

UNCERTAINTY SCOPE OF THE FORCE CALIBRATION MACHINES

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Abstract: Using the method specified in EAL-G22 (new ref. EA-10/04) with corresponding mathematical model functions for each type of force calibration machine and providing a practical example, the paper describes the approach adopted to apply EAL-R2 (new ref. EA-4/02) to the statement of the best measurement capability achievable for forces. The scope of the calibration laboratory can thus be defined according to the criteria of the accreditation body ensuring that the normalized error remains < 1 , when inter-laboratory comparisons are performed.

Keywords: Measurement uncertainty, Force calibration machine, Transfer Standard, Inter-laboratory comparison

1 INTRODUCTION

The EAL-G22 (new ref. EA-10/04) describes the measurement and evaluation method for the determination of the best measurement capability (uncertainty of link-up measurements) achievable when forces are measured with a force calibration machine [1]. Since publication of the EA Document "Expression of the Uncertainty of Measurement in Calibration" (EAL-R2, new ref. EA-4/02), a harmonized method for the determination and statement of the measurement uncertainty in calibration has been available for calibrations performed in all areas of physical measurements [2,3]. With a view to ensuring uniform assignment of the best measurement capability, it is, therefore, demanded that the accredited calibration laboratories generally apply the evaluation method described in EA-4/02. Using the method specified in EA-10/4 and providing practical examples, the paper describes the approach adopted to apply EA-4/02 to the statement of the best measurement capability achievable for forces.

2 DIFFERENT TYPES OF FORCE CALIBRATION MACHINES AND THEIR TRACEABILITY

For the calibration of force measuring devices, accredited calibration laboratories use in general force calibration machines (FCM) designed to realize forces by one of the methods stated below:

1. Force generation by means of deadweights (direct loading)
2. Hydraulic amplification
3. Lever amplification
4. Comparison with a calibrated force reference transducer or build-up system

In several publications the different design principles of these types of calibrations machines are described. Within the frame work of German Calibration Service (DKD) the traceability of a force calibration machine should be defined in two steps applying the method of Guideline EA-10/04. In the 1st step the input quantities of the force step generated should be directly traceable. The force step generated should then be traced back in the 2nd step with the aid of a transfer standard (several transfer standards, if necessary) on a force standard machine (FSM) through comparison measurements in compliance with EA-10/04 (link-up with the national force standard machine).

3 INPUT QUANTITIES

In the following the input quantities for different types of force calibration machines are listed. In cases of some force calibration machines it may be necessary to take additional influencing quantities into account as input quantities, in addition to the input quantities listed below:

Input quantities of step 1:

- a) Force generation by means of deadweights (direct loading)
 - the mass of the deadweight (m),
 - the local acceleration due to gravity at the place where the machine has been installed (g_{loc}),

- the air density (r_L),
 - the density of the deadweights used (r_m).
- b) Hydraulic or lever amplification
- the mass of the deadweight (m),
 - the local acceleration due to gravity at the place where the machine has been installed (g_{loc}),
 - the air density (r_L),
 - the density of the deadweights used (r_m),
 - the multiplication ratio (Q).
- c) Comparison with a calibrated force reference transducer
- the mean calibration value as a result of the reference force transducer calibration (F_{RefTra}).

Input quantities of step 2:

- the relative deviation ($\Delta_{Traceability}$) determined for the force generated, which is due to tracing it back to the primary force standard machine.

4 MATHEMATICAL MODEL

4.1 Force generation by means of deadweights (direct loading)

$$F_{FCM} = m \cdot g_{loc} \left(1 - \frac{r_L}{r_m}\right) \cdot (1 - \Delta_{Traceability}) \quad (1)$$

