

# VECTORIAL METHOD FOR MEASUREMENT OF SHORT-CIRCUIT LOOP IMPEDANCE IN NETWORK WITH OPERATING LOAD

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*Abstract: This paper is concerned with the measurement of short-circuit loop impedance in electrical low voltage network with operating loads. The paper discusses and analyzes the influence of the operating network load on the accuracy of loop impedance measurement when are use classical instruments and methods. The results of analysis indicate that the effect of the operating load is significant especially when the measurement point is close to the induction motors. The paper presents the new vectorial method for measurement loop impedance in which the influence of operating loads on measurement result is eliminated. The relations are derived for determining the values of magnitude loop impedance  $Z$  and its orthogonal component resistance  $R$  and reactance  $X$ .*

*Keywords: Short-Circuit Impedance, Measurement, Errors*

## 1 INTRODUCTION

Measurement of short-circuit loop impedance in low voltage network has fundamental meaning to determine the effectiveness of protective neutral earthing. The protection is effected by the short-circuit current causing operation of the fuse or circuit breaker in the supplying line when phase to earth fault occurs. In practice the prospective the short-circuit current is determined by measurement the short-circuit loop impedance  $Z$ . An accurate impedance measurement is needed for reliable operation of protective device and consequently for safety of personnel and apparatus.

All meters and methods in use presently for measuring short-circuit loop impedance show remarkable differences in metrological properties. The fundamental parameter of any method or meter is the measurement error. Negative errors are the most dangerous when testing the effectiveness of protective neutral earthing, i.e. in the case where the measured value of loop impedance is less than the actual value. This leads to the situation where machinery with ineffective protection is exploited.

The accuracy demanded by the regulations is not high; for example from [1] the permissible error margin is as high as 30%. Such mild requirements are the result of the fact that the measurement is affected by many factors which lead to inaccuracies. Some of the causes of inaccuracies are due to the method itself (simplification of the equivalent tested circuit, approximations applied in the measuring techniques of the meters), as well as inaccuracies in the construction of the instrument.

The main sources of loop impedance measurement error are: the phase difference between loop impedance  $\text{Arg}(Z)$  and measuring load impedance  $\text{Arg}(Z_0)$ , electrical transient due to inductance in a tested circuit, voltage fluctuation and the operating network loads.

## 2 PRINCIPLE OF SHORT-CIRCUIT LOOP IMPEDANCE MEASUREMENT

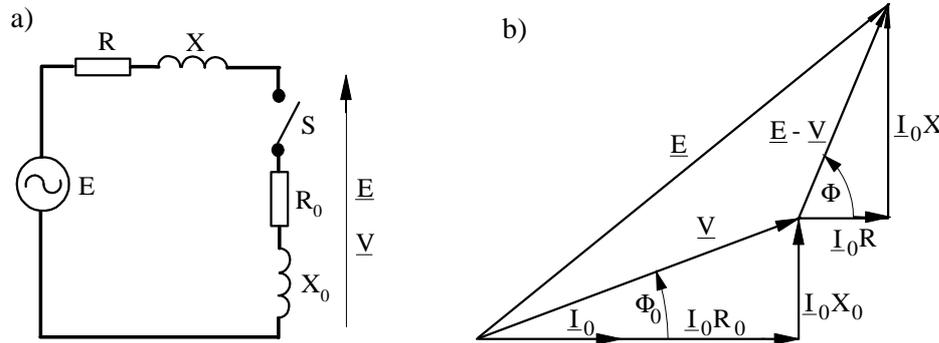
The classical methods [2,3,6] for measuring loop impedance  $Z = R + jX$  are based on the assumption that the tested system can be represented by following simplified equivalent circuit, composed of linear elements as shown in Fig. 1a. To measure impedance  $Z$ , the two values of voltage are determined at the measurement point: the first,  $E$  - when switch  $S$  is OFF, the second,  $V$  - when the switch  $S$  is ON. When switch  $S$  is closed, measurement load of known impedance value  $Z_0$  is switched ON.

