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MODIFIED VOLTERRA SERIES FOR NONLINEAR SYSTEM CHARACTERISATION

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Abstract: This paper describes different solutions, all based on a modified Volterra series, for the characterisation of nonlinear dynamic systems.
Keywords: Volterra series, nonlinear system, system characterisation

1 INTRODUCTION

In recent years there has been an increasing growth in the study and modelling of nonlinear systems with memory because they are largely used in modern electronic engineering. The Volterra series representation of a nonlinear system with memory was amongst the first to be postulated and it is also been one of the most widely applied [1,2,3,4]. The limitations of the Volterra series are well documented and they include the inability to model hysteresis [5]. This approach introduces kernels to which are associated one-dimensional as well as multidimensional transfer functions of different orders. However the measurement of higher-order kernels is an obstacle to efficient system modelling [6,7,8,9,10]. The authors introduced previously a modified Volterra series based on the dynamic deviations of the input signal with respect to its value assumed in the instant in which the output is evaluated [11]; the modified Volterra series is characterised by different convergence properties with respect to the conventional Volterra series [12]. This paper compares the different solutions, based on the classical Volterra series or on a Volterra-like approach, in order to reduce the number of kernels which must be identified for an efficient modelling of a nonlinear system with memory.

2 DIFFERENT REPRESENTATIONS OF THE NONLINEAR SYSTEMS THROUGH A VOLTERRA-LIKE APPROACH

The input-output behaviour of a nonlinear system with memory can be described through the Volterra series as follows [2]:

$$u(t) = y_0 + \sum_{r=1}^{+\infty} \frac{y_r(t)}{r!} \tag{1}$$

where $u(t)$ is the output signal, y_0 the offset and:

$$y_r(t) = \int ..\overset{+\infty}{\underset{-\infty}{}}..\int h_r(t_1, t_2, ..., t_r)\left[\prod_{i=1}^{r} s(t - t_i)\mathrm{d}t_i\right] \tag{2}$$

the multidimensional convolution integral of this series, $h_r(t_1, t_2, ..., t_r)$ being the r^{th}-order Volterra kernel. By separating the contribution of the first order term and by pointing out the remainder Δ_m of this series truncated to the m^{th}-order term we can write:

$$u(t) = y_0 + \int_{-\infty}^{+\infty} h_1(t_1)s(t - t_1)\mathrm{d}t_1 + \sum_{r=2}^{+m} \frac{1}{r!}\int ..\overset{+\infty}{\underset{-\infty}{}}..\int h_r(t_1, t_2, ..., t_r)\prod_{i=1}^{r} s(t - t_i)\mathrm{d}t_i + \Delta_m \tag{3}$$

Therefore the Volterra series can be interpreted as an extension to the non-linear operation (for $r \geq 2$) of the well-known linear convolution integral which describes the time-domain response of a linear time-invariant dynamic system; in order to take into account also the non-linear dynamic effects, at least the second order kernel must be taken into account. Equation 3 suggests that a non-linear dynamic system can be described, through the Volterra series expansion, as the sum of two blocks which take into account respectively the linear and non-linear effects (fig. 1). The linear system with memory and the non-linear system with memory are characterised respectively by the following equations:

$$u_L(t) = y_0 + \int_{-\infty}^{+\infty} h_1(t_1) s(t - t_1) \mathrm{d}t_1 \tag{4}$$

$$u_{NL}(t) = \sum_{r=2}^{+m} \frac{1}{r!} \int .. \int_{-\infty}^{+\infty} .. \int h_r(t_1, t_2, ..., t_r) \prod_{i=1}^{r} s(t - t_i) \mathrm{d}t_i + \Delta_m \tag{5}$$

The remainder depends on the product of the non-linear effects amplitude and of the input signal maximum amplitude [12]. Since it is very difficult to identify kernels of an order greater than 3 or 4, the Volterra series can be practically used only for weakly non-linear systems, both with long or short memory [7,8,9,10].

