

# SUPER-HIGH PRECISION DC CURRENT MAGNETIC COMPARATOR

**R.J. Berkman**

Laboratory of Electromagnetic Investigations  
Lviv Centre of Institute of Space Research, Naukova St., 5a, Lviv, Ukraine

*Abstract: The high precision comparison of DC currents in electrically insulated circuits using magnetic modulators was not yet realized in practice; the nature of arising errors and their values were not described in literature. The physical model is proposed, which explains the error as a result of non-homogeneous magnetic fields influence, produced by comparison and output windings along the ring core of magnetic modulator. The expression for comparison error calculation in such devices is deduced, experimentally verified and methods of its reducing are proposed. The comparison error about  $10^{-7}$  is realized.*

*Keywords: DC comparator, magnetic modulator, flux-gate sensor*

## 1 INTRODUCTION

The method of DC high precision comparison using two input windings on magnetic modulator (MM) creating magnetic fields in opposite directions was described long ago [1-3]. Such a method in electrically insulated circuits allows to realise arbitrary current comparison coefficient by simple changing of winding turns number. Calculations showed that variations of sensitivity and zero drift of MM produced negligible errors, while comparison coefficient, equal to numbers of turns ratio, seems to be known absolutely exactly.

In real devices the comparison accuracy has found to be moderate, at any case not better than, e.g., in bridge arrangements – the magnetic comparator (MC) error reached 0.001% and more. Then MC did not receive further development and practical use.

The physical reasons of additional error appearance, corresponding calculation methods and methods of their elimination were not considered in known papers, whereas to solve this problem for the precise MC development could be of a great importance.

Present report analyzes the physical and mathematical model of non-ideal MC as a combination of ideal MM and “very bad” ring-core flux-gate sensor (FGS), using the same core. According to this model, FGS operates in the mode of measurement of heterogeneous magnetic field created by the system of non-ideal comparison windings along the core when the useful signal is applied. Output winding of FGS is also non-homogeneous; as a result the additional error appears in dependence on combination of all said three heterogeneity

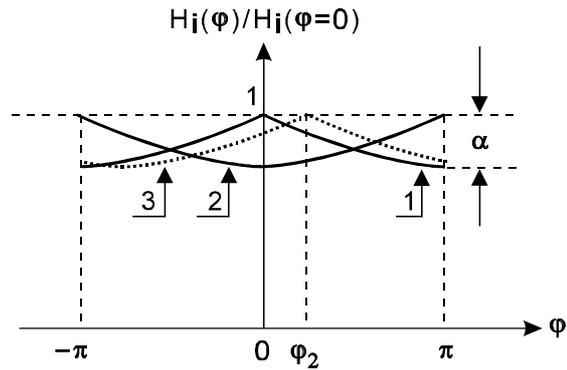
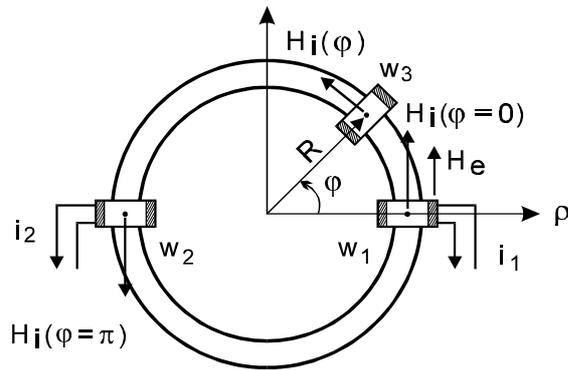
## 2 MATHEMATICAL ANALYSIS

The full theory of ring-core FGS in heterogeneous measuring fields is complicated and needs the presentation of cumbersome transformations. For simplifying the analysis it was accepted that excitation mode ensures periodic rectangular change of core permeability from its maximum value to the negligible minimum. For this model the equations for relative output signal calculation (both for MM and FGS) are the same, as for inner homogeneous and heterogeneous magnetic flux distribution. As a result the error appeared is fully determined with winding turns density along the core ring [4]. The “super-bad” FGS with maximum possible comparison winding heterogeneity (two narrow comparison windings and narrow output winding) is presented at Fig. 1. Magnetic field structure of narrow comparison windings is presented at Fig. 2. It was shown [4] that the inner field  $H_i(r)$  distribution is close to parabolic – see Fig. 2, curves 1, 2 and 3 (dotted line) for winding position  $w_1$ ,  $w_2$  and  $w_3$  correspondingly.

