

OPTIMAL SAMPLING AND PROCESSING OF NOISY SIGNALS

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Abstract: This paper deals with Digital Signal Processing techniques of noisy current signals from electric motors in order to form reasonably reliable energy estimates applied to the plant control of rubber mixing room processes. The algorithms proposed are characterized with a good result rate. They are implemented in practice and can be successfully used for technological experiments of new products.

Keywords: noisy signals, measurement of electrical quantities, optimal sampling

1 BASIC TERMINOLOGY AND POSTULATION

The characteristics of many objects are represented in model-based measurements by random functions, depending on time.

Besides, the informative parameters, which are used for all description of these characteristics, on the other hand, can be dependent on time, too.

Hereof follows the main properties of the stochastic signals:

- stationarity / non-stationarity
- ergodicity / non-ergodicity

A stochastic signal $x(t)$ can be called stationary if every its probability distribution keeps one and the same character while replacing the parameter t with $t+t_0$, as follows:

$$W_{(n)}(x_1, t_1; x_2, t_2; \dots; x_n, t_n) \equiv W_{(n)}(x_1, t_1 + t_0; x_2, t_2 + t_0; \dots; x_n, t_n + t_0), \quad (1)$$

when $t_0 \in (-\infty, +\infty)$, for any $n = 1, 2, \dots$

where

W - is the n -order Probability Density Function (PDF),
depending on $n-1$ finite differences,
 t_1, t_2, \dots, t_n - is a set of separability of the signal $x(t)$.

A stationary stochastic signal $x(t)$ will be also of ergodic type if for every measurable function $f[x(t_1), \dots, x(t_n)]$, the probability that its time average value, expressed as

$$\langle f \rangle = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^{+T} f[x(t_1+t), \dots, x(t_n+t)] dt \quad (2)$$

equals the observation ensemble average value

$$\mathbf{M}[f] = \int_{-\infty}^{+\infty} \dots \int_{-\infty}^{+\infty} f(x_1, \dots, x_n) dW_{(n)}(x_1, t_1; \dots; x_n, t_n) \quad (3)$$

within a given confidence interval, i.e.

$$P\{\langle f \rangle = \mathbf{M}[f]\} = 1. \quad (4)$$

In other words, every stochastic signal, in terms of the above considerations can be named ergodic if its statistical characteristics, found by making an observation ensemble average coincide with the time average of data obtained with single observation scheme. However, in order to achieve a certain measurement convenience, the experimental data must fit in with a presumed hypothesis of ergodicity, although, in most cases it can be one only possibility for determination of stochastic signal characteristics, because of impossibility to carry out the observations again.

In this paper, the methods of determination of main statistical parameters and characteristics of stochastic signals are examined. The knowledge of such characteristics is of high importance to choose an optimal sampling period, minimizing both the error dispersion, as well as the amount of measurement information.

2 DATA ACQUISITION PROCESSES IN A RUBBER MIXING ROOM SYSTEM

The batch production of tires requires rubber mixtures, produced by strictly defined recipes. The mixture, in industrial conditions, is executed in mixing rooms using pressing pistons, moved by electromotors. The topology, operations and performance of a rubber mixing system was presented in reference [5].

In the designed μP - based system there are provided analog input channels with corresponding instrument transducers. The connecting cables are with a length about 80-100 m, but their characteristic impedance Z fits in with the input impedance of the normalization electronic devices, i.e.:

$$\dot{Z}(w) = \sqrt{\frac{R_o + jwL_o}{G_o + jwC_o}} \approx \dot{Z}_{in}(w), \quad (5)$$

$$g(w) = \sqrt{(R_o + jwL_o)(G_o + jwC_o)} = a(w) + jb(w) \quad (6)$$

where

- ω - is the modulation frequency of the signal,
- R_o - is the real resistance of the channel per unit length,
- L_o , C_o , and G_o - are the inductance, capacitance and conductivity of the channel per unit length,
- γ - is the propagation constant,
- $\alpha(\omega)$ - is the frequency-dependent loss factor,
- $\beta(\omega)$ - is the frequency-dependent phase constant.

