

A SIX-COMPONENT STANDARD: A FEASIBILITY STUDY

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Abstract: Two schemes of possible transducer-based six-component standards are presented. The equations describing the functioning of the apparatuses have been formulated and the analysis of the uncertainty has been performed. Thereupon, information on the achievable metrological behaviours and on the criteria to be adopted for designing some of the mechanical elements is provided.

Keywords: Multicomponent Standard; Measurement of Force, Mass and Torque

1 INTRODUCTION

The aim to achieve a world-wide unitary value for the quantity force had determined, in the past decades, a demand for the investigation into the effect of the parasitic components that force primary standards apply to dynamometers during their calibration [1]. Furthermore, in both the fields of force and torque measurement, a comprehensive comparison among different transducers cannot leave the evaluation of their sensitivity to parasitic components aside [2, 3]. To serve this purpose, standards capable of generating the desired components with uncertainties in the order of a few per cent constitute very often a satisfying solution. However, recent years have seen arising the demand for standards capable of generating all the components with relative uncertainties smaller than 1% (e.g., for the development of multicomponent transducers for monitoring stress in buildings and bridges and to ensure traceability in robotics). In such cases the full calibration in vectorial terms is often required, rather than the evaluation of the sensitivity to one or more components of a single output transducer [4].

In this work, two schemes of transducer-based six-component standards are proposed. By applying the laws of statics, a generalized mathematical model describing the functioning of such apparatuses has been formulated. Thereupon, the analysis of the uncertainty has been performed by assuming reliable values for the uncertainties of the various input quantities. On the basis of the obtained results, in order to optimise the performances of the system, some criteria to be adopted to design the critical mechanical elements have been deduced.

2 TRANSDUCER-BASED, MULTICOMPONENT STANDARDS

A system capable of applying known values of all the components of the force and moment vectors (\mathbf{F} and \mathbf{M} , respectively) to multicomponent transducers (MCT) constitutes a multicomponent standard (MCS). This work is devoted to systems adopting force and/or torque transducers. A MCS can be realised by connecting the two ends of the MCT to two plates, the first one (loading plate) connected to a set of actuators capable to generate the components, and the second (measuring plate) equipped with a set of – minimum – six force and/or torque devices D_j ($j = 1, \dots, 6$). Fig. 1 shows schemes of the measuring plate of two such MCS. The first one (fig. 1.a) adopts six dynamometers (F), whereas the second (fig. 1.b) is equipped with three dynamometers (F) and three torque transducers (M).

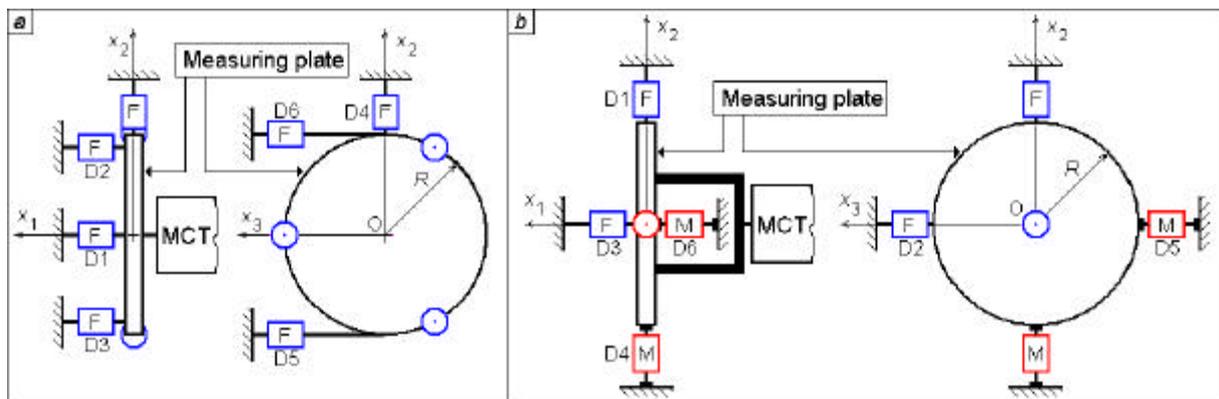


Figure 1. Schemes of MCS adopting: a) dynamometers; b) dynamometers and torque transducers

Depending on the nature of the j^{th} device D_j (dynamometer or torque transducer), the j^{th} output represent a measurement of either the local axial force or the local torque. The remaining five local components that are to be defined constitute a set of spurious components that influence the uncertainty in use of the devices D_j . In order to enhance the metrological performances of the MCS, a reduction of the spurious components below given values is mandatory.

