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COMPARISON OF TWO SAMPLING METHODS FOR SINE WAVE MEASUREMENT

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Abstract: Two methods, one approximate and one theoretically (mathematically) exact, for calculation of parameters of measured low frequency sinusoidal signal are thoroughly studied and compared. Both methods perform sampling of measured signal by HP 3458A Digital Multimeter in DC mode in order to achieve the best possible accuracy. Comparison was made firstly on calculated, ideal sinusoid (computer simulation) what has shown us theoretical limits of accuracy due to calculating inaccuracy and approximations. Then a series of measurements in real, laboratory conditions was made to confirm or refute theoretical foresights. For both methods, advantages and disadvantages are noticed and main influences on the final result uncertainty are explained.
Keywords: voltage measurement, sampling, high accuracy

1 INTRODUCTION

Development of fast and accurate digital voltmeters, i.e. A/D converters, such as HP 3458A digital multimeter, which was used in our experiments, enabled implementation of (theoretically) old ideas about periodical alternating signal measurement, that is, it enabled sampling of signal. Soon, two basically different approaches have risen. One is to take as many as possible samples of one period, and the other is to take only one sample per period, but the whole measurement (sampling) is spread along many periods. Each approach has its advantages and this article is not intended to com-pare these two philosophies, because that problem was already well treated by other authors [1,2]. In our work, we have used the second approach and this paper reveals our conclusions about theoretical and practical limits of uncertainty for measurement of low frequency sinusoidal signals in that way.

2 THEORETICAL BASIS OF COMPARED METHODS

The final aim of our endeavor is to determine the effective value (that is, the amplitude) of the unknown sinusoidal signal. We use HP 3458A to take one sample of signal per period along few hundreds of periods, each time in the other moment, beginning at the point as which the precedent sample ended. Although it measures the alternating signal, HP is set to the DC mode in order to achieve the best possible accuracy. Denoting duration of one sample (in fact, integration time) as T_m, the time between two consecutive samples as t_s and the period of signal as T, fig.1 is clear illustration of the whole process. If we wish to have n samples per period, it follows that $t_s = T + T/n = T(1+1/n)$.

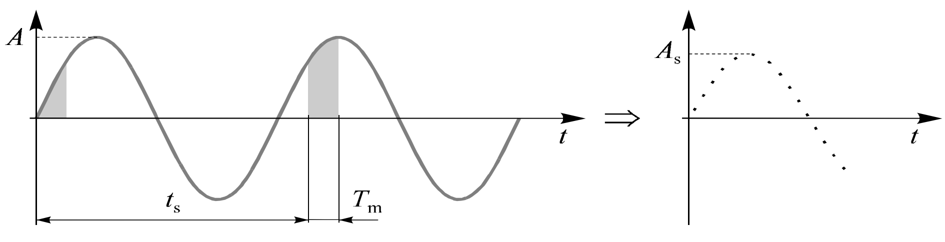

Figure 1. Left - sampling method (samples are shadowed) and results (right)

The mathematical form of samples obtained in that manner is [1,2]

$$U(i) = \frac{A}{p}\frac{T}{T_m}\sin\frac{pT_m}{T}\sin\left(2p\,\frac{t_s + \Delta t_s}{T}i + 2p\,\frac{T_d}{T} + p\,\frac{T_m}{T} + j\right) \qquad i = 0,1,2,\ldots,n-1 \qquad (1)$$

where symbols have the following meanings : *U*(*i*) is the value of i^{th} sample, *n* is the total number of all samples, *A* is the amplitude of the measured signal, *j* is phase of the measured signal, T_d is delay time (i.e. time in which the sampling instrument does not start measurement after it has arrived the signal to start), Δt_s represents inaccuracy of t_s (explained few sentences latter in the text) and the other symbols have the same meaning as explained before. Obviously, samples are also points of sinusoid (we shall name it "derived sinusoid") of the same period (if $\Delta t_s = 0$) as the measured one, but of less amplitude whose value is firmly mathematically related to the amplitude of measured signal by formula

