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PECULIARITIES OF CYLINDER DIAMETER DETERMINATION BY DIFFRACTION METHOD

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Abstract: The detailed analysis of diffraction pattern in far zone for circular absolutely reflecting metal cylinder with regard to the influence of wave reflected from its surface is presented. The method of stationary phase was used when calculating the reflected component of the input field. The behaviour of diffraction pattern minima for wide range of angles is studied. The formula for angular position of minima in case of small observation angles was found. Three algorithms of cylinder diameter reconstruction using its diffraction pattern were proposed. They allow theoretically to increase appreciably the accuracy of the dimensional measurements.
Keywords: Fraunhofer diffraction, cylinder, optical inspection, laser metrology

1 INTRODUCTION

The metallic bodies (or their fragments) with reflecting cylindrical surfaces are frequently used as objects for dimensional geometrical parameters measurement. While precision measuring the geometrical parameters of such extended objects (with error about 1 μm) by shadow and diffraction methods as opposed to plane objects (zero thickness) it arises the necessity of taking into account the diffraction phenomena peculiarities on 3D bodies. Since the strict solutions of light diffraction on metallic circular cylinder with *R* radius [1] are very consuming for engineering applications and known works [2 - 4] in this field are devoted to the exclusively experimental investigations of diffraction on cylinder, an urgent demand in the constructive theory of Fraunghofer phenomena on such bodies appears. This theory must be simple and at the same time sufficiently strict, analogous to the proposed early for the extended object of constant thickness [5].

It is essential, that for the cylinders with perfectly reflecting surfaces the main contribution to far zone field (when the wavelength $l << R$) gives the component, caused by the incident wave reflection from the front part of cylinder [2 - 4]. This field component is the additional to the known one, corresponding to diffraction on plane screen the same size $D = 2R$ (plane analogue of cylinder). It brings about the shift of position of diffraction pattern minima. As a result in determination (reconstruction) of cylinder diameter by the same algorithms as for the plane objects, deviations take place between the experimental data and expected ones. In so doing the measurement error of cylinder diameter by diffraction method (versus *D* and spatial bandwidth) can be found from units to tens microns [2, 3]. In our opinion the absence of the above mentioned constructive theory of diffraction phenomena on cylinders gives no way to receive the satisfactory explanation to the experimentally observed effects.

Preliminary results of investigations of diffraction phenomena on circular perfectly reflecting cylinders were presented by us in [6]. The aim of this work is more detailed study of peculiarities for Fraunhofer diffraction phenomena on perfectly reflecting circular cylinder, including calculation and analysis of influence of reflected component on its diffraction pattern (spectrum), as well as its comparison with the case of plane screen, study of behaviour of the spectrum minima and then the search for more precise and constructive (applicable for engineering calculations) algorithms for diameter determination by its diffraction pattern.

2 DIFFRACTION PATTERN FOR ABSOLUTELY REFLECTING CYLINDER

Let us consider the light diffraction in far zone (Fraunhofer diffraction) by metal circular cylinder, illuminated by monochromatic plane wave with length l (Fig. 1). Suppose that the main contributions into the diffraction field $Å(q)$, observed by angle q, give two components: $Å_{ref}(q)$ and $Å_{trans}(q)$, determined by diffraction of the wave, reflected from the front part of cylinder, and usual diffraction of wave in transmission light by plane screen with the size $D = 2R$, i.e.:

$$Å(q) = Å_{trans}(q) + Å_{ref}(q), \qquad (1)$$

Taking into consideration the peculiarities of incident wave reflection from cylinder surface [4], one can obtain in optogeometrical approximation the following equivalent input distribution $f(x)$ (in diametral plane of cylinder):

$$f(x) = 1 - rect\left(\frac{x}{2R}\right) - b\ exp\left\{ j2kR[\frac{2}{3}(|x| - R)]^{\frac{3}{2}} \right\} \cdot Y(|x| - R), \qquad (2)$$

where $k = 2p/l$ – wave number, $Y(x)$ is the step Heaviside function, β is some coefficient.

To receive in analytical form the field in far zone $Å(q)$, which is Fourier transformation of the input distribution $f(x)$, we used under $R/l >> 1$ the stationary phase method when calculating the reflected component $Å_{ref}(q)$. As a result, the normalized field intensity in far zone (when $q ^1 0$) is described by the expression:

$$I(q) = \mathrm{sinc}^2(k|q|R) + 0.125b^2\frac{l}{R}|q| + 0.25b \times$$
$$\mathrm{sinc}(k|q|R)\sqrt{|q|\frac{l}{\mathrm{R}}}\cos(k|q|^3 R/8 + k|q|R - \frac{p}{4}), \qquad (3)$$

where $\mathrm{sinc}(z) = \sin z / z$.