2.1 Equations

A reference system of co-ordinates $\{O; x_k\}$ is defined for the MCS. Its origin O corresponds to a well-identified point located at the end of the MCT connected to the MCS (here supposed to be coincident with the centre of the measuring plate), and its axes and unitary vectors are x_1, x_2, x_3 and $\hat{i}_1, \hat{i}_2, \hat{i}_3$, respectively. Making x_1 match the MCT axis makes F_1 and M_1 represent the axial force and the torque, respectively, and F_2, F_3 and M_2, M_3 represent the components of the transverse force and of the bending moment, respectively.

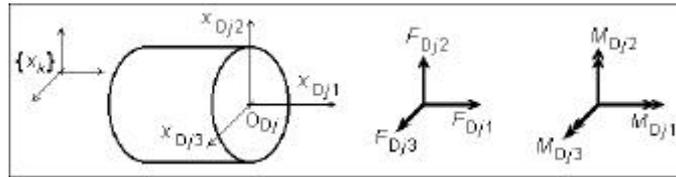


Figure 2. Local system of co-ordinates

Furthermore, a local system of co-ordinates $\{O_{Dj}; x_{Dji}\}$ (fig. 2) is defined for each device D_j . Its origin O_{Dj} is coincident with a well-identified point located at the end of the device connected to the measuring plate, and its axes and unitary vectors are $x_{Dj1}, x_{Dj2}, x_{Dj3}$ and $\hat{i}_{Dj1}, \hat{i}_{Dj2}, \hat{i}_{Dj3}$, respectively. The vector

$$\mathbf{r}_{Dj} = \sum_{k=1}^3 r_{Djk} \hat{i}_k \quad (1)$$

represents the position of the j^{th} device with respect to $\{O; x_k\}$. All the $\{O_{Dj}; x_{Dji}\}$ are supposed to be parallel to $\{O; x_k\}$, i. e.

$$\hat{i}_{Dji} \cdot \hat{i}_k = 0 \text{ or } 1. \quad (2)$$

Making x_{Dj1} match the axis of the j^{th} device makes F_{Dj1} represent the local axial force, F_{Dj2} and F_{Dj3} the components of the local transverse force, M_{Dj1} the local torque, M_{Dj2} and M_{Dj3} the components of the local bending moment.

With respect to $\{O; x_k\}$, \mathbf{F} and \mathbf{M} can be expressed as

$$\mathbf{F} = \sum_{k=1}^3 F_k \hat{i}_k; \quad (3)$$

$$\mathbf{M} = \sum_{k=1}^3 M_k \hat{i}_k. \quad (4)$$

The force and moment vectors transmitted to the plate by the j^{th} device can be written as

$$\mathbf{F}_{Dj} = \sum_{i=1}^3 F_{Dji} \hat{i}_{Dji}; \quad (5)$$

$$\mathbf{M}_{Dj} = \sum_{i=1}^3 M_{Dji} \hat{i}_{Dji}. \quad (6)$$

respectively, and the corresponding components with respect to $\{O; x_k\}$ assume the following expressions

$$F_{Djk} = \mathbf{F}_{Dj} \cdot \hat{i}_k = \sum_{i=1}^3 F_{Dji} \cos \alpha_{k,Dji}; \quad (7)$$

$$M_{Djk} = \mathbf{M}_{Dj} \cdot \hat{i}_k = \sum_{i=1}^3 M_{Dji} \cos \alpha_{k,Dji}, \quad (8)$$

where $\alpha_{k,Dji}$ is the angle between \hat{i}_k and \hat{i}_{Dji} .