$$A_s = A\frac{T}{pT_m}\sin\frac{pT_m}{T} \tag{2}$$

It is clear that, if the time between samples is not exactly t_s (and it never is) because of some "slipping" Δt_s, the samples will not be points of sinusoid of the same period as the measured one and that fact presents a heavy problem in such measurements. Already now we have to point out the fact that the origin of Δt_s is something we can never know because we can sample ideal sinusoid with non-perfect sampling device (unstable t_s) or we can have perfect sampling device (i.e. exact and stable t_s) and the signal with unstable frequency. In both cases the result is the same, that is, after determining Δt_s (its average value), we can not say is it consequence of unstable sampling period or of unstable frequency of the measured signal. In reality, of course, Δt_s will be consequence of both of these influences.

Such sampling has one, in the case of signal with higher harmonics, very useful peculiarity. It can be easily derived that samples of k^{th} harmonic are a little modified formula (1).

$$U_k(i) = \frac{A}{kp}\frac{T}{T_m}\sin\frac{kpT_m}{T}\sin\left(k2p\,\frac{t_s+\Delta t_s}{T}\,i + k2p\,\frac{T_d}{T} + kp\,\frac{T_m}{T} + j\right) \qquad i=0,1,2,\dots,n-1 \tag{3}$$

We see that the amplitude of the k^{th} harmonic's derived sinusoid is

$$A_{k_s} = A\frac{T}{kpT_m}\sin\frac{kpT_m}{T} \tag{4}$$

so choosing T_m to be *T/k* we eliminate k^{th} harmonic's and all its multiplyers' influence on the signal. As usually only odd harmonics present some problem, setting $T_m/T = 1/3$ we can eliminate 3^{rd} , 6^{th} , 9^{th} etc. harmonics, that is, only those whose ordinal number is not dividable by 3 remain and that should not be problem in laboratory conditions.

We have tried a few methods of calculation to extract parameters of signal from values of samples and two of them have shown the strongest "resistance" to the above mentioned problem ($\Delta t_s \neq 0$). One is approximate, already known and described in [2]. As the reference [2] has been written in Croatian, what might be problem for most of readers, we shall briefly explain theoretical basis of that method. More details are always available contacting the authors of this paper. The stress here will be on little modification we have made in the original method presented in [2], but significant improvement which has been achieved by that modification. The other method is new, completely calculating one, essentially much simpler then approximate, but superior under certain conditions.

2.1 Approximate method

In [2] it has been shown that, in the case of ideal sampling (no Δt_s), the effective value of derived sinusoid can be calculated as

$$U_{efs}^2 = \frac{\sum_{i=0}^{n-1} U_i^2}{n} \qquad n \geq 3 \tag{5}$$

where *n* is the total number of all samples and U_{efs} is the effective value of derived sinusoid.

If the time between samples varies during measurement, solution proposed in [2] comprises two approximations. Firstly, the last 10 samples are used to determine (calculating a sinusoid according to the least squares theory) index (ordinal number) of sample which is of equal value as the first one. Generally, it is not an integer number and let us denote it as *m*. The next step is to find a 5^{th} grade polynom (again according to the least squares theory) that approximates the curve of the partial sums in the nominator of (5) around intersection of approximate sinusoid and time axis near n^{th} sample (index *n*-1). Substituting *m* as the variable in that polynom, we obtain value of the nominator in (5), which divided with m yields an approximate value of U_{efs}. We can see that there are two complicated and error prone approximations which even theoretically (as explained latter in the text) limit accuracy to about 10^{-9} (0,001 ppm). There are two main causes of uncertainty. The first one is the large number of mathematical operations needed to carry out two least squares approximations. The second one is the second approximation, that is, it seems that the 5^{th} grade polynom is the function that best approximates the curve of partial sums in nominator of (5), but it is not exactly that curve and approximation is accurate enough only in very narrow area around (n - 1). Any error in the nominator of (5) has essential influence on the final result, so it is clear that the second approximation brings the largest part of the final error into the whole process. We succeeded to eliminate this second approximation and replace it with mathematically exact correction in formula (5) what lowered theoretical boundary of error to 10^{-12} (10^{-6} ppm) or for even 3 orders of quantity. The new (corrected) formula (5) is

$$U_{efs}^2 = \frac{\sum_{i=0}^{n-1} U_i^2}{n - \sum_{i=0}^{n-1} \cos 2\left(2p\,\frac{t_s + \Delta t_s}{T}\,i + j_s\right)} \tag{6}$$