The normalized analog signals enter in controller for A/D conversion. Such signals are generated by the following objects:

- scale for oil dosing,
- scale for soot dosing,
- scale for grainy rubber dosing,
- main electrical driver.

All of these signals are obtained in the conditions of powerful industrial disturbances. Because of the fact that the noise filtering depends mainly by the ratio between spectral densities of the signal and the noise (but in our case $T_N \ll T_S$) we are introduce an integral estimate about the data acquisition process.

Such an integration was realized in a digital mode by the equation:

$$\int_0^{kT} f(t)dt = \sum_{k=0}^n A_k \cdot f(t_k) + R \cdot n, \quad (7)$$

where

- t_k - are the measurement moments,
- A_k - are coefficients depending on the method applied for interpolation,
- R - is the integration error, between two neighboring moments depending on the method applied for interpolation.

The noise influence is of an additive type. It can be reported by an integration closely before each stage of the process during a given time interval while the electrical current is under the threshold level (for example, m - steps). Such an integral value can be extrapolated over the full time interval (n - steps). The integral criterion W will not be including the component R , because the both integrations use one and the same method.

$$W = \sum_{k=0}^n A_k \cdot f(t_k) + R \cdot n - \frac{R \cdot m}{m} \cdot n - \frac{S_o}{m} \cdot n. \quad (8)$$

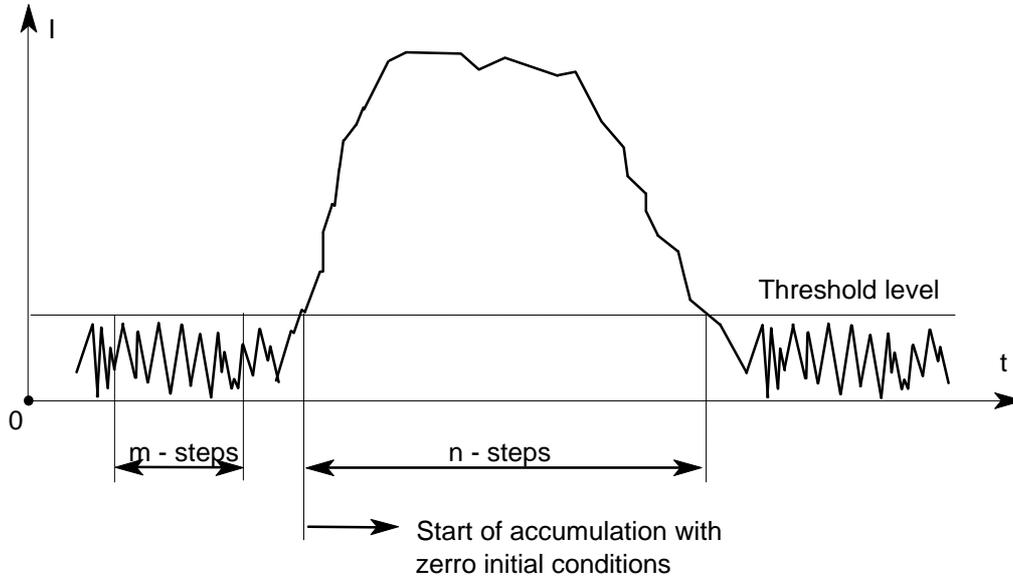


Figure 1. The curve of the main electromotor current corrupted by noises.

Here of follows the conclusion about our experiment design that an interpolation equation of 0 - order (method of the rectangles) also will be giving a good result rate. That is why the use of Simpson, Tchebycheff, or Gauss methods is not justified.

3 OPTIMAL SAMPLING AND FLOATING-POINT QUANTIZATION TECHNIQUES

The classical approach - uniform signal sampling is a broadly developed mathematical tool, but it needs to be further improved. It is important to know that the error of periodical sampling depends on the law of probability distribution of the input signals. The Sampling Theorem does not take account of any stochastic characteristics. It is evident, that the problem of optimal sampling can be successfully solved via an irregular in time sampling and by means of a floating-point quantization. Special attention in the paper is also emphasized to the determination and analysis of the correlation function. Such a function can be either object of investigation or be used as applied method. Usually the analysis and processing are conducted in frequency domain. In the paper the autocorrelation function is analyzed in time domain versus the parameter of delay - τ , as in (9) and a discussion about some pros and cons of such an approach can take place.

$$\text{Cor}_{xx}(t) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T x^*(t) \cdot x^*(t+t) dt \quad (9)$$

There are possible two cases during the data acquisition processes:

- absence of correlation between individual observations,
- availability of correlation between individual observations.