In order to find the expression for the resulting  $M\tilde{N}$  core internal field let us consider the magnetic flux distribution when the ring core is magnetised with external field  $H_e$  of the narrow section  $w_1$  near the point  $r=R$ ,  $f=0$  (Fig. 1). It is evident that the tangential component of the internal field intensity has maximum at  $f=0$  and minimum at  $f=p$  (Fig. 2).

The relation between tangential components of inner magnetic field density  $H_i$  at the arbitrary point with polar coordinates ( $r=R$ ,  $f=var$ ), and the point with magnetic field density  $H_0$  and coordinates of

winding  $w_1$  ( $r=R, f=0$ ) can be written as follows as follows:



**Figure 1.** Magnetizing coil positions at ring core **Figure 2.** Relative inner field of magnetizing coils

$$\frac{H_i(\varphi)}{H_i(0)} = S(\varphi) = 1 - 2\alpha \frac{\varphi}{\pi} + \alpha \frac{\varphi^2}{\pi^2}; \quad (1)$$

where  $\alpha$  – parameter, depending on the core dimensions ratio and magnetic permeability of the material,

$$\alpha = \frac{\pi^2 \lambda^2}{\mu_m \ln 4\lambda}; \quad \lambda = R \sqrt{\frac{\pi}{s}}; \quad (2)$$

$\mu_m$  - maximum magnetic permeability of core material,  $R$  and  $s$  - correspondingly radius and summarising cross-sectional area of MC cores.

For arbitrary  $w_2$  position

$$S_2(\varphi) = 1 - 2\alpha \frac{\varphi_2 - \varphi}{\pi} + \alpha \frac{(\varphi_2 - \varphi)^2}{\pi^2} \quad \text{for } f_2 \neq f \neq 0, \quad (3)$$

and

$$S_2(\varphi) = 1 - 2\alpha \frac{\varphi - \varphi_2}{\pi} + \alpha \frac{(\varphi - \varphi_2)^2}{\pi^2} \quad \text{for } 2p \neq f \neq f_2 \quad (3a)$$

The resulting relative inner field:

$$S_0(\varphi) - S_2(\varphi) = -4\alpha \frac{\varphi}{\pi} + 2\alpha \frac{\varphi_2}{\pi} - \alpha \frac{\varphi_2^2}{\pi^2} + 2\alpha \frac{\varphi_2 \cdot \varphi}{\pi^2} \quad \text{for } f_2 \neq f \neq 0, \quad \text{and} \quad (4)$$

$$S_0(\varphi) - S_2(\varphi) = -2\alpha \frac{\varphi_2}{\pi} - \alpha \frac{\varphi_2^2}{\pi^2} + 2\alpha \frac{\varphi_2 \cdot \varphi}{\pi^2} \quad \text{for } 2p \neq f \neq f_2 \quad (4a)$$

is shown at Fig.3 for two combinations of comparison windings –  $w_1$  ( $f=0$ ),  $w_2$  ( $f=p$ ) - solid line, and  $w_1$  ( $f=0$ );  $w_3$  ( $f=p/4$ ) -dotted line.

The MM output signal is proportional to the resulting magnetic field in the output winding. Then, even if number of turns and flowing current in both comparison windings are exactly the same, output signal may be equal to zero only for output winding position at the points with resulting magnetic field equal to zero:  $f=p/2$  or  $f=3p/2$  for the solid line, and  $f=p/8$  or  $f=9p/8$  for the dotted line, Fig. 3. In all other cases an error appears, positive for positive resulting field and negative in another case, with maximum relative value:

$$d_{\max} = \alpha \frac{\varphi_2}{\pi} \left( 2 - \frac{\varphi_2}{\pi} \right) \quad (5)$$