Now, the first approximation gives us index m which contains information about Δt_s and it also provides value of j_s ($j_s = f(T_d, T_m, j)$). After calculating Δt_s, we can use (6) to find U_{efs}, so there are (theoretically) no more errors in the process.

2.2 Calculating method

When a man encounters problem of extracting parameters of sinusoid from its samples, the first idea might be to calculate the least squares approximation through all the samples. Theoretically, everything is clear, but difficulties arise from limitations in calculating with computer. Samples are defined with formula (1) and are, generally, of the form $U(i)=A_s\cdot\sin(xi+y)$ (please note that $y = j_s$). Taylor series of this function is (all members of order higher than first are neglected)

$$U(i) = A_{s0}\cdot\sin(x_0 i + y_0) + \sin(x_0 i + y_0)\cdot dA_s + A_{s0}\cdot i\cdot\cos(x_0 i + y_0)\cdot dx + A_{s0}\cdot\cos(x_0 i + y_0)\cdot dy \quad , \tag{7}$$

where A_{s0} is supposed value of A_s , x_0 is supposed value of x and y_0 is supposed value of y. The system of equations (written in Gaussian notation), which gives us corrections dA_s, dx, dy for supposed values of parameters A_s, x and y, is

$$\begin{aligned} [aa]\cdot dA_s + [ab]\cdot dx + [ac]\cdot dy &= [aV] \\ [ab]\cdot dA_s + [bb]\cdot dx + [bc]\cdot dy &= [bV] \quad \Rightarrow \quad dA_s,\ dx,\ dy \\ [ac]\cdot dA_s + [bc]\cdot dx + [cc]\cdot dy &= [cV] \end{aligned} \tag{8}$$

where variables a, b, c and V have the following meaning.

$$\begin{aligned} V_i &= U(i) - A_{s0}\cdot\sin(x_0 i + y_0) \\ a &= \sin(x_0 i + y_0) = (U(i) - V_i) / A_{s0} \\ b &= A_{s0}\cdot i\cdot\cos(x_0 i + y_0) \\ c &= A_{s0}\cdot\cos(x_0 i + y_0) = b / i \end{aligned} \tag{9}$$

Problem is in values that some coefficients get in the calculating process. Namely, the first coefficient [*aa*] is the sum of squares of *sin* function values while, for example, coefficient [*bb*] contains sum of products of the amplitude and index i (which can be few thousands). In reality, the ratio of the largest and the lowest coefficient can be of order 10^{10}. Ordinary PC computers, which calculate on 15 digits, can not resolve such system with error lower than 10^{-6},even in one step (pass through the system) and usually few (tenths) steps are needed. So large error in calculation, without any measurement errors, makes Gaussian approximation through all the samples completely unacceptable.

Because of these difficulties, we had to find another way to calculate unknown parameters. We have found extremely simple, mathematically exact and, in the meaning of the computer implementation, very effective solution. Although derivation of needed formulas is not long, we shall omit it and here are only final relations. Frequency and phase parameters (x and y) of derived sinusiod can be calculated as follows.

$$x = \arccos\frac{U(i-1)+U(i+1)}{2\cdot U(i)} \quad , \quad y = arcctg\,\frac{U(i+1)-U(i-1)}{2\sin x\cdot U(i)} - xi \tag{10}$$

Finding x and y (x contains information about Δt_s), amplitude can be calculated from any sample. The first idea how to calculate the amplitude was to extract it from every three samples and then to take the average value as the final result. During experiments with computer simulation this approach proved not to be the best one, especially in real conditions. The origin of the problem and its solution are explained in the next fragment of text.