The components of \mathbf{F} and \mathbf{M} with respect to $\{O; x_k\}$ can be written as follows:

$$F_k = \sum_{j=1}^6 F_{Djk} ; \quad (9)$$

$$M_k = \sum_{j=1}^6 [M_{Djk} + \varepsilon_{klm} r_{Djl} F_{Djm}] , \quad (10)$$

where $\varepsilon_{klm} r_{Djl} F_{Djm}$ is the k^{th} component of the vector $r \times F$ (ε_{klm} is the Ricci symbol).

Eqs. (5) to (10) constitute the mathematical model of the measurement. They establish a relationship between the measurand and the forces and/or the moments measured by a set of six devices. The model accounts even for the effect of the spurious components transmitted from the devices to the measuring plate and takes into account the geometry of the system.

2.2 Uncertainties

By applying the ISO-GUM to eq. (9) in the hypothesis of uncorrelated quantities

$$u^2(F_k) = \sum_{j=1}^6 \sum_{i=1}^3 \left(\frac{\partial F_k}{\partial F_{Dji}} \right)^2 u^2(F_{Dji}) + \sum_{j=1}^6 \sum_{i=1}^3 \left(\frac{\partial F_k}{\partial \alpha_{k,Dji}} \right)^2 u^2(\alpha_{k,Dji}) , \quad (11)$$

and by substituting the derivatives obtained from eqs. (7) and (9), the following relationship can be written for the uncertainty of the force components F_k ($k = 1, \dots, 3$)

$$u^2(F_k) = \sum_{j=1}^6 u^2(F_{Djk}) , \quad (12)$$

where

$$u^2(F_{Djk}) = \sum_{i=1}^3 \left[\cos^2 \alpha_{k,Dji} u^2(F_{Dji}) + F_{Dji}^2 \sin^2 \alpha_{k,Dji} u^2(\alpha_{k,Dji}) \right] . \quad (13)$$

As can be observed, $u(F_k)$ depends on the uncertainty in use of the devices D_j , on the spurious forces applied to the measuring plate by the devices and on the accuracy in identifying the lines of action of the forces.

By applying the ISO-GUM to eq. (10)

$$u^2(M_k) = \sum_{j=1}^6 \left\{ \left(\frac{\partial M_k}{\partial M_{Djk}} \right)^2 u^2(M_{Djk}) + \sum_{t \neq k} \left[\left(\frac{\partial M_k}{\partial r_{Djt}} \right)^2 u^2(r_{Djt}) + \left(\frac{\partial M_k}{\partial F_{Djt}} \right)^2 u^2(F_{Djt}) \right] \right\} , \quad (14)$$

and by substituting the derivatives calculated from eqs. (7), (8) and (10), the following relationship can be obtained for the uncertainty of the moment components M_k ($k = 1, \dots, 3$)

$$u^2(M_k) = \sum_{j=1}^6 \left[u^2(M_{Djk}) + \varepsilon_{klm}^2 (F_{Dji} u(r_{Djm}))^2 + \varepsilon_{klm}^2 (r_{Dji} u(F_{Djm}))^2 \right] , \quad (15)$$

where $u^2(F_{Djm})$ is given by eq. (13) and

$$u^2(M_{Djk}) = \sum_{i=1}^3 \left[\cos^2 \alpha_{k,Dji} u^2(M_{Dji}) + M_{Dji}^2 \sin^2 \alpha_{k,Dji} u^2(\alpha_{k,Dji}) \right] . \quad (16)$$

2.3 Results

For both the proposed MCS schemes, the uncertainties of the six components have been calculated. In both cases, the MCS has been supposed capable of generating forces and moments up to ± 10 kN and ± 1 kN·m, respectively, and values of 0,5 m and 0 m have been assumed for R and the thickness of the measuring plate, respectively. Furthermore, the following hypotheses have been formulated.