with

$$\begin{aligned} \Delta_{Traceability} &= \frac{\bar{F}_{FCM} \cdot (1 - \Delta_{RelDev}) \cdot (1 - \Delta_{HysFCM}) - \bar{F}_{FSM} \cdot (1 - \Delta_{Drift_TraStd}) \cdot (1 - \Delta_{Realization})}{\bar{F}_{FSM} \cdot (1 - \Delta_{Drift_TraStd}) \cdot (1 - \Delta_{Realization})} \\ &\approx \frac{\bar{F}_{FCM} \cdot (1 - \Delta_{RelDev}) \cdot (1 - \Delta_{HysFCM})}{\bar{F}_{FSM}} - 1 + \Delta_{Drift_TraStd} + \Delta_{Realization} \end{aligned} \quad (2)$$

where

- \bar{F}_{FCM} : mean value of forces indicated by the transfer standard in the force calibration machine (FCM)
- \bar{F}_{FSM} : mean value of the forces indicated by the transfer standard in the force standard machine (FSM)
- Δ_{Drift_TraStd} : relative long-term drift of the transfer force transducer (interval between measurements at NMI and in the calibration laboratory)
- $\Delta_{Realization}$: relative standard uncertainty of force realization at PTB
- Δ_{HysFCM} : relative hysteresis of the FCM determined taking the hysteresis of the transfer standard in the FSM into account
- Δ_{RelDev} : relative deviation of the mean force values indicated between FCM and FSM

The measurement method used for the link-up measurement is described in section 4.1 of EA-10/04. F_{FCM} and F_{FSM} are determined by measurements performed in n mounting positions (preferably $n=4$). The repeatability error is determined in at least one mounting position by at least one repeat measurement. This is necessary to assess to qualify the force calibration machine basically. However, the contribution of the repeatability error to the uncertainty is not taken into account in the model, as this contribution for the qualified FCM is normally negligible.

4.2 Force generation by means of hydraulic or lever multiplication

$$F_{FCM} = m \cdot g_{loc} \left(1 - \frac{r_L}{r_m}\right) \cdot Q \cdot (1 - \Delta_{Traceability}) \quad (3)$$

where

- $\Delta_{\text{Traceability}}$: calculation according to equation 2,
 Q : multiplication ratio (The change of the multiplication ratio, for example due to temperature change, can be considered into the uncertainty of the multiplication ratio.)

4.3 Force generation using a reference force transducer system

$$F_{\text{FCM}} = F_{\text{RefTra}} \cdot (1 - \Delta_{\text{Drift_RefTra}}) \cdot (1 - c_J) \cdot (1 - \Delta_{\text{Traceability}}) \quad (4)$$

where

- F_{RefTra} : result of the reference force transducer calibration
 $\Delta_{\text{Drift_RefTra}}$: relative long-term drift of the reference force transducer calibration
 c_J : relative change in the calibration result due to the temperature change
 $\Delta_{\text{Traceability}}$: calculation according to equation 2

5 EVALUATION OF UNCERTAINTY OF MEASUREMENT

On the assumption that the quantities are uncorrelated, the combined relative variance is obtained from the random error propagation law. The relative standard uncertainty u_{rel} is expressed as w [2].

5.1 Force generation by means of deadweights

$$w(F_{\text{FCM}}) = \sqrt{w^2(m) + w^2(g_{\text{loc}}) + \left(-\frac{r_L}{r_m}\right)^2 \cdot w^2(r_L) + \left(\frac{r_L}{r_m}\right)^2 \cdot w^2(r_m) + w^2(\Delta_{\text{Traceability}})} \quad (5)$$

where

$$w^2(\Delta_{\text{Traceability}}) = w^2(\bar{F}_{\text{FCM}}) + w^2(\Delta_{\text{HysFCM}}) + w^2(\Delta_{\text{Drift_TraStd}}) + w^2(\bar{F}_{\text{FSM}}) + w^2(\Delta_{\text{RelDev}}) + w^2(\Delta_{\text{Realization}}) \quad (6)$$

with

$$w^2(\bar{F}_{\text{FCM}}) = \frac{\sum_{i=1}^n (x_i - \bar{x}_{\text{FCM}})^2}{n(n-1) \cdot \bar{x}_{\text{FCM}}^2} \quad (7)$$