3 COMPUTER SIMULATION AND ITS RESULTS

In order to find theoretical limits of error for both methods, we have made computer simulation of sampling and calculating signal parameters. Firstly, we have simulated sampling of ideal sinusoid with perfect sampling device. Samples obtained in such experiment would be of values given by (1), which can be rewritten as follows.

$$U(i) = A_s \sin\left[2p\left(\frac{t_s}{T} + \frac{\Delta t_s}{T}\right)\cdot i + 2p\,\frac{T_d}{T} + p\,\frac{T_m}{T} + j\right] = A_s \sin\left[2p\left(\frac{t_s}{T} + \frac{\Delta t_s}{t_s}\,\frac{t_s}{T}\right)\cdot i + y\right]$$

$$= A_s \sin\left[2p\,\frac{t_s}{T}\left(1 + \frac{\Delta t_s}{t_s}\right)\cdot i + y\right] = A_s \sin\left[2p\,\frac{n+1}{n}\left(1 + \frac{\Delta t_s}{t_s}\right)\cdot i + y\right] \qquad (11)$$

$$= A_s \sin(xi + y) \quad ; \quad x = 2p\,\frac{n+1}{n}\left(1 + \frac{\Delta t_s}{t_s}\right) \quad , \quad y = 2p\,\frac{T_d}{T} + p\,\frac{T_m}{T} + j$$

In ideal sampling $\Delta t_s = 0$ and $T_d = 0$, but because T_d causes only additional phase shift between derived and original sinusoid, its value is not important, although we have entered the estimated value $T_d = 50$ ns into the program. Approximate method has been implemented with new formula (6) and as mentioned before, calculating method was realised by finding amplitude as the average value of amplitudes obtained from every three samples. Implemented in that way, the calculating method showed larger error than approximate, sometimes reaching 10^{-9}. The main reason was that the average value form every three samples was not the amplitude of average sinusoid through all samples, but average amplitude of many independent sinusoids. Although samples were calculated by formula (11), slight deviations due to calculation limitations on PC computer taken into account many times caused so large error value. This is obvious by looking at the Table 1 that contains results of *sin*($x\cdot\pi$) (here *x* is only a number) function returned by Microsoft Excel97. Although (mathematically) all values should be equal and zero, the difference among them reaches the order of 10^3. This is error only in samples and to calculate amplitude, we need to find *x* and *y* which are obtained from inverse trigonometric functions, that is, these steps also bring some error into the final result.

Table 1. Values of *sin* function returned by Microsoft Excel 97

x	1	5	10	30	60	100	500	1000
sin($x\cdot\pi$)	$1{,}2\cdot10^{-16}$	$6{,}1\cdot10^{-16}$	$-1{,}2\cdot10^{-15}$	$-1{,}1\cdot10^{-14}$	$-2{,}2\cdot10^{-14}$	$1{,}9\cdot10^{-15}$	$-1{,}6\cdot10^{-13}$	$-3{,}2\cdot10^{-13}$

This problem enforced us to use formula (6) to calculate the amplitude of measured signal instead of direct calculation of amplitude from samples. The result was impressive. The only error remained, whose influence on the final result was still notable, was the error caused by computer calculation limitations, because the relative error of amplitude for $\Delta t_s = 0$ was $6\cdot10^{-14}$ and even less for some values where $\Delta t_s \neq 0$. Complete comparison with approximate method is in Table 2 (column titled with “app. c.” contains result for the new approximate method, that is, it uses formula (6) and column “app.nc.” means “not corrected”, i.e. original approximate method).