- All the spurious components, localised in the origins O_{Dj} of the local systems of co-ordinates, have been regarded as generated by friction-like phenomena. Therefore, they have been considered as unbiased quantities. A maximum admissible value has been assumed for each kind of spurious component, and the corresponding uncertainty has been supposed equal to a third of such value (i. e., $u(F_{Dj1,sp}) = F_{Dj1,sp,max}/3$, $u(F_{Dj2-3}) = F_{Dj2-3,max}/3$, etc.).
- The uncertainty of the main local component of each transducer D_j ($u(F_{Dj1})$ or $u(M_{Dj1})$ for dynamometers and torque transducers, respectively) has been regarded as a constant over the entire measuring range.
- All the mechanical elements of the MCS (measuring plate, transducers, couplings, etc.) have been

considered as rigid bodies.

Table 1. Uncertainties

		Fig. 1.a				Fig. 1.b			
		case A		case B		case A		case B	
$u(F_{D1}) / N$		2		1		2		1	
$F_{D1,sp,max} / N$		---		---		1		$1 \cdot 10^{-1}$	
$F_{D2-3,max} / N$		1		$1 \cdot 10^{-1}$		1		$1 \cdot 10^{-1}$	
$u(M_{D1}) / N \cdot m$		---		---		$2 \cdot 10^{-1}$		$1 \cdot 10^{-1}$	
$M_{D1,sp,max} / N \cdot m$		1		$1 \cdot 10^{-1}$		1		$1 \cdot 10^{-1}$	
$M_{D2-3,max} / N \cdot m$		1		$1 \cdot 10^{-1}$		1		$1 \cdot 10^{-1}$	
$u(\alpha_{k,Dij}) / rad$		$2 \cdot 10^{-3}$		$4 \cdot 10^{-4}$		$2 \cdot 10^{-3}$		$4 \cdot 10^{-4}$	
$u(r_{Djk}) / m$		$1 \cdot 10^{-4}$		$2 \cdot 10^{-5}$		$1 \cdot 10^{-4}$		$2 \cdot 10^{-5}$	
C_f / u	Value	$u(C_f) / u$	$u_r(C_f)$	$u(C_h) / u$	$u_r(C_f)$	$u(C_f) / u$	$u_r(C_f)$	$u(C_h) / u$	$u_r(C_f)$
F_1 / N	$1 \cdot 10^3$	4	$4 \cdot 10^{-3}$	2	$2 \cdot 10^{-3}$	4	$4 \cdot 10^{-3}$	1	$1 \cdot 10^{-3}$
F_2 / N	$1 \cdot 10^3$	3	$3 \cdot 10^{-3}$	1	$1 \cdot 10^{-3}$	4	$4 \cdot 10^{-3}$	1	$1 \cdot 10^{-3}$
F_3 / N	$1 \cdot 10^3$	4	$4 \cdot 10^{-3}$	1	$1 \cdot 10^{-3}$	4	$4 \cdot 10^{-3}$	1	$1 \cdot 10^{-3}$
$M_1 / N \cdot m$	$1 \cdot 10^2$	2	$2 \cdot 10^{-2}$	$8 \cdot 10^{-1}$	$8 \cdot 10^{-3}$	2	$2 \cdot 10^{-2}$	$3 \cdot 10^{-1}$	$3 \cdot 10^{-3}$
$M_2 / N \cdot m$	$1 \cdot 10^2$	1	$1 \cdot 10^{-2}$	$6 \cdot 10^{-1}$	$6 \cdot 10^{-3}$	1	$1 \cdot 10^{-2}$	$2 \cdot 10^{-1}$	$2 \cdot 10^{-3}$
$M_3 / N \cdot m$	$1 \cdot 10^2$	2	$2 \cdot 10^{-2}$	$7 \cdot 10^{-1}$	$7 \cdot 10^{-3}$	1	$1 \cdot 10^{-2}$	$2 \cdot 10^{-1}$	$2 \cdot 10^{-3}$
F_1 / N	$1 \cdot 10^4$	20	$2 \cdot 10^{-3}$	5	$5 \cdot 10^{-4}$	30	$3 \cdot 10^{-3}$	6	$6 \cdot 10^{-4}$
F_2 / N	$1 \cdot 10^4$	20	$2 \cdot 10^{-3}$	4	$4 \cdot 10^{-4}$	30	$3 \cdot 10^{-3}$	6	$6 \cdot 10^{-4}$
F_3 / N	$1 \cdot 10^4$	20	$2 \cdot 10^{-3}$	5	$5 \cdot 10^{-4}$	30	$3 \cdot 10^{-3}$	6	$6 \cdot 10^{-4}$
$M_1 / N \cdot m$	$1 \cdot 10^3$	10	$1 \cdot 10^{-2}$	2	$2 \cdot 10^{-3}$	10	$1 \cdot 10^{-2}$	3	$3 \cdot 10^{-3}$
$M_2 / N \cdot m$	$1 \cdot 10^3$	2	$2 \cdot 10^{-3}$	$6 \cdot 10^{-1}$	$6 \cdot 10^{-4}$	10	$1 \cdot 10^{-2}$	2	$2 \cdot 10^{-3}$
$M_3 / N \cdot m$	$1 \cdot 10^3$	10	$1 \cdot 10^{-2}$	3	$3 \cdot 10^{-3}$	10	$1 \cdot 10^{-2}$	2	$2 \cdot 10^{-3}$