The measured loop impedance is given by the following definite vector relationship

$$\underline{Z} = \frac{\underline{E} - \underline{V}}{\underline{I}_0} = \underline{Z}_0 \frac{\underline{E} - \underline{V}}{\underline{V}} = \underline{Z}_0 \left( \frac{\underline{E}}{\underline{V}} - 1 \right) \quad (1)$$

Due to the technical difficulties in practice the following approximation is used during measurement

$$Z_m = Z_0 \frac{E - V}{V} = Z_0 \left( \frac{E}{V} - 1 \right) \quad (2)$$



**Figure 1.** The principle of measurement of loop impedance  
a) simplified equivalent tested circuit  
b) phasor diagram

The tested short-circuit loop impedance  $Z$  and the measurement load impedance  $Z_0$  consist of a series connected resistance and inductance reactance. The modules of impedance  $Z$  and the impedance  $Z_0$  are given by

$$Z = \sqrt{R^2 + X^2} \quad Z_0 = \sqrt{R_0^2 + X_0^2} \quad (3)$$

Where:  $\phi = \arctan X/R$  and  $\phi_0 = \arctan X_0/R_0$

The measurement error is given by dependence

$$\Delta Z_m = \frac{Z_m - Z}{Z} \quad (4)$$

Inaccuracies resulting from replacing vectors ( $\underline{E}$ ,  $\underline{V}$ ,  $\underline{Z}_0$ ) with their moduli ( $E$ ,  $V$ ,  $Z_0$ ) are in view of the VDE [1] regulations, acceptable so long as the tested circuit and load impedance are characterised by a difference in impedance arguments of  $\phi_r = |\phi - \phi_0| \leq 15^\circ$ ,

The values measured and eventually converted are usually maximum values of both voltages  $E_m$  and  $V_m$  or their rms values  $E$  and  $V$ .

### 3 MEASUREMENT OF LOOP IMPEDANCE NEAR THE NETWORK LOAD

The short-circuit loop impedance measurements using classical methods and instruments are often performed in the network initially loaded by static power equipment, such as induction motors or welders, as is shown in Fig. 3a. When the measurement point is near the induction motor, this motor will affect the measured loop impedance value viewed from the terminals at the testing point. One of the most significant and apparent causes of measurement error are changes in equivalent source voltage  $E$ , which is caused by normal changes of drop voltage across loop impedance  $Z$ . Its source is operating induction load ( $Z_M = R_M + jX_M$  and  $\phi_M = \arctan X_M/R_M$ ) which is connected to the network. When the measuring load  $Z_0$  is switched OFF, the measured open circuit voltage across motor terminal is  $V = V_M$ . After switching ON the measuring load  $Z_0$  the voltage  $V$  is decreasing and the motor is subject to a braking action. The magnetic and mechanical energy stored in the motor will be transferred to the tested circuit. After some time which depends on the type and the rating parameters of the motor, the measured values of load voltage ( $V = V_1$ ) and current ( $I = I_0$ ) will reach steady state. The short-circuit loop impedance is obtained by measurement of two voltages at testing point. When switch  $S$  is open the open circuit voltage  $\underline{V}_M$  is given by

$$\underline{V}_M = \underline{E} \frac{\underline{Z}_M}{\underline{Z} + \underline{Z}_M} \quad (5)$$

When switch S is closed the dependence for load voltage  $\underline{V}_1$  at the testing point is given by

$$\underline{V}_1 = \underline{E} \frac{\frac{\underline{Z}_M \cdot \underline{Z}_0}{\underline{Z}_M + \underline{Z}_0}}{\underline{Z} + \frac{\underline{Z}_M \cdot \underline{Z}_0}{\underline{Z}_M + \underline{Z}_0}} = \frac{\underline{E}}{1 + \frac{\underline{Z}}{\underline{Z}_0} + \frac{\underline{Z}}{\underline{Z}_M}} \quad (6)$$

To simplify the mathematical description to calculate on the base of voltage magnitudes the measured value of loop impedance  $Z_m$ , the relative denotation is introduced as

$$B = \sqrt{1 + \frac{S^2 S_M^2 + 2SS_M(S_M \cos\phi + S \cos\phi_M)}{S^2 + S_M^2 + 2SS_M \cos(\phi - \phi_M)}} \quad (7)$$

Where

$$S = \frac{Z}{Z_0}, \quad S_M = \frac{Z_M}{Z_0} \quad \phi_M = \Phi_M - \Phi_0, \quad \phi = \Phi - \Phi_0 \quad - \text{are the ratios and the difference of}$$