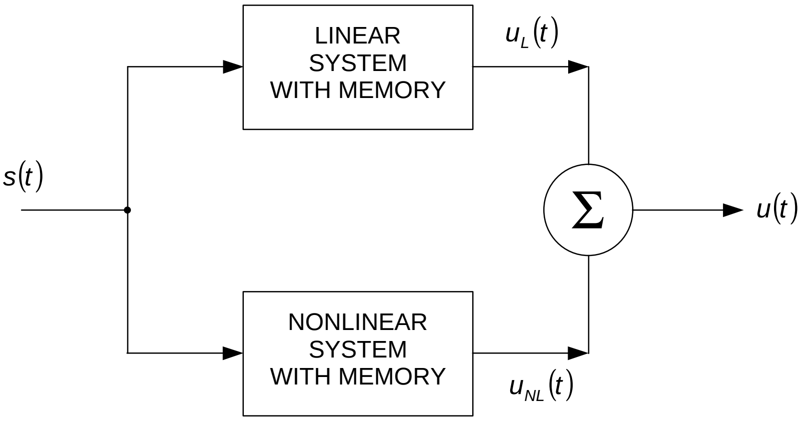

Figure 1. Non-linear dynamic system described, through the Volterra series expansion, as the sum of two blocks which take into account respectively the linear and non-linear effects.

By considering the *dynamic deviation* $e(t,t)$

$$e(t,t) = s(t - t) - s(t) \tag{6}$$

where $e(t,0) = 0$, which represents the deviation of the input signal $s(t - t)$ with respect to $s(t)$, i.e. to the value assumed by the input at the instant *t* in which the output is evaluated, the output signal can be expressed through the following dynamic-deviation-based Volterra-like series [11]:

$$u(t) = z_0\{s(t)\} + \sum_{r=1}^{+\infty} \frac{z_r(t)}{r!} \tag{7}$$

where $z_0\{s(t)\}$ represents the static characteristic of the system and $z_r(t)$ the multiple convolution integral with respect to the dynamic deviations:

$$z_r(t) = \int ... \int_{-\infty}^{+\infty} ... \int g_r\{s(t), t_1, ..., t_r\} \prod_{i=1}^{r} e(t, t_i) \mathrm{d}t_i \quad \text{with } r \geq 1 \tag{8}$$

being the algebraic function $g_r\{\}$ with respect both to the punctiform value $s(t)$ and the shifts $t_1,..t_r$ called the r^{th}-dimensional kernel of the dynamic-deviation-based Volterra-like series. The static characteristic can be expressed as a power series in the variable $s(t)$:

$$z_0\{s(t)\} = y_0 + \sum_{r=1}^{+\infty} \frac{a_r}{r!} s^r(t) \tag{9}$$

where each coefficient of the series coincides with the multiple integral of the corresponding Volterra kernel of the same dimension:

$$a_r = \int .. \int_{-\infty}^{+\infty} .. \int h_r(t_1 ... t_r) \prod_{i=1}^{r} \mathrm{d}t_i \tag{10}$$

The modified Volterra kernel $g_r\{\}$ can be expressed as a function of the Volterra kernels and of the input signal $s(t)$ as follows:

$$g_r\{s(t),t_1,\dots,t_r\} = h_r(t_1,\dots,t_r) + \sum_{m=1}^{+\infty}\frac{b_{r+m}}{m!}s^m(t) \tag{11}$$

with $r = 1,2,\dots\dots$. Therefore $g_r\{\}$ is the sum of the original Volterra kernel of the same dimension plus a power series in the variable $s(t)$. Each coefficient b_{r+m} of the power series can be expressed as a function of the Volterra kernels as follows:

$$b_{r+m}(t_1,\dots,t_r) = \int..\overset{+\infty}{\underset{-\infty}{}}..\int h_{r+m}(t_1,..,t_r,t_{r+1},..,t_{r+m})\prod_{s=1}^{m}\mathrm{d}t_{r+s} \tag{12}$$