Two loading conditions have been considered. Firstly, both MCS have been supposed to generate $F_1 = F_2 = F_3 = 1 \cdot 10^4 N$ and $M_1 = M_2 = M_3 = 1 \cdot 10^3 N \cdot m$; secondly, all the components have been reduced to a tenth of the values assumed formerly. Furthermore, two distinct sets of reliable values have been assumed for the uncertainties of the input quantities. In the latter case all values have been reduced by given amounts with respect to the former case. The results are shown in tab. 1.

Table 2. Effect of reducing single input quantity uncertainty

		X_h													
		F_{D1}	F_{D2-3}	$M_{D1,sp}$	M_{D2-3}	$\alpha_{k,Dij}$	r_{Djk}	F_{D1}	$F_{D1,sp}$	F_{D2-3}	M_{D1}	$M_{D1,sp}$	M_{D2-3}	$\alpha_{k,Dij}$	r_{Djk}
C_f / u	Value	$\Delta_r u(C_f) / \Delta_r u(X_h) / \% - Fig. 1.a$							$\Delta_r u(C_f) / \Delta_r u(X_h) / \% - Fig. 1.b$						
F_1 / N	$1 \cdot 10^3$	57	1	0	0	21	0	25	0	1	0	0	0	47	0
F_2 / N	$1 \cdot 10^3$	40	3	0	0	30	0	25	0	1	0	0	0	47	0
F_3 / N	$1 \cdot 10^3$	48	1	0	0	26	0	25	0	1	0	0	0	47	0
$M_1 / N \cdot m$	$1 \cdot 10^2$	39	1	4	4	21	0	0	0	2	1	2	9	57	0
$M_2 / N \cdot m$	$1 \cdot 10^2$	61	0	2	15	0	0	0	0	1	1	3	14	45	0
$M_3 / N \cdot m$	$1 \cdot 10^2$	32	1	3	6	27	0	0	0	1	1	3	14	45	0
F_1 / N	$1 \cdot 10^4$	1	0	0	0	94	0	0	0	0	0	0	0	98	0
F_2 / N	$1 \cdot 10^4$	0	0	0	0	96	0	0	0	0	0	0	0	98	0
F_3 / N	$1 \cdot 10^4$	1	0	0	0	95	0	0	0	0	0	0	0	98	0
$M_1 / N \cdot m$	$1 \cdot 10^3$	1	0	0	0	92	0	0	0	0	0	0	0	96	0
$M_2 / N \cdot m$	$1 \cdot 10^3$	40	0	2	10	0	19	0	0	0	0	0	0	93	1
$M_3 / N \cdot m$	$1 \cdot 10^3$	0	0	0	0	93	0	0	0	0	0	0	0	93	1