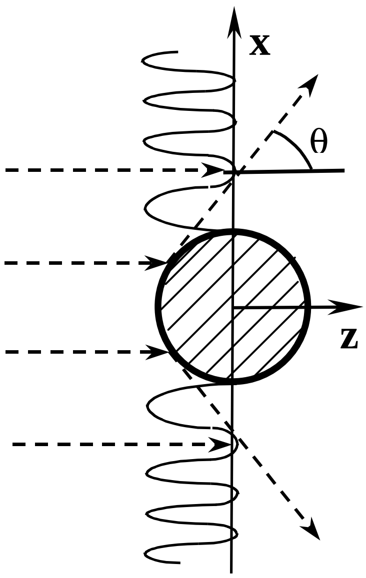

Figure 1. Ray configuration for light diffraction by metal circular cylinder of radius R.

Except the first term, $I_0(q)=sinc^2(kqR)$, corresponding to the usual diffraction by plane screen (plane screen spectrum), there are two additional terms, due to the influence of wave, reflected from the front part of cylinder. It is substantially, that their influence on distribution $I(q)$ appreciably increases at angles $q ^3 q_{cr.}^{(0)} = \sqrt[3]{l / R}$, when volume effects are strong enough, i.e. when $kRq^3/8 = p/4$.

As it will be indicated below the most essential contribution to the field gives the last interference component, which brings to the appearance of low-frequency oscillations in spectrum among the high-frequency ones. As a result, instead of equidistant located minima in spectrum (that is valid for plane analogue of cylinder) one can observe more complicated dependence.

3 CYLINDER DIFFRACTION PATTERN ANALYSIS

Let's study the peculiarities of behaviour of distribution (3) and compare it with plane screen spectrum $I_0(q)=sinc^2(kqR)$.

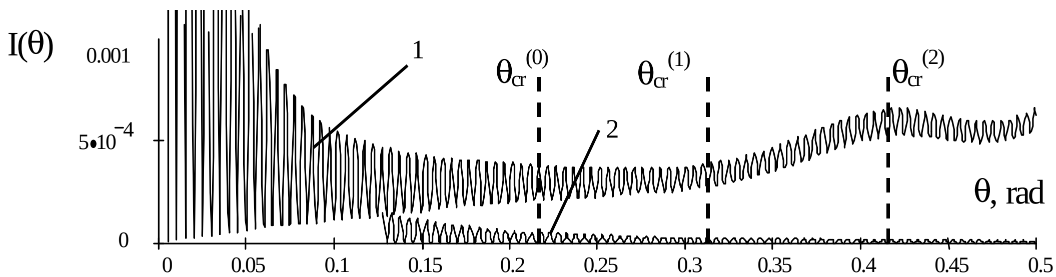

Figure 2. The typical diffraction patterns for the circular cylinder (curve 1) and plane screen of the same size $D = 2R$ spectrum (curve 2) at large angles (R = 60 µm and λ = 0,633 µm).

In Fig. 2 (curve 1) the typical diffraction pattern for the circular cylinder with R = 60 µm and λ = 0,633 µm at angles $q > q_{cr}^{(0)}$ is shown. One can see that as compared to plane screen spectrum (curve 2) under $q > q_{cr}^{(0)}$ there is the intensity increasing due to the influence of reflected component in diffraction pattern. In so doing, simultaneously with the high-frequency modulation (like Fraunhofer type), caused by the interference of two fields from opposite cylinder parts, there is the low-frequency modulation (by the influence of reflected component $Å_{ref}(q)$), the depth of which is decreasing for $q > q_{cr}^{(0)}$.

From the optical inspection point of view the position of diffraction minima is the main characteristic for the determining the cylinder radius *R* by diffraction method. The behaviour of cylinder diffraction pattern minima positions and their deviations from plane object's case were investigated in details.