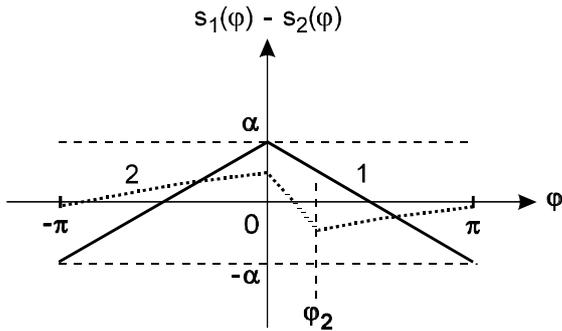


Figure 3. Relative inner field distribution

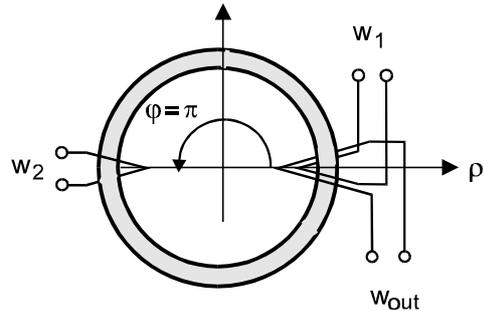


Figure 4. Narrow winding positions for the test

In the worst case (comparison windings are placed at the end of ring diameter,  $f_2 = \rho$ ) maximum relative error is equal to  $a$ , what is clear directly from Fig. 2. For small values of  $f_2$   $d_{\max} \gg 2af_2/\rho$ . For comparison and output windings, arbitrary distributed along the core ring, the winding density may be presented as Fourier set. Let us write the expressions connecting the internal and known external field intensities with the help of Fourier transform of  $S(f)$  and  $H_e$ , produced by comparison currents:

$$S(\varphi) = \sum_{n=0}^{\infty} S_n \cos n\varphi = 1 - \frac{2\alpha}{3} + \sum_{n=1}^{\infty} \frac{4\alpha}{\pi^2 n^2} \cos n\varphi; H_e = H_0 + \sum_{m=1}^{\infty} (H_{mc} \cos m\varphi + H_{ms} \sin m\varphi); \quad (6)$$

$$H_i(\varphi) = \frac{\int_0^{2\pi} H_e(\varphi_e) * S(\varphi - \varphi_e) d\varphi_e}{\int_0^{2\pi} S(\varphi) d\varphi} = H_0 + \frac{4\alpha}{\pi^2 \left(1 - \frac{2\alpha}{3}\right)} \sum_{n=1}^{\infty} \left( \frac{H_{nc}}{n^2} \cos n\varphi + \frac{H_{ns}}{n^2} \sin n\varphi \right),$$

where  $H_{mc} = H_{mc1} - H_{mc2}$ ;  $H_{ms} = H_{ms1} - H_{ms2}$  - the resulting Fourier components of heterogeneity of windings  $w_1$  ( $H_{mc1}$ ,  $H_{ms1}$ ) and  $w_2$  ( $H_{mc2}$ ,  $H_{ms2}$ ), correspondingly.

These equations show that average values of external and internal constant components of Fourier transform along the core ring are equal, but external field spatial harmonics are transformed into the internal field with the coefficients which are reciprocally proportional to the square harmonic number and with the common factor depending from the ratio of core size and magnetic permeability of core material.

Let us accept for simplifying that compared current  $I$  and number of turns are the same for both current windings,  $w_1 = w_2 = w$ . Then

$$H_e = I k_0 \left( 1 + \sum_{m=1}^{\infty} \left[ \frac{k_{mc}}{k_0} \cos m\varphi + \frac{k_{ms}}{k_0} \sin m\varphi \right] \right); \text{ where } k_0 = \frac{w}{2\pi R}; \frac{k_{mc}}{k_0} = \frac{H_{mc}}{H_0} \quad (7)$$

Then the interlinkage of  $M\tilde{N}$  output winding

$$\begin{aligned} \Psi_{out} &= \int_0^{2\pi} \dot{i}_o \dot{i}_m S H_i(\varphi) k(\varphi) R d\varphi = \\ &= \dot{i}_o \dot{i}_m H_0 S w \left( 1 + 4\pi\alpha \sum_{m=1}^{\infty} \left[ \frac{(k_{mc1} - k_{mc2}) k_{mc\ out}}{k_0 m^2} + \frac{(k_{ms1} - k_{ms2}) k_{ms\ out}}{k_0 m^2} \right] \right) \end{aligned} \quad (8)$$