In the first case the variations in each measured value will be function of the errors inherent to the same measurement process. Then, the final result (in our case - the integral criterion - W) will be function of the mathematical expectations - E of the measured arguments, i.e.:

$$W = F\{E[x_1], E[x_2], \dots, E[x_n]\}. \quad (10)$$

In the second case the possible variance of every measured value will be defined not only by the error of the corresponding measurement, but also by the variances of the rest arguments. Then the above mentioned integral criterion will be function, as follows:

$$W = F[E(x_1, x_2, \dots, x_n)]. \quad (11)$$

That is why while digital signal processing the main task is to state whether there is any correlation between consecutive measured values. It is convenient to use the following criterion:

$$\frac{R\sqrt{n}}{1-R^2} < t(P, k), \quad (12)$$

where

$t(P, k)$ - are Student's coefficients with confidence probability - P and k -degrees of freedom ($k=n-1$, and n is the full number of the measured values),
 R - is the correlation coefficient between arguments, and

$$R = \frac{\frac{1}{n} \left[\sum_{i=1}^n x_i^h \cdot x_i^l - \frac{1}{n} \left(\sum_{i=1}^n x_i^h \right) \left(\sum_{i=1}^n x_i^l \right) \right]}{s(x_i^h) s(x_i^l)}, \quad (13)$$

where

x_i^h and x_i^l are values of the h -th and l -th arguments,
 $s(x_i^h)$ and $s(x_i^l)$ are rms estimates of the h -th and l -th arguments.

If equation (12) is fulfilled, then it means that there is no correlation between h -th and l -th argument.

In this connection the investigations are directed to give a possibility for estimation of both random and deterministic components of the signals, and also to find out ways improving the noise immunity of the measuring devices. It can be effectively done by analyzing the graph-plot of $Cor_{xx}(\tau)$, and in particular its amplitude decrement. Besides, while processing data of model-based measurements it is necessary to check up if there is no correlation between individual observations - a fact that is often ignored.

There is also a logical necessity of algorithms adapting both the quantization amplitude and the sampling rate to the stochastic characteristics of the observed signals, in accordance with a given criterion.

This task is solved by some authors, by applying Bayesian estimate (minimal average risk):

$$\mathfrak{R}_{AV} = \int \int_{\Gamma \Theta} Var(x, x_o) \cdot p(x/x_o) \cdot p(x) \cdot dx \cdot dx_o \rightarrow \min, \quad (14)$$

where

$Var(x, x_o)$ - is variance of the measured values x with the real value x_o ,
 $p(x/x_o)$ - is the conditional probability density function for getting x under the condition that x_o is given,
 $p(x)$ - is the probability density function of x ,
 Γ - is the spread domain of the possible values of x_o ,
 Θ - is the spread domain of the possible values of x .
 \mathfrak{R}_{AV} - is with a meaning of an average risk.

An algorithm with a good result rate can be obtained by next approach: the magnitude of the quantization step has to be irregular and it has to correspond to the following constrain:

$$S = \int_{x_1}^{x_2} p(x_{in}) dx \approx const., \quad (15)$$

where

$p(x_{in})$ - is the PDF of the input signal.

4 PRACTICAL IMPLEMENTATIONS

Section "Practical implementations" shows the results of a successful Digital Signal Processing of a noisy signal - the main electric motor current of mixing room for pneumatic tire production [5]. The main aim was to form reasonably reliable energy estimates of the works done for every technology stage in order to go from time control to energy control of the mixing processes. Such an approach gives a good result rate and allows a strict quality control in production.

The problems solved and the algorithms proposed were examined in the context of subjective university teaching of Stochastic Measurements in Technical University of Sofia, but the obtained results are applicable to the design both of Program Base of Intelligent Measurement Instrumentation and mainly of VXI-bus Operating Systems.

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