$$w^2(\Delta_{\text{HysFCM}}) = \frac{a_{\text{HysFCM}}^2}{3} \quad (8)$$

$$w^2(\Delta_{\text{Drift_TraStd}}) = \frac{a_{\text{Drift_TraStd}}^2}{3} \quad (9)$$

$$w^2(\bar{F}_{\text{FSM}}) = \frac{\sum_{i=1}^n (x_i - \bar{x}_{\text{FSM}})^2}{n(n-1) \cdot \bar{x}_{\text{FSM}}^2} \quad (10)$$

$$w^2(\Delta_{\text{RelDev}}) = \frac{(\bar{x}_{\text{FCM}} - \bar{x}_{\text{FSM}})^2}{24 \cdot \bar{x}_{\text{FSM}}^2} = \frac{a_{\text{RelDev}}^2}{6} \quad (11)$$

$$w(\Delta_{\text{Realization}}) = 1 \cdot 10^{-5} \text{ (relative standard uncertainty of realization)}$$

where

- a : relative half-width of the maximum deviation of the respective influence quantity
- n : number of rotational positions
- x_i : readings indicated for the measurements with a transfer transducer (one value per rotational position)
- \bar{x}_{FCM} : mean measurement indication of a transfer standard determined from n rotational positions in the force calibration machine
- \bar{x}_{FSM} : mean measurement indication of a transfer standard determined from n rotational positions in the force standard machine

5.2 Force generation by hydraulic or lever amplification

$$w(F_{FCM}) = \sqrt{w^2(m) + w^2(g_{loc}) + \left(-\frac{r_L}{r_m}\right)^2 \cdot w^2(r_L) + \left(\frac{r_L}{r_m}\right)^2 \cdot w^2(r_m) + w^2(Q) + w^2(\Delta_{Traceability})} \quad (12)$$

$w^2(\Delta_{Traceability})$ being calculated according to equations 6 to 11.

5.3 Force generation using a reference force transducer system

$$w(F_{FCM}) = \sqrt{w^2(F_{RefTra}) + w^2(\Delta_{Drift_RefTra}) + w^2(c_J) + w^2(\Delta_{Traceability})} \quad (13)$$

$w^2(\Delta_{Traceability})$ being calculated according to equations 6 to 11. $w^2(F_{RefTra})$ is calculated in compliance with EA-10/04; the uncertainty of the influence quantity 'hysteresis' of the reference force transducer can either be neglected or at least be substantially reduced when the systematic component of the hysteresis is appropriately taken into account when the reference force transducer is used in the force calibration machine.

Table 1. Inter-laboratory comparison results – FCM amplification or comparator type

Measurement carried out in a force standard machine					
F_{FSM}	x_{0°	x_{90°	x_{180°	x_{270°	\bar{x}_{FSM}
600 kN	1222663	1222670	1222655	1222663	1222663
800 kN	1630128	1630130	1630130	1630125	1630128
1000 kN	2037463	2037488	2037480	2037467	2037475
800 kN	1630365				
600 kN	1223093				
F_{FSM}	hysteresis		spread	$w(F_{FSM})$	
600 kN	430		15	2,5E-06	
800 kN	237		5	7,2E-07	
1000 kN			25	2,8E-06	
Measurement carried out in force calibration machine, comparison					
F_{FCM}	x_{0°	x_{90°	x_{180°	x_{270°	\bar{x}_{FCM}
600 kN	1222415	1222423	1222405	1222393	1222409
800 kN	1629783	1629775	1629770	1629768	1629774
1000 kN	2037013	2037020	2037007	2037003	2037011
800 kN	1630015				
600 kN	1222825				
F_{FCM}	hysteresis	Δ_{HysFCM}	spread	$w(F_{FCM})$	Δ_{RelDev}
600 kN	410	-1,6E-05	30	5,3E-06	-2,1E-04
800 kN	232	-3,1E-06	15	2,1E-06	-2,2E-04
1000 kN			17	1,8E-06	-2,3E-04