This coefficient is obtained as the original Volterra kernels of higher order integrated with respect to the arguments subsequent to the order of the kernel considered. By comparing eqns.10 and 12 it results:

$$a_{r+m} = \int..\overset{+\infty}{\underset{-\infty}{}}..\int b_{r+m}(t_1,..,t_r,t_{r+1},..,t_{r+m})\mathrm{d}t_1...\mathrm{d}t_r \tag{13}$$

By considering the non-linear dynamic model represented by the modified Volterra series truncated to the first term we can write:

$$u(t) = z_0\{s(t)\} + \int_{-\infty}^{+\infty} g_1\{s(t),t_1\}e(t,t_1)\mathrm{d}t_1 + \Delta\{s(t)\} \tag{14}$$

where (eqn.11 and 12):

$$g_1\{s(t),t_1\} = h(t_1) + \sum_{m=1}^{+\infty}\frac{s^m(t)}{m!}b_{1+m}(t_1) \tag{15}$$

$$b_{1+m}(t_1) = \int..\overset{+\infty}{\underset{-\infty}{}}..\int h_{1+m}(t_1,..,t_{1+m})\mathrm{d}t_2...\mathrm{d}t_{1+m} \tag{16}$$

Due to the superior order products of the dynamic deviations, it can be shown that the remainder $\Delta\{s(t)\}$ is negligible only if the considered non-linear system has a finite memory $T_A + T_B$ relatively short with respect to the signal period for a given shape-factor [11]. In this hypothesis the response of the non-linear system can be adequately approximated by the sum of the static characteristic plus the non-linearly controlled convolution integral of the dynamic deviation of the signal in the short memory interval weighted by the first-order kernel $g_1\{\}$ of the modified Volterra series, which is a non-linear function of the reference input signal.

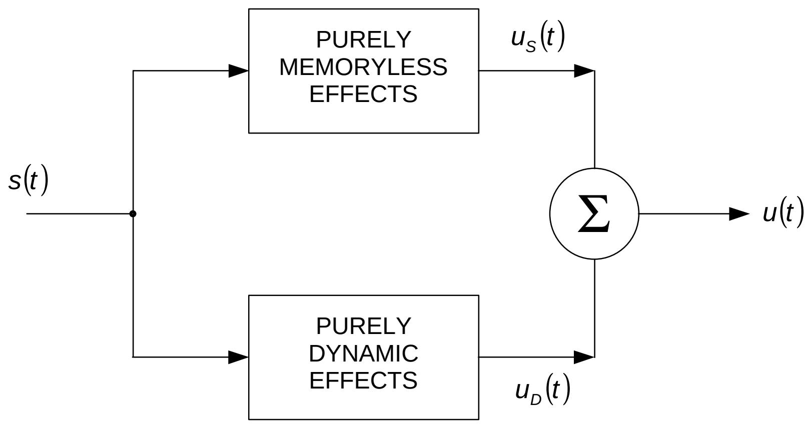

Figure 2. Non-linear dynamic system described as the sum of two blocks which take into account respectively the memoryless and the purely dynamic effects.

A non-linear dynamic system can therefore be described, through the dynamic deviation-based Volterra-like series, as the sum of two blocks which take into account respectively the memoryless and the dynamic effects (fig.2). The purely memoryless system and the purely dynamic system are characterised respectively by the following equations:

$$u_S(t) = y_0 + \sum_{r=1}^{+\infty}\frac{a_r}{r!}s^r(t) \tag{17}$$

$$u_D(t) = \int_{-T_A}^{+T_B} g_1\{s(t), t_1\} e(t, t_1) dt_1 + \Delta\{s(t)\} \quad (18)$$

It is important to emphasise that, in order to make negligible the remainder, the purely dynamic block must be characterised by relatively short memory effects with respect to the signal period apart from the signal amplitude for a given shape factor.