In order to evaluate the influence of each input quantity on both the proposed MCS schemes, another calculation procedure has been followed. One at a time, the uncertainty of each kind of input quantity has been reduced from the value hypothesised in case A to the one of case B (see tab. 1). Thereupon, for both the considered loading conditions, the uncertainties of all the components have been recalculated. The values assumed by the ratio

$$\Delta_r u(C_f) / \Delta_r u(X_h). \tag{17}$$

of the relative reduction in the uncertainty of each component $-\Delta_r u(C_i) = \Delta u(C_i)/u(C_{i,\text{case A}})$ – to the relative reduction in the uncertainty of the considered input quantity $-\Delta_r u(X_h) = \Delta u(X_h)/u(X_{h,\text{case A}})$ – can quantify the corresponding effect. Tab. 2 reports the results obtained from eq. (17), expressed as a percentage.

In the case of the MCS adopting only dynamometers, as can be observed, $u(M_2)$ results smaller than both $u(M_1)$ and $u(M_3)$. Besides, $u(M_2)$ results insensitive to reductions in $u(\alpha_{k,Dji})$, whereas it shows a high sensitivity to $u(F_{Dj1})$ and to the uncertainty in the spurious components. Such behaviours, enhanced by having assumed a null thickness of the measuring plate, are peculiar to the particular geometry of the system. Indeed (see fig. 1.a), for all the devices it is $r_{Dj1} = 0$, and $r_{D43} = r_{D53} = r_{D63} = 0$. By writing eq. (15) for $k = 2$, it can be noticed that twelve of the thirty terms at the right hand side result equal to zero, thus producing the effects evidenced.

3 CONCLUSION

A comparison between the proposed MCS schemes shows that systems equipped with both force and torque transducers can achieve higher performances than apparatuses adopting only dynamometers. Indeed, given that in the former case the moment components are measured directly by means of torque transducers, the accuracy in generating the moments results less sensitive to the uncertainty in use of the dynamometers. In addition, it is very unlike an actual transducer to behave such that its uncertainty can be considered as a constant over all the measuring range. From this point of view, whereas torque standards can allow the calibration for both clockwise and anticlockwise torque without disconnecting the transducer from the calibration facility, dynamometers cannot be calibrated in the region close to the unloaded condition. As a consequence, in the former case it is possible to quantify the uncertainty of calibration within a region that results wider than in the latter one [5].

As can be observed from the values reported in tab. 2, the smaller the components, the greater the effect of the spurious components on the accuracy of the MCS. Moreover, spurious components degrade the metrological performances of both force and torque transducers. It is therefore mandatory to give the matter a careful consideration, especially by means of a shrewd design of the couplings connecting the transducers to the measuring plate. For either a dynamometer or a torque transducer, such mechanical elements have to allow all the displacements but the translational or the rotational one, respectively, with respect to x_{Dj1} . Moreover, because of the noticeable influence exhibited by $u(\alpha_{k,Dji})$, a careful design of the couplings has even to optimise the accuracy in defining the position of the lines of action of both forces and moments. By adopting particular mountings of cross-knives elements associated with friction-reducing systems, satisfying solutions can be obtained for dynamometers. Efforts have to be put into the research on adequate solutions in the case of torque transducers.

As formulated above, all the mechanical elements of the MCS (measuring plate, transducers, couplings, etc.) have been considered as rigid bodies. Such hypothesis makes the positions of the lines of actions of force and moment vectors do not depend on the loading condition. Therefore, $u(\alpha_{k,Dji})$ and $u(r_{Djk})$ have been regarded as constants over the entire measuring range. Actually, compliance can produce sensible effects on both $\alpha_{k,Dji}$ and r_{Djk} . In light of the meaningful contribution to the uncertainty of the MCS shown by $\alpha_{k,Dji}$, a preliminary evaluation of the effect determined by the compliance of the system is suitable.

The results presented in this work show that transducer-based MCS can generate force and moment components with relative uncertainties smaller than 1%. Such goal can be achieved on condition that all the meaningful contributions to the uncertainty are maintained below reasonable values. To serve this purpose, the previous elements are to be taken into careful consideration.

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