

CALIBRATION DESIGNS OF MASS STANDARDS FOR 2 COMPARATORS IN A DECADE SERIES

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Abstracts: The decade series of 10-5-2-2'-1 and 10-5-3-2-1 for submultiples of the unit of mass have been studied to find designs where two comparators are used. The criterion used in finding the designs was to minimize, using the method of least squares, the sum of the absolute values of the elements in the inversion matrices of the normal equations. To date researchers only have reported that when more than one comparator is used in a decade using an orthogonal design for one comparator the orthogonality is lost to produce covariances. However this study has found several cases where orthogonal designs can be used for using two comparators in a decade. The designs could reduce both the nonorthogonality by 50 % and the overall variances by about 45 %. Even though the orthogonal designs are for the case where the ratio of the comparator's standard deviation is 2, the combination of most comparators could be used with the orthogonal designs.

Keywords : mass standards, orthogonal, calibration design

1 INTRODUCTION

Much attention has been paid to the cases where only one comparator is used for comparison of mass standards of a decade series. Researchers just have mentioned that when an orthogonal design for one comparator is used for the case of two comparators the orthogonal property is lost to produce covariances.[1,2] Therefore it is necessary to make sure that if there are designs for the case of two comparators which still keep the orthogonality. Also it is natural to extend the number of weighing more than the usual 12 or 14 weighings in 10-5-2-2'-1 series or 10-5-3-2-1 series, as fully automated commercial mass comparators are available. The objective in the search for better designs is to those weighing designs that give a minimum value of variance or covariance with an increased number of weighing. Addition and repetition of elementary measurements increase the number of weighing.

General treatments of the least-squares method relevant to the present study have already been discussed [1,2,3]. Here the statistical procedure and notation of Cameron et al.[4] are applied. The decade series 10-5-2-2'-1 and 10-5-3-2-1 were studied to find designs that reduce the variances at increased number of weighing[5]. Here it is considered that the case where two comparators of significantly different precision are used. In the original least-squares method when two comparators are used the errors due to the comparator of lower degree of precision are going to be minimized at the cost of the comparator of higher degree of precision. To straighten this bias, the concept of weighting is employed through a matrix, W , which has only diagonal elements which are $1, 1/c_2^2, \dots, 1/c_n^2$ for n observations, where c_n is a ratio of standard deviation to the comparator of higher degree of precision. Therefore design matrix X and observation matrix Y are transformed to weighted matrices, $Y^* = W^{1/2}Y$ and $X^* = W^{1/2}X$. The variances and covariances of the mass values are directly proportional to the inversion matrices of the newly transformed normal equation, C_{ij} and to the variance of the comparator of $n=1$. Therefore the criterion used in finding the designs was to minimize, using the method of least squares, the sum of the absolute values of the elements in the inversion matrices of the normal equations. Such designs could be evaluated by computation as in the previous study, where all the possible cases of weighing are considered [5].

2 DESIGNS FOR WEIGHING CALIBRATION

The elementary measurements in the series of 10-5-2-2'-1 and 10-5-3-2-1 are given in Table 1[5]. For simplicity the designs can each be expressed as a row vector G whose elements are the numbers of weighings in the corresponding row of Table 1.

Table 1. Elementary measurements in each series

Series 10-5-2-2'-1							Series 10-5-3-2-1						
Row	10	5	2	2'	1	1	Row	10	5	3	2	1	1
1	1	-1	-1	-1	-1		1	1	-1	-1	-1	1	-1
2	1	-1	-1	-1		-1	2	1	-1	-1	-1	-1	1
3		1	-1	-1	-1		3	1	-1	-1		-1	-1
4		1	-1	-1		-1	4	1	-1	-1	-1		
5			1	-1	1	-1	5		1	-1	-1		
6			1	-1	-1	1	6		1	-1	-1	1	-1
7			1	-1			7		1	-1	-1	-1	1
8			1		-1	-1	8		1	-1		-1	-1
9				1	-1	-1	9			1	-1	-1	
10					1	-1	10			1	-1		-1
							11				1	-1	-1
							12					1	-1

Two comparators could be used in as one of the following patterns according to their capacities.