Table 2. Sampling of ideal sinusoid with perfect ($\Delta t_s = 0$) and non-perfect ($\Delta t_s \neq 0$) sampling device

	$\Delta t_s/t_s = 0$			$\Delta t_s/t_s = 10^{-4}$			$\Delta t_s/t_s = 10^{-5}$			$\Delta t_s/t_s = 10^{-6}$		
	calc.	app. c.	app.nc.	calc.	app. c.	app.nc.	calc.	app. c.	app.nc.	calc.	app. c.	app.nc.
rel.err.	$6\cdot10^{-14}$	$-6\cdot10^{-12}$	-10^{-14}	$3\cdot10^{-13}$	-10^{-12}	$-7\cdot10^{-10}$	$2\cdot10^{-14}$	$-4\cdot10^{-12}$	$-1\cdot10^{-09}$	$4\cdot10^{-15}$	$-6\cdot10^{-12}$	-10^{-10}

We see that error of calculating method slightly depends on Δt_s , probably following the error of trigonometric functions implemented on computer, but is never (for any reasonable value of Δt_s) greater than 10^{-13}. The corrected approximate method showed practically total independence of Δt_s , but its error is at least one order of quantity greater than the calculating method’s one. The original approximate method showed, as expected, great dependence on Δt_s and its error is even three orders of quantity greater than the error of improved approximate method, so in our further experiments we have compared only calculating and improved approximate method.

4 RESULTS OF REAL LABORATORY MEASUREMENT

After first selection and determining of theoretical limits of considered methods, we made a series of real laboratory measurements using only calculating and improved approximate method. All measurements were made in Croatian Primary Electromagnetic Laboratory, so climatically conditions satisfied all accepted standards. As the signal source, we used FLUKE 5700 AC Calibrator, sampling device was HP3458A DMM and there was one ordinary PC computer as controller. These experimental measurements were made with signal of 7 V_{rms} and with HP on DC 10 V range. The measurement procedure was to take five series of samples, then to calculate effective voltage for every series and the final result was the average value of these five partial results. Typical results are in the Table 3.

Table 3. Typical results obtained with calculating and improved approximate method

	series 1	series 2	series 3	series 4	series 5	average val.	st.dev.
Calc. U_{rms} / V	7,000010	6,999933	6,999905	7,000013	6,999974	6,999967	$4{,}74\cdot10^{-5}$
App. U_{rms} / V	6,999975	6,999950	6,999974	6,999958	6,999973	6,999966	$1{,}1\cdot10^{-5}$

As the first, we notice excellent repeatability of partial results. The largest difference among them for calculating method is about 125 µV, i.e. 125 µV / 7 V ≈ 18 ppm (standard deviation is ≈ 7 ppm) and this example is one of the worst. This method is so precise, and theoretically error free, that we may ascribe these differences either to the sampling device or to the signal source, but not to the method itself. The same stands for the improved approximate method which shows almost identical characteristics as the calculating one. In this example its standard deviation is a little less, but in all our experiments we have irrefutably established that it is of the same order for both methods. This is logical, having in mind that theoretical limit for both methods lie deeply under errors of used equipment.

5 CONCLUSION

Our main conclusion would be: “We have made only the first step, but in right direction.”. About methods themselves, we can not say much for one and against the other, because they are both much more accurate than sources and sampling devices. Of course, we may assume that Fluke’s short time stability is a few ppm, so results should be, if not absolutely accurate, at least enough close to each other and repeatable. From Table 3 we see that both of these expectations are met, so in future we must examine and compare absolute accuracy. However, authors are personally “supporters” of calculating method, because of its mathematical foundation and simplicity of implementation. Approximate method requires incomparably more complicated program and sometimes, depending on obtained signal samples, it diverges what forces us to break program execution and repeat measurement. Even if there are no problems during running of program, it is much slower than the program for calculating method. Finally, theoretical accuracy of calculating method is nevertheless one order of quantity better then approximate method’s one.

We said: ”…the first step,…”, and the next one is marked by slight differences among results in Table 3 which point to the problem of Δt_s and ΔT_m. Furthermore, all these considerations assume ideal sinusoid, i.e. signal without disturbations and higher harmonics, so in future we shall have to examine “resistance” of these methods to such problems. Anyway, already now we may state that in laboratory measurements, using “clean” and stable signals from calibrators, this new calculating method presents a great improvement in accuracy using relatively simple and not too expensive equipment. This is especially true having in mind that, by adjusting integration time, we can neutralise some most problematic signal components.

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