Additional item after the «one» in round brackets of equation (8) is the relative error of current comparison, produced by heterogeneous output and current windings of  $M\tilde{N}$ :

$$\delta = 4\pi\alpha \sum_{m=1}^{\infty} \left[ \frac{(k_{mc1} - k_{mc2}) k_{mc\ out}}{k_0 m^2} + \frac{(k_{ms1} - k_{ms2}) k_{ms\ out}}{k_0 m^2} \right] \quad (9)$$

It is seen that the “heterogeneity error” appears only with simultaneously existing three heterogeneity of every Fourier component - two of current windings and the third - of output winding. It is essential that the part of full error, what depends on higher harmonics of heterogeneity, decreases quickly with number of harmonic. Then, to achieve perfection of symmetry it is enough to make approximately equal number of turns in every one fourth or sixth part of core length.

The most important result of analysis is that the values of parameters  $\mu_m$  and  $\sqrt{s}/R$  produce reciprocal changes in the comparison error and coefficient of proportionality  $a$  may be done very small.

### 3 EXPERIMENTAL RESULTS

Laboratory test of the obtained results was made on the MÑ with current and measuring windings having maximum possible heterogeneity - the narrow sections shifted on the angle  $180^\circ$  on the core ring (Fig. 4, see upper).

For this construction all the parameters  $k_{ms}$ ,  $k_{mc}$  are close to  $d$  - function and the resulting error, calculated by the equation (9), aspires to  $d_{max} = a$  (see also Fig. 2). The parameters of MÑ were: ring cores  $D \cdot d = 30 \cdot 27$  mm,  $2 \cdot 10$  rings with  $D=0.05$  mm; core material mumetal 80HM;  $w_1=w_2=w_3=10$  turns; maximum relative magnetic permeability, calculated with using the value of sensitivity - 67000. Calculated and measured errors of MÑ -  $d_{meas} = 14\%$  and  $d_{ct} = 1.7\%$  respectively. Taking into account rough simplification in the mathematical analysis the coincidence between measured and calculated error can be considered as good enough.

The laboratory model of MÑ with varying coefficient of comparison  $K_c$  was made on the base of MM with built-in air inductance [5]. The comparison error  $\sim 10^{-7}$  (0.00001 %) was realised by comparison equal currents  $I_1=I_2=10 \dots 50$  mA. The set of  $K_c=M/N$ , where  $M$  and  $N$  are arbitrary numbers each having values from 1 to 1000 with a step equal to 1, is realized. The comparison error achieved  $2 \cdot 10^{-6}$  in the worst case but this value may include additional error of measuring equipment. The measurements of minimal errors were made by connection of both current windings in opposite direction; calculated additional error produced by leakage resistance for such a method was found negligible.

In perspective the MM with 4 cores and  $90^\circ$  shifted excitation currents (QMM) may be recommended [6]. The simplified functional diagram of MC using QMM is given on Fig.5

QMM includes two pairs of cores  $C_{11}, C_{12}$  and  $C_{21}, C_{22}$  (ring cores are shown schematically as linear) with two pairs of excitation windings  $w_{e11}, w_{e12}$  and  $w_{e21}, w_{e22}$ , producing opposite magnetic fluxes. Two excitation channels, containing blocks 1,2,3 and 1,4,5,6 produce excitation currents, which phases are shifted at  $90^\circ$ . The phase of double-frequency output voltage at two output windings  $w_{out1}$  and  $w_{out2}$  is shifted at  $180^\circ$ . Output windings are connected in opposite, then (with taking into account  $180^\circ$  phase shift) their output voltages are added.