By substituting eqn.15 into eqn.14 and separating the contribution of the first order Volterra kernel $h_1(t_1)$, we can write:

$$u(t) = y_0 + \int_{-\infty}^{+\infty} h_1(t_1) s(t - t_1) \mathrm{d}t_1 + \sum_{r=2}^{+\infty} a_r \frac{s^r(t)}{r!} + \sum_{m=1}^{+\infty} \frac{s^m(t)}{m!} \int_{-\infty}^{+\infty} b_{1+m}(t_1) e(t, t_1) \mathrm{d}t_1 + \Delta\{s(t)\} \quad (19)$$

where the remainder of this new modified Volterra series $\Delta\{s(t)\}$ coincides with that of eqn.14. The response of a non-linear system with memory is now expressed by the sum of the following contributions:

- the offset y_0,
- the linear memory effects expressed through the linear convolution integral with respect to the signal,
- the non-linear static effects,
- the first order purely non-linear memory effects expressed through the non-linear convolution integral with respect to the dynamic deviations,
- the remainder which takes into account the superior order non-linear memory effects.

Therefore a non-linear dynamic system can be described as the sum of three blocks which take into account separately of the linear memory effects, of the purely non-linear respectively memoryless and dynamic effects (fig.3). The linear system with memory, the purely non-linear memoryless system, and the purely non-linear with finite memory system are described by the following equations:

$$u_L = y_0 + \int_{-\infty}^{+\infty} h_1(t_1) s(t - t_1) \mathrm{d}t_1 \quad (20)$$

$$u_{SNL} = \sum_{r=2}^{+\infty} \frac{s^r(t)}{r!} a_r \quad (21)$$

$$u_{DNL}(t) = \sum_{m=1}^{+\infty} \frac{s^m(t)}{m!} \int_{-T_A}^{+T_B} b_{1+m}(t_1) e(t, t_1) \mathrm{d}t_1 + \Delta\{s(t)\} \quad (22)$$

By comparing eqn.17 with eqn.1 we observe that the Volterra series cumulates all the non- linear effects, both static and purely dynamic; we must emphasise that the remainder $\Delta\{s(t)\}$ in eqn.17 depends on the behaviour of the dynamic deviations within the memory interval, while the remainder Δ_M in eqn.1 depends on the behaviour of the entire signal within the same interval. Therefore Volterra series can be used uniquely for weakly non-linear system, independently from the duration of the memory effects; instead, in the case of short-memory non-linear effects, eqn.17 allows to discover the

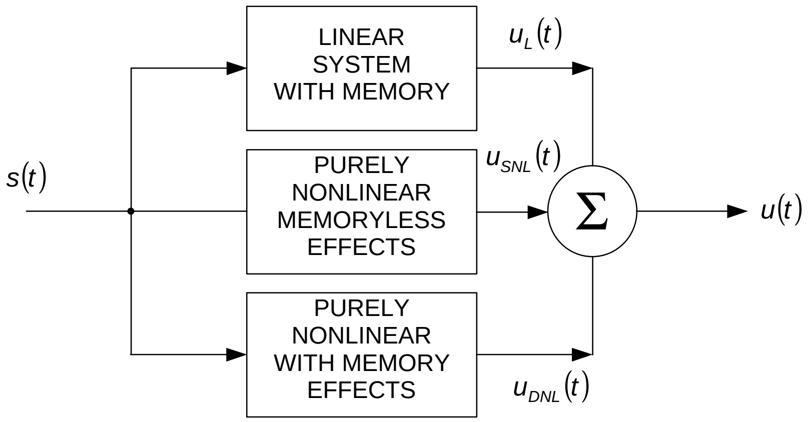

Figure 3. Non-linear dynamic system described as the sum of three blocks which take into account separately of the linear memory effects and respectively of the purely non-linear memoryless and dynamic effects.

corresponding different contributions taking into account only the first order purely non-linear memory effects. By comparing instead eqn.17 with eqn.14, we observe that eqn.14 cumulates the linear and nonlinear memory effects while eqn.17 separates the two contributions; therefore in this last case the short memory effects regard uniquely the purely non-linear ones. If the entire system is characterised by great linear memory and by finite nonlinear memory effects, this solution allows to take into account only the first order non-linear kernel in order to characterise the non-linear dynamic effects.