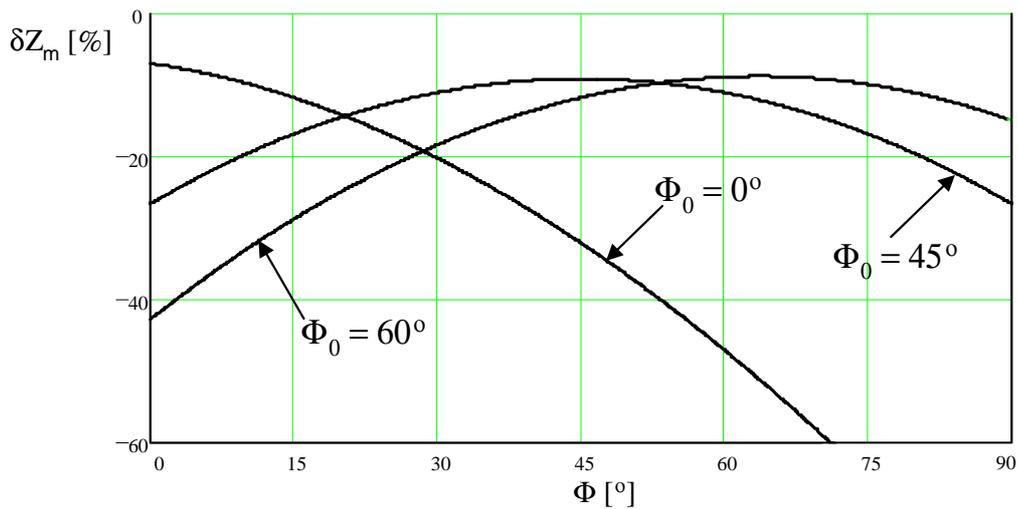
phase angles of the impedances of the tested circuit

Taking into consideration the equs (5), (6), (7), now according to equ. (2) the dependence for measured value of loop impedance  $Z_m$  we may write as

$$Z_m = \frac{V_M - V_1}{I_0} = Z_0 \left( \frac{V_M}{V_1} - 1 \right) = Z_0 (B - 1) \quad (8)$$

The loop impedance measuring error  $\delta Z_m$  due to operating load is given by dependence

$$\delta Z_m = \frac{Z_m}{Z} - 1 = \left[ \frac{1}{S} (B - 1) - 1 \right] \quad (9)$$



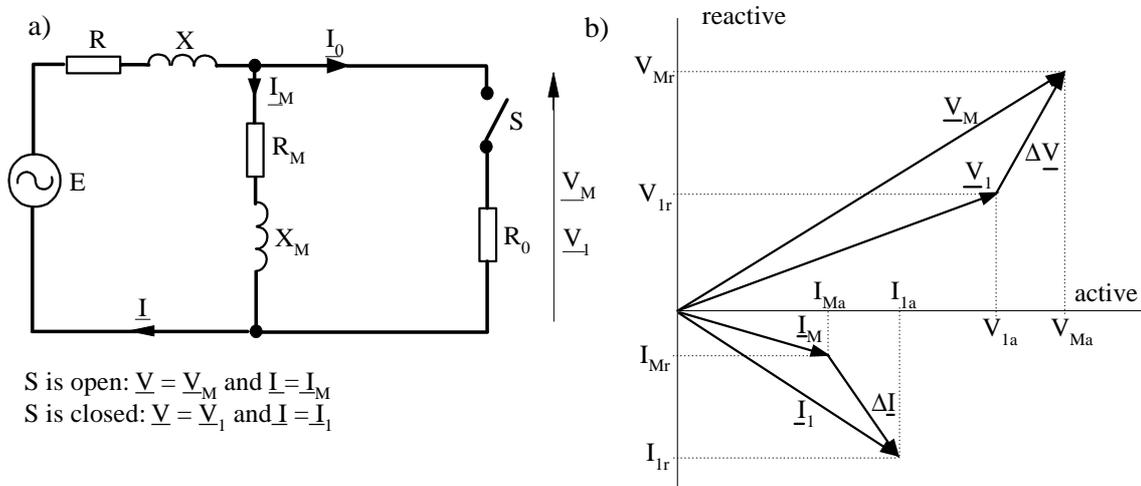
**Figure 2.** Example of graphs of measuring errors  $\delta Z_m = f(\phi)$  for  $\phi_s = 45^\circ$  and  $\phi_0 = 0^\circ, 45^\circ, \text{ and } 60^\circ$  and for a given impedances magnitude:  $Z = 1 \Omega, Z_M = 10 \Omega, Z_0 = 10 \Omega$

Measuring errors  $\delta Z_m$  depends on the parameters of the tested circuit. An analysis of the error resulting from operating network load and the difference in phase angles of the tested circuit impedances are presented in previous authors articles [4, 5]. As an example Figure 2 shows the

graphs of measuring error  $\delta Z_m = f(\phi)$  when measurement point is close to the network load  $Z_M$ . and the measurement is performed using loop impedance meter with three values of instrument impedance phase angel  $\phi_0$ . It follows from the analysis that loop impedance measuring error may exceed the permissible value with respect to the VDE Standards [1], so there is a need to search for a techniques to eliminate the influence of dominated network load on measurement result.