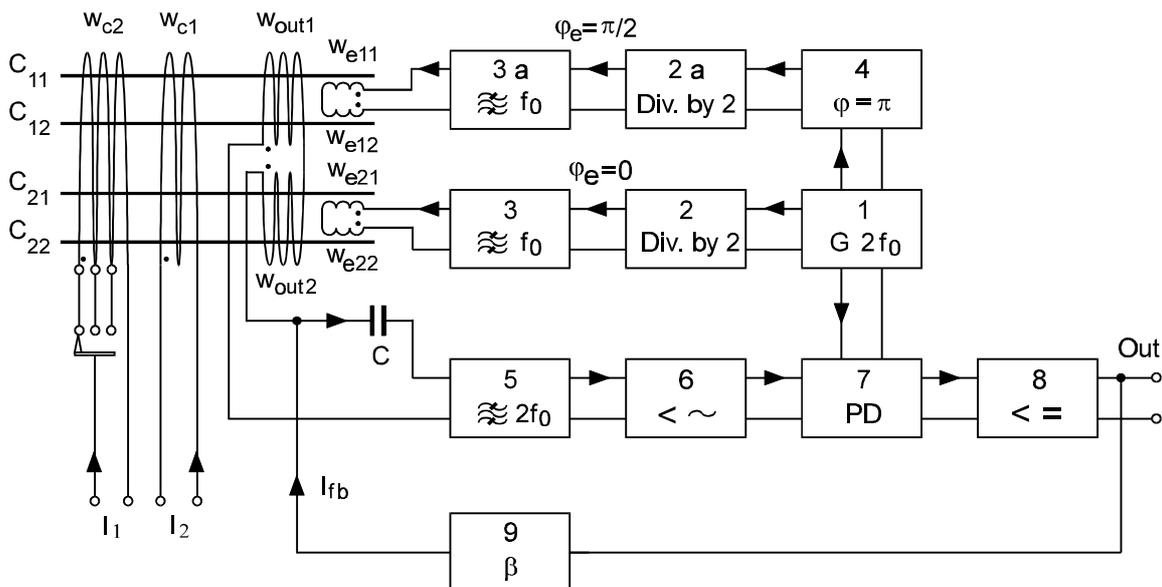


Figure 5. Magnetic comparator using quad-shifted excitation mode for magnetic modulator

The measuring channel (blocks 5,6,7,8) includes filter at double frequency, alternating voltage amplifier, phase detector PD and output direct voltage amplifier. Feedback circuit is used for stabilising the sensitivity of MC with QMM. Capacitor C is used for the prevention of short-circuiting feedback current  $I_{fb}$  through the input of filter 5.

Windings  $w_{c1}, w_{c2}$  with comparison currents  $I_1, I_2$  are connected in opposite directions. Number of turns of winding  $w_{c1}$  may be changed by switching the sections.

The main advantage of QMM is that equivalent summarised magnetic permeability of all cores is approximately constant. Then, summarising magnetic flux produced by comparison windings is also constant (it is only "switched" between two pairs of cores). By this one pair of cores works as magnetic shield for another pair and magnetic heterogeneity of every pair decrease.

Comparison measurements of MC errors at the laboratory samples with narrow windings by using usual MM and QMM were made. The decreasing of heterogeneity error in about 30 times was realised at different cases. The obtained results show that heterogeneity error less than  $10^{-7}$  may be realised in MC with multisectional comparison windings and consequently wide choice of comparison coefficients.

#### 4 CONCLUSION

The practical recommendations for MC design were found on the basis of mathematics analysis which allow to decrease the additional comparison errors to negligible values. A new construction of MC was proposed using magnetic modulators with double set of cores and with excitation of each couple by voltages shifted in phase by  $90^\circ$ . It allows to create the precise MC, on the basis of which it is possible even in the case of moderate requirements to the homogeneity of signal and measuring windings and also to heterogeneity of magnetic material properties along the core (non-homogeneity of the order of some percents) to create the MC with the error lower than  $10^{-7}$  in the wide range of comparison coefficients. The proposed MC can be used, besides to the metrologically high class systems, for the realization of current followers in the insulated circuits with given  $K_C$ , stabilizers of strong currents and other devices which have to operate long time with as high as possible precision.

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**AUTHOR:** Dr. R.Ja. BERKMAN, Department of Electromagnetic Investigations, Lviv Centre of Institute of Space Research, 5A Naukova St., 79000 Lviv, Ukraine, Phone int. +380 0322 63 91 63, Fax int. +380 0322 63 82 44, E-mail: [berkman@isr.lviv.ua](mailto:berkman@isr.lviv.ua)