As mentioned above, for plane screen the minima of its spectrum are equidistant, i.e. $\Delta q^{(0)} = q^{(0)}_{n+1} - q^{(0)}_{n} = l\ /\ D$. In case of small observation angles ($q << q_{cr}^{(0)}$) it was found [6] that for absolutely reflecting metal cylinders the angular position of n-th diffraction minima θ_n (under $b = 1/\sqrt{3}$) is determined by the following expression:

$$\hat{q}_n = q_n^{(0)} - \Delta q_n = ln/D - 0.25(l\ /\ D)^2 n^{3/2}, \qquad (4)$$

In Eq. (4) first term $q_n^{(0)} = ln/D$ corresponds to the light diffraction on plane screen, and the second one $-\Delta q_n = 0.25(l\ /\ D)^2 n^{3/2}$ – describes the contribution of reflected component. For example,

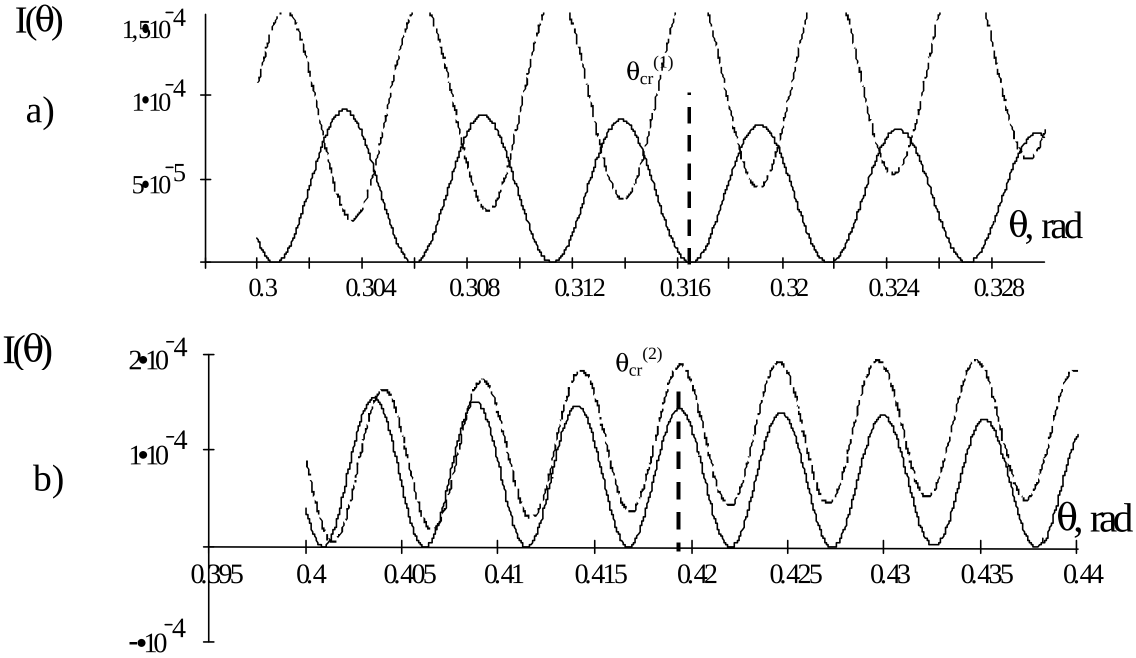

Figure 3. The diffraction patterns of the circular cylinder with diameter *D* (dash line) and plane screen with the size *D* (solid line) in the vicinity of the critical angles $\theta_{cr}^{(1)}$ (a) and $\theta_{cr}^{(2)}$ (b) at R = 60 µm, λ = 0.633 µm. For simplicity the positions and the scales of dash plots were changed.

at *R = 60* µm, *l = 0.633* µm, *n = 5* the value $\Delta q_n / q_n^{(0)}$ is $0.25 l \sqrt{n}/D \approx 0.3\%$. As the observation angle increases $\theta > \theta_{cr}^{(0)}$, the difference in minima position for the plane screen and the cylinder is growing. Moreover, at the certain angle $\theta = \theta_{cr}^{(1)}$ this difference is substantial so the position of minimum of distribution I(θ) coincides with the position of maximum of function $I_0(q) = sinc^2(kqR)$ (Fig. 3a).

The angle $\theta_{cr}^{(1)}$ can be obtained from the following condition: $\Psi = k q_{cr}^{(1)^3} R/8 - p/4 = p/2$. As a result, it equals to $q_{cr}^{(1)} = \sqrt[3]{3l/R}$. Under the given parameters R and λ the value $\theta_{cr}^{(1)}$ is about 18.1^0.