#### 4 PRINCIPLE OF THE VECTORIAL METHOD

Accurate measurement of loop impedance according to equation (1) is a result of the determination of geometrical difference between voltages, which do not occur simultaneously. All mathematical operations involving these voltages should treat them as vectors. This is quite easily achieved with digital technique. The present growth in the field of electronics technology gives the idea to find a solution to the defined problem in that direction [6]. The principle of the method is based on applying simulated fault trough resistive instrument load  $R_0$  and to measurement the changes of the voltage  $\Delta V$  and current  $\Delta I$  at the testing point. A system with an analogue-digital converter is use to store and convert the orthogonal components of the voltage and the current vectors in the tested circuit, as shown in Fig. 3



**Figure 3.** Vectorial measurement of short-circuit loop impedance  
a) equivalent tested circuit  
b) vector diagram of voltages and current

To present the principle of the vectorial method for loop impedance measurement a mathematical description are introduced for the equivalent tested circuit shown in Fig 3a, Fig. 3b shows the vector diagram of the voltages and currents.

When switch S is open the voltage ( $\underline{V} = \underline{V}_M$ ) across the load terminals and the current ( $\underline{I} = \underline{I}_M$ ) in loop impedance branch are given by

$$\begin{aligned} \underline{V}_M &= V_{Ma} + jV_{Mr} \\ \underline{I}_M &= I_{Ma} + jI_{Mr} \end{aligned} \tag{10}$$

When switch S is closed the voltage ( $\underline{V} = \underline{V}_1$ ) across  $R_0$  and the current ( $\underline{I} = \underline{I}_1$ ) in the loop impedance branch are given by

$$\begin{aligned} \underline{V}_1 &= V_{1a} + jV_{1r} \\ \underline{I}_1 &= I_{1a} + jI_{1r} \end{aligned} \tag{11}$$

Where

$V_{Ma}$ ,  $V_{1a}$  and  $I_{Ma}$ ,  $I_{1a}$  are active components of the voltage and current vectors  $\underline{V}_M$ ,  $\underline{V}_1$ ,  $\underline{I}_M$ ,  $\underline{I}_1$

$V_{Mr}$ ,  $V_{1r}$  and  $I_{Mr}$ ,  $I_{1r}$  are reactive components of the voltage and current vectors  $\underline{V}_M$ ,  $\underline{V}_1$ ,  $\underline{I}_M$ ,  $\underline{I}_1$

According to the circuit diagram in Fig. 3b the value of loop impedance is given by

$$\underline{Z} = R + jX = \frac{\Delta V}{\Delta I} = \frac{(V_{Ma} - V_{1a}) + j(V_{Mr} - V_{1r})}{(I_{1a} - I_{Ma}) + j(I_{1r} - I_{Mr})} \quad (12)$$

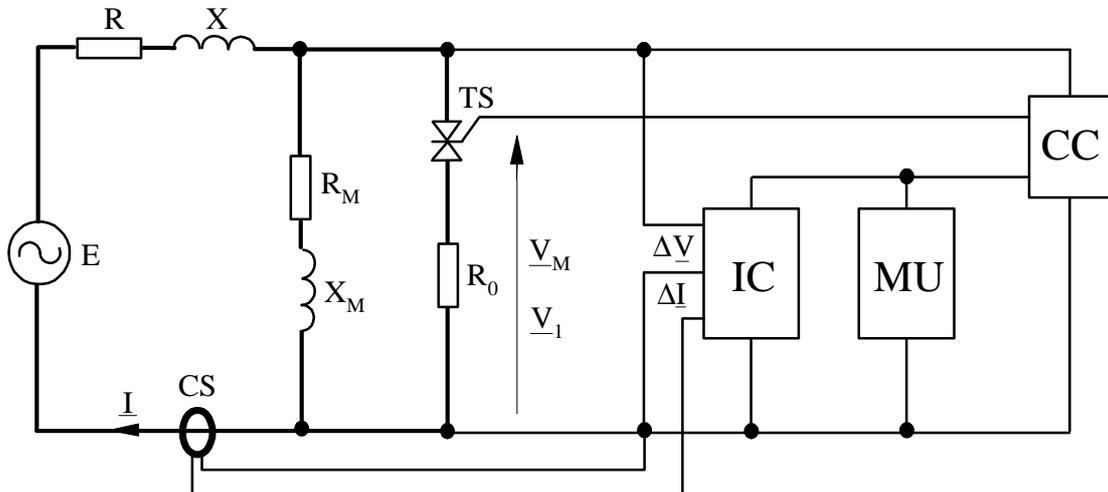
Rearranging above expression we may write dependence to determine the loop impedance magnitude  $Z$  and its orthogonal components resistance  $R$  and reactance  $X$  as