- Pattern A : (10, 5, 3, 2), (1)
- Pattern B : (10, 5, 3), (2, 1)
- Pattern C : (10, 5), (3, 2, 1)
- Pattern D : (10), (5, 3, 2, 1)

Each pattern consists of 2 blocks with 2 parentheses where the second parenthesis requires the second comparator of better precision. For example, Row 5 of the Series 10-5-2-2'-1 of Table 1 requires the first comparator in Pattern B, but requires the second comparator in Pattern C. As the pattern goes from A to D, the precision of the overall measurements are improved because most of the measurements are carried out by the better comparator of precision.

The designs which are searched are to give minimized sums of the absolute values of the elements in the inversion matrices of the normal equations where two comparators of different variances are used.

3 RESULTS

The results of search have produced designs that reduce the variances and covariances for the case where two comparators are used in a decade for submultiples of the unit of mass. Number of weighing was increased up to 20 for all cases to find designs. The tested inverse "ratio of standard deviation" of the comparators, c_n , were 2, 4, and 8. Table 2 shows the designs which have orthogonality for 10-5-2-2'-1 series. Table 3 also shows the designs, which have orthogonality for 10-5-3-2-1 series. Both Table 2 and Table 3 are for $1/c_n$ of 2. Other values of $1/c_n$ did not produce cases of orthogonality. In the tables Sum $C_{ij, Total}$ means the sum of the absolute values of all the elements in the inversion matrices of the normal equations. And the values of "Sum $C_{ij, Total}$ " are normalized by those of the orthogonal design of 12 weighings and c_n of 1.

Table 2. Orthogonal designs for 10-5-2-2'-1 series with 2 comparators

Pattern	Number of weighing	Sum $C_{ij, Total}$	G'
A			None
B	14	0.500	2 2 2 2 0 0 2 1 1 2
	16	0.500	2 2 2 2 2 2 1 1 1 1
	18	0.500	2 2 2 2 4 4 0 1 1 0
C	12	0.500	2 2 2 2 1 1 0 1 1 0
	14	0.500	2 2 2 2 0 0 2 1 1 2
D	18	0.250	4 4 1 1 2 2 0 2 2 0
	20	0.250	4 4 1 1 1 1 2 2 2 2

Table 3. Orthogonal designs for 10-5-3-2-1 series with 2 comparators

Pattern	Number of weighing	Sum $C_{ij, Total}$	G'
A			None
B			None
C	16	0.500	2 2 2 0 4 0 0 2 1 1 1 1
D			None

Figure 1 shows the case where two comparators are used while the design is for one comparator. The lower line indicates the sum of off-diagonal elements of C_{ij} , and the upper line indicates the sum of all elements of C_{ij} . As the inverse ratio of standard deviation increased on from 1, orthogonality disappears and the upper line shrinks by a little amount.

Figure 2 shows the case where two comparators are used with the designs in Table 2. As the inverse ratio of the comparator's standard deviation increases from 1, the upper line shrinks to 0.5 and the lower line reduces to zero when the ratio is 2. As the inverse ratio increases on, the upper line keeps the lowered value and the lower line increases to 0.1 when the inverse ratio reaches to 8. Comparing Figure 1 and 2 we could find that the designs in Table 2 reduce the nonorthogonality by about 50 % and the over-all variances indicated by the upper lines by about 45 %.

Figure 2 is for not only integer values of the ratio of comparator's standard deviation but also decimal numbers. Therefore it indicates that the combination of most commercial comparators could be used with the designs in Table 2 and 3.

