, a reduction by a factor of 2 has been obtained at all laboratories.

Measurements by the automatic test system standard procedure showed that, with a single exception, the impedance value estimated by the polynomial fit agrees with the spectrum analyser measurement to less than 5 Ω. It should be noticed that the proposed algorithm also yields a phase estimate, which the spectrum analyser cannot provide.

Table 1. Impedance towards ground of three reference loads: experimental results.

	Laboratory n.1			**Laboratory n.2**			**Laboratory n.3**		
Nominal value [Ω]	980	1000	1020	980	1000	1020	980	1000	1020
Measured impedance [Ω]	987	1003	1023	967	994	1011	978	1000	1018
Standard deviation [Ω]	9,9	10.1	12	19.3	15.3	14.8	5.3	5.4	4.7
Measured phase [deg]		-0.20			-0.06			-0.22	
Standard deviation [deg]		0.6			0.7			0.3	
Measured impedance (avg) [Ω]		1005			997			1001	
Standard deviation (avg) [Ω]		5,7			8,4			2,3	

4 CONCLUSIONS

The purpose of this work was to analyse the feasibility of measuring vector quantities by comparatively simple and low-cost instrumentation, obtaining an acceptable degree of accuracy. Within the limits placed by the signal-to noise ratio of the considered signals, the approach based on signal processing by Fourier transformation has been proved to achieve this goal. Results confirm the theoretical predictions of measurement variance, while comparison with a different measurement procedure has shown a good agreement of experimental data.

Summarising, the choice of measurement approach is the result of a trade-off between accuracy and cost. While spectrum analyser accuracy is better, there is no dramatic improvement over the performances obtained by a digital oscilloscope, but equipment cost is approximately one order of magnitude lower in the latter case. It should also be noticed that measurement accuracy of the digital oscilloscope could be improved by averaging techniques, at the cost of a longer measurement time.

A possible limitation of the proposed approach in the application to conformance testing is the comparatively large variability of measurements. It can be easily seen from Fig. 3 that, even when the impedance estimated by a polynomial fit is larger than the 1 kΩ limit value, point-to-point variations easily cross the limit. If a pass/fail verdict is expected from the test, this would be invariably a fail. One way to overcome the problem could be the inclusion in the processing algorithm of a data fitting procedure employing a low-order model: this would allow to still account for variations in the measured quantity, but at the same time would reduce statistical fluctuations. Acceptance of such a procedure by laboratory accreditation bodies remains, however, open to question.

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