6 CALCULATION OF THE NORMALIZED ERROR

A fraction of the relative deviation Δ_{RelDev} determined at the end of the inter-laboratory comparison measurement can be taken into account computationally for compensation in the form of an estimate (for example, in Table 2, column 2) affected by a corresponding measurement uncertainty, provided a reproducible characteristic influence caused by the force calibration machine is approved [4]. Generally, the relative deviation determined, or at least a fraction left, cannot be compensated. On the one hand, the relative deviation is, therefore, treated as a *random variable* and, on the other, it is used to calculate the normalized error. In compliance with EA-2/03 (previously EAL-P7), the *normalized error* E_n is calculated as follows [5]:

$$E_n = \frac{\Delta_{\text{RelDev}}}{W} \quad (14)$$

Table 2. Uncertainty for a multiplication type FCM and normalized error for the 600 kN force step

quantity	estimate	relative half-width value <i>a</i>	probability distribution	relative standard uncertainty	sensitivity coefficient	relative uncertainty contribution
<i>m</i>	6117,584 kg	1,0E-05	rectangular	5,8E-06	1	5,8E-06
<i>g_{loc}</i>	9,80923 m/s ²	5,0E-07	rectangular	2,9E-07	1	2,9E-07
ρ_L	1,150 kg/m ³	3,0E-02	rectangular	1,7E-02	1,50E-04	2,6E-06
ρ_m	7850 kg/m ³	1,0E-02	rectangular	5,8E-03	1,50E-04	8,7E-07
<i>Q</i>	10	4,0E-05	rectangular	2,3E-05	1	2,3E-05
$\Delta_{\text{Realization}}$	0 kN		normal	1,0E-05	1	1,0E-05
F_{FSM}	600 kN		normal	2,5E-06	1	2,5E-06
F_{FCM}	600 kN		normal	5,3E-06	1	5,3E-06
Δ_{HysFCM}	0 kN	8,2E-06	rectangular	4,7E-06	1	4,7E-06
$\Delta_{\text{Drift_TraStd}}$	0 kN	3,0E-05	rectangular	1,7E-05	1	1,7E-05
Δ_{RelDev}	- 0,09 kN	2,9E-05	triangular	1,2E-05	1	1,2E-05
F_{FCM}	599,91 kN					3,4E-05
expanded rel. uncertainty $W = k w(F_{\text{FCM}})$ for $k = 2$						6,8E-05
normalized error E_n related to W						0,85
specification of best measurement capability W_{bmc}						1,0E-04
normalized error E_n related to best measurement capability						0,58

Table 3. Uncertainty for a compactor type FCM and normalized error for the 600 kN force step

quantity	estimate	relative half-width value <i>a</i>	probability distribution	relative standard uncertainty	sensitivity coefficient	relative uncertainty contribution
F_{RefTra}	600 kN		normal	5,0E-05	1	5,0E-05
$\Delta_{\text{Drift_RefTra}}$	0 kN	2,0E-04	rectangular	1,2E-04	1	1,2E-04
c_J	0 kN	$\alpha * \Delta_T = 5,0E-5$	rectangular	2,9E-05	1	2,9E-05
$\Delta_{\text{Realization}}$	0 kN		normal	1,0E-05	1	1,0E-05
F_{FSM}	600 kN		normal	2,5E-06	1	2,5E-06
F_{FCM}	600 kN		normal	5,3E-06	1	5,3E-06
Δ_{HysFCM}	0 kN	8,2E-06	rectangular	4,7E-06	1	4,7E-06
$\Delta_{\text{Drift_TraStd}}$	0 kN	3,0E-05	rectangular	1,7E-05	1	1,7E-05
Δ_{RelDev}	0 kN	1,1E-04	triangular	4,3E-05	1	4,3E-05
F_{FCM}	600 kN					1,4E-04
expanded rel. uncertainty $W = k w(F_{\text{FCM}})$ for $k = 2$						2,8E-04
normalized error E_n related to W						0,76
specification of best measurement capability W_{bmc}						5,0E-04
normalized error E_n related to best measurement capability						0,42

The expanded uncertainty of measurement W should not be greater than the best measurement capability (bmc) of the calibration laboratory. At the end of the evaluation, *the relative best measure-*

ment capability W_{bmc} is, however, specified such that the condition $E_n < 1$ is always guaranteed for the normalized error.