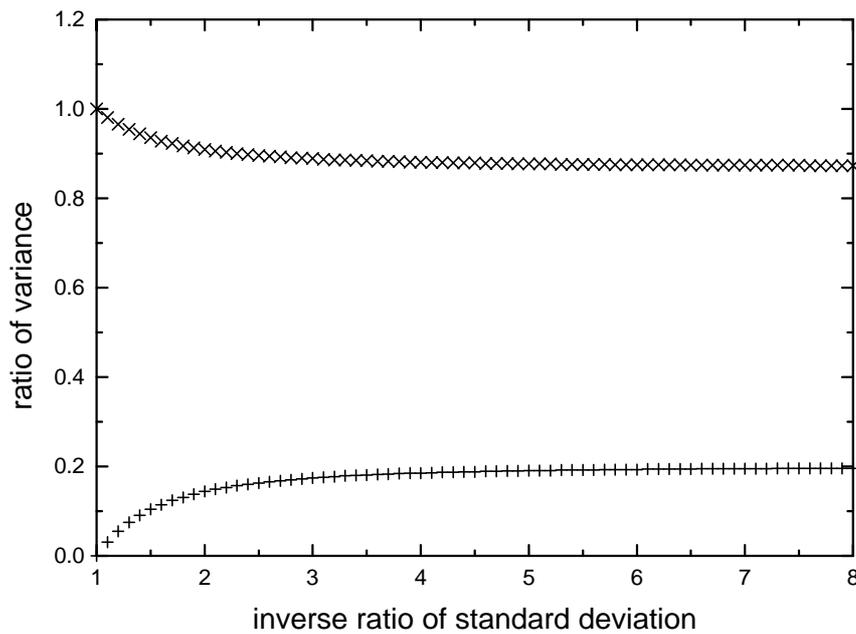


Figure 1. Application of the design for one comparator to the cases for two comparators

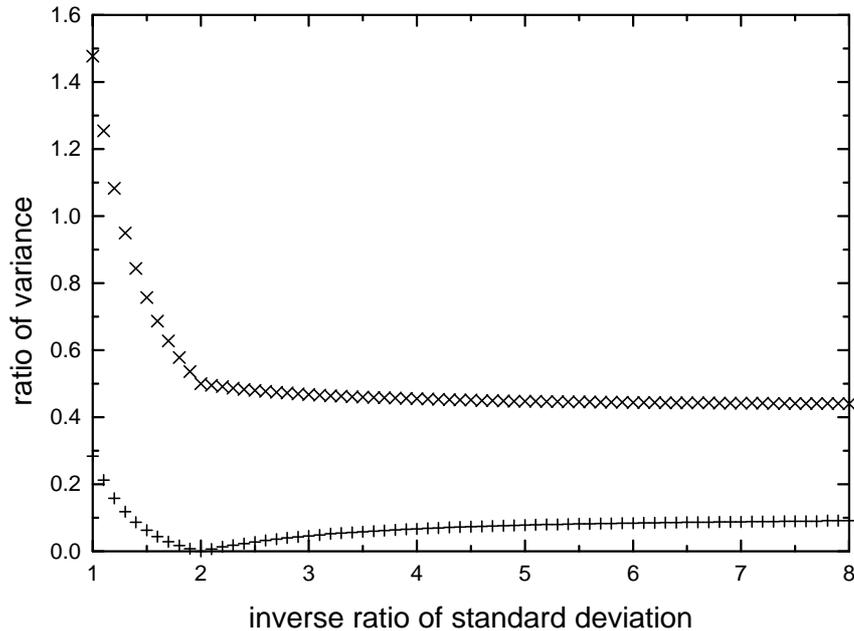


Figure 2. Application of the design for two comparator to the cases for two comparators

4 CONCLUSIONS

Orthogonal designs have been found where two comparators are used in a decade series of 10-5-2-2⁻¹ and 10-5-3-2-1 for submultiples of the unit of mass.

The designs could reduce both the nonorthogonality by 50 % and the over-all variances by about 45 %.

Even though the orthogonal designs are for the case where the ratio of the comparator's standard deviation is 2, the combination of most comparators could be used with the orthogonal designs.

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