A different solution can be obtained by partitioning the nonlinear block with memory into the cascade of a linear block with memory and a nonlinear one with shorter, residual memory. To this end we can impose that the behaviour of the nonlinear block is memoryless in small signal operation and zero bias [13]. In this hypothesis, according to fig.3, the nonlinear block can be represented as the sum of two blocks, one takes into account the purely static effects and the other the purely nonlinear dynamic effects; previously in fact it was shown that all these effects can be considered separately (eqn.19). This solution is represented in fig.4 as the cascade of a linear block with memory and the sum of the purely static and nonlinear dynamic blocks.

By indicating with s_O the output of the linear block with memory of fig.4, the different blocks of this figure can be described by the following equations:

$$s_O = \int_{-\infty}^{+\infty} h(x)s(t-x)\mathrm{d}x \tag{23}$$

$$u_S(t) = y_0 + \sum_{r=1}^{+\infty} a_r s_O^r(t) \tag{24}$$

$$u_{DNL}(t) = \sum_{m=1}^{+\infty} \frac{s_O^m(t)}{m!} \int_{-T_A}^{+T_B} b_{1+m}(t_1) e_O(t,t_1)\mathrm{d}t_1 + \Delta\{s(t)\} \tag{25}$$

where:

$$e_O(t,t_1) = s_O(t-t_1) - s_O(t) \tag{26}$$

and b_{1+m} can be deduced from eqn.16. The remainder in eqn.25 can be disregarded if the nonlinear memory effects are sufficiently short with respect to the period of the signal.

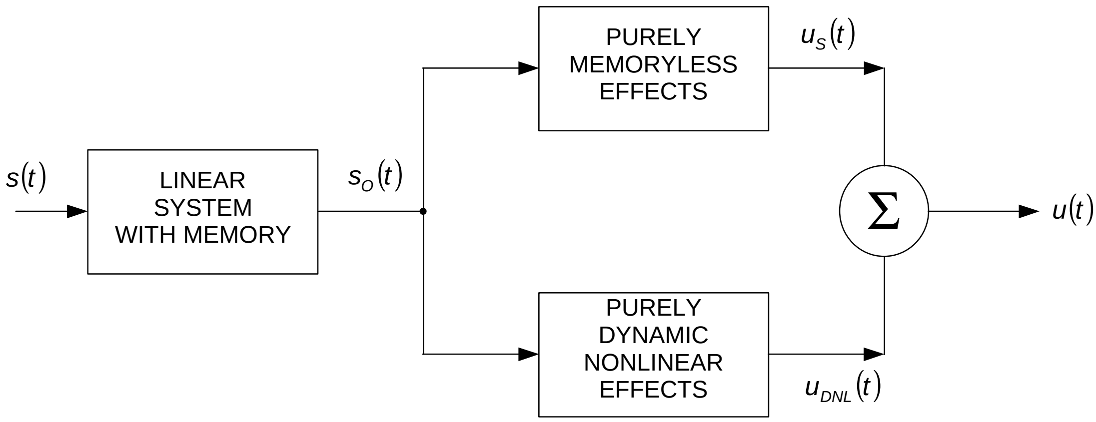

Figure 4. Nonlinear dynamic system described as the cascade of a linear block with memory and the sum of a nonlinear memoryless effects and the purely dynamic nonlinear effects.

3 CONCLUSIONS

The Volterra series, which is in principle capable of describing a very large class of non-linear systems with memory, can be practically used only for mildly non-linear systems (or, equivalently, for limited signal amplitude in strongly non linear systems), i.e. when the kernels of higher order (typically greater than 3) can be neglected. The dynamic-deviation-based Volterra-like series can be instead truncated to the first order kernel when the nonlinear memory effects are sufficiently short with respect to the period of the signal; this happens, for example, when the non-linear phenomena are strong while the non-linear memory effects are quite limited. To utilise this truncated series also in presence of relevant linear memory effects, it is convenient to separate the linear and nonlinear memory effects;

two different models are proposed which can be used when this condition is satisfied. In this way it is possible to select the model which is more convenient for any particular application.

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