Table 4. Consideration of systematic deviation for all steps of table 1 and statement of the bmc

F_{FCM}	Δ_{RelDev}	W	$E_n = \Delta_{RelDev} / W$	W_{bmc}	$E_{n,bmc} = \Delta_{RelDev} / W_{bmc}$
599,91 kN	5,8E-05	6,8E-05	0,85	1,0E-04	0,58
799,88 kN	6,8E-05	6,9E-05	0,98	1,0E-04	0,68
999,85 kN	7,8E-05	7,1E-05	1,10	1,0E-04	0,78

7 MEASUREMENT RESULTS AND BEST MEASUREMENT CAPABILITY

Table 1 shows as an example results of a transfer standard achieved in the inter-laboratory comparison measurements. Three to five transfer standards are normally required for the total range of a FCM to be calibrated and verified in one force direction. Independent of the type of the FCM similar presentation of the results can be achieved for different types of force calibration machines. The time schedule of the loading cycles in force standard machine and in force calibration machine should be kept very similar. The readings achieved at each rotational position are indicated values of a high precision amplifier. The upper part of the table shows the measurement results in a force standard machine and the corresponding uncertainty of the mean indication value for the increasing forces. The lower part of the table shows the measurement results in a FCM and the corresponding uncertainty for the increasing forces. The values shown for the Δ_{HysFCM} are the relative differences of the hysteresis between both machines. Finally, the table shows the relative deviation of the mean indications between FCM and FSM.

Table 2 shows for this example computations performed to determine the best measurement capability for the 600 kN force step in case of a FCM with lever or hydraulic multiplication. The estimates for $\Delta_{Realization}$, $\Delta_{Drift_TraStad}$ and Δ_{HysFCM} are considered to have the values of 0 kN, but their uncertainty cannot be neglected as indicated in the 3rd and 5th column. Although no hysteresis can be measured at the nominal force of the transfer standard, it is expedient to use the highest value of Δ_{HysFCM} for both, the nominal force and all other steps in the range. The results of the relative deviations in the last column of Table 1 show that a systematic influence is concerned here which can be compensated by computation. Accordingly, a realistic fraction of $1,5 \cdot 10^{-4}$ (which is in this example - 0,09 kN) is taken into account in the computation of the uncertainty of measurement as an estimate of Δ_{RelDev} (see Table 2, column 2) with the remaining relative deviation of $5,8 \cdot 10^{-5}$ (half-width $a = 2,9 \cdot 10^{-5}$) as uncertainty contribution.

In the case of deadweight machines the Δ_{RelDev} should not be compensated, unless it is caused by an incorrect calibration of the deadweights and its amount is known.

Table 3 shows the results of computations in case that the results of Tabel 1 have been achieved in FCM with a reference force transducer system. The basis for the operation of the FCM is the calibration equation of the reference force transducer determined in a FSM. The uncertainty contributions are the uncertainty of the calibration results of the reference force transducer, its long-term drift and temperature sensitivity. The estimates for $\Delta_{Drift_TraStad}$ and Δ_{HysFCM} are considered in the same way as in the case of an amplification machine, but as in the case of the deadweight machines the relative deviations will not be considered as systematic deviation. In order to achieve expanded measurement uncertainty $W <$ best measurement capability and simultaneously achieve $E_n < 1$, the specified best measurement capability should be increased as shown in the last two lines of the Table 3.

Finally, Table 4 shows how the best measurement capability (column 5) for all force steps of an amplification FCM can be clearly reduced when the new estimates are taken into consideration. For the last force step it is additionally necessary to increase W_{bmc} from $7,1 \cdot 10^{-5}$ to $1,0 \cdot 10^{-4}$ to fulfil $E_n < 1$.

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