$$Z = \sqrt{R^2 + X^2} = \sqrt{\frac{(V_{Ma} - V_{1a})^2 + (V_{Mr} - V_{1r})^2}{(I_{1a} - I_{Ma})^2 + (I_{1r} - I_{Mr})^2}} \quad (13)$$

$$R = \frac{(V_{Ma} - V_{1a})(I_{1a} - I_{Ma}) + (V_{Mr} - V_{1r})(I_{1r} - I_{Mr})}{(I_{1a} - I_{Ma})^2 + (I_{1r} - I_{Mr})^2} \quad (14)$$

$$X = \frac{(V_{Mr} - V_{1r})(I_{1a} - I_{Ma}) - (V_{Ma} - V_{1a})(I_{1r} - I_{Mr})}{(I_{1a} - I_{Ma})^2 + (I_{1r} - I_{Mr})^2} \quad (15)$$

Figure 4 shows the schematic arrangement for vectorial measurement of the short-circuit loop impedance  $Z$  in the low voltage network with operating load  $Z_M$ . This arrangement consists of measurement load branch with thyristorized switch  $TS$ , instrument resistance  $R_0$  and the measuring branch with microprocessor measuring unit  $MU$ , the input circuit  $IC$ , the current sensor  $CS$ , and the control circuit  $CC$ .



**Figure 4.** Schematic diagram for vectorial measurement of short-circuit loop impedance

The measured values of the orthogonal components of the voltage and current vectors active;  $V_{Ma}$ ,  $V_{1a}$  and  $I_{Ma}$ ,  $I_{1a}$  and reactive;  $V_{Mr}$ ,  $V_{1r}$  and  $I_{Mr}$ ,  $I_{1r}$  are obtained by sampling the sinusoidal voltage and current waveforms. The measurement results are obtained by processing the components of both voltages in appropriate microprocessor unit  $MU$  on the base of equation (13), (14) and (15) for the moduli of loop impedance  $Z$ , and its active and reactive components resistance  $R$  and reactance  $X$  respectively.

## 5 CONCLUSION

The classical methods and instruments which are use nowadays for measurement short-circuit loop impedance should take into account the influence of operating network load on measuring accuracy.

The vectorial method as results from above consideration enable measurement of short-circuit loop impedance magnitude  $Z$  and its orthogonal components resistance  $R$  and reactance  $X$  regardless the values of arguments  $\text{Arg}(Z)$ ,  $\text{Arg}(Z_0)$  and  $\text{Arg}(Z_M)$  of the impedances of tested circuit. The use of resistive instrument measurement load  $R_0$  has positive effect on the dimensions and mass of the loop impedance meter.

Instruments build on the base of proposed vectorial method can be applied in electrical power system to determine the short-circuit current. At present the model of the vectorial loop impedance meter is elaborated as a prototype which will be tested under laboratory and industrial conditions and the results of that investigation will be presented in future paper.

## REFERENCES

- [1] DIN 57413 Teil 3. Deutsche Normen Schleifenwiderstands – Messgerate.
- [2] J. Masny: "*Measurement of short-circuit loop impedance*", Gospodarka Paliwami i Energi<sup>1</sup>, nr 7, 1982. (in polish)
- [3] M. Voit: "*Messpractis Schutzmassnahmen DIN VDE 0100.3. bearbeitete Auflage*" ABB Metrawatt, Pflaum, München 1992.
- [4] S. Czapp, R. Roskosz, A. Skiba: "*Earth Loop Impedance Measurement In Electrical Network With Operating Loads*", KKM98 Gdańsk, Poland 1998, (in polish).
- [5] R. Roskosz, "*Error in earth loop impedance Measurement due to the duration of measurement process*", Proceeding of IMEKO 6<sup>th</sup> TC-4 Symposium, Brussel, 1993.
- [6] R. Roskosz: "*A New Method for Measurement of Earth Loop Impedance*". IEEE Trans. on Power Delivery. Vol. 6. 1991.

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