While further increasing the observation angle up to $q = q_{cr}^{(2)}$ number of minima of interference pattern for plane object and cylinder differs by one, i.e. the function $sinc^2(kqR)$ (the plane object case) has N minima and the distribution ***I(q)*** includes *N+1* minima (Fig. 3b). As for the critical angle $q_{cr.}^{(2)}$, it equals to $\sqrt[3]{7 l\ /\ R}$ (Ψ=3/2π) and for the same parameters *R* and *l* corresponds to $q_{cr}^{(2)}$ » 24^0.

In practice it's more convenient to deal rather with the number of diffraction minima than with angular minimum position. Let's define critical parameter N_{cr} as the number of diffraction minima of distribution $I_0(q)$ (with angular distance between them $q_0=l/D$), included in an angular range $0 \le q \le q_{cr}^{(2)}$. It is equal to $N_{cr}^{(2)} = q_{cr}^{(2)}/q_0 = 2\sqrt[3]{7(R/l)^2}$. Critical number for the same l and R equals to N_{cr} = 82.

4 CYLINDER DIAMETER RECONSTRUCTION ALGORITHMS

While reconstruction the cylinder diameter D by its diffraction pattern let's take as a zero approximation the standard algorithm applied to the case of plane screen. According to this algorithm the reconstructed cylinder diameter equals to $D_0 = ln/q_n$ (q_n is the position of n-th minimum in cylinder diffraction pattern). We have investigated in detail the behaviour of error, taking place in this case:

$$\Delta D = D_0 - D = ln/q_n - D, \tag{5}$$

In Fig.4 there is the dependence of error ΔD versus number *n.* One can see that the behaviour of

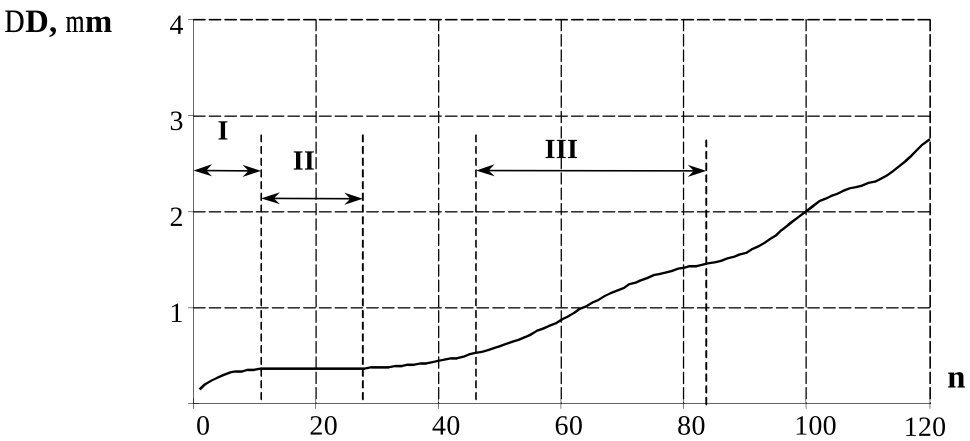

Figure 4. Error *DD(n)* versus number of diffraction minimum *n* under the cylinder diameter *(D)* reconstruction using the standard algorithm applied to the plane screen with the size *D,* zones I, II and III are spectrum regions for the reconstruction (l=0,633 µm, *D* =120 µm).

error *DD(n)* strongly depends on the observed region. So, in region of small angles (I) the error, according to Eq. (4) increases proportionally to $n^{1/2}$. In case of middle angles (region II) this error is a constant value. Finally, in the region of large angles (III) the error is $\Delta D \sim n^2$. Three algorithms of parameter *D* determination by cylinder diffraction pattern were developed.

The first algorithm uses some first minima of diffraction pattern and is based on Eq. (4). By solving this equation regarding to *D* (with known q_n and *n*) one may receive the following expression for the cylinder diameter *D* evaluation:

$$D_I = ln/q_n - 0.25 l \sqrt{n} \tag{6}$$

In such a way the error *DD* of diameter *D* is described by the simple equation:

$$\Delta D_I = 0.25 l \sqrt{n} \tag{7}$$

and, for example, at n = 4 equals to 0,5λ. Account of error *DD* accordingly to Eq.(7) allows to improve the measurement accuracy. The allowable upper limit for angle q_I, used for the reconstruction *D* equals to $q_I \le 0.1 q_{cr}^{(2)} = 0.1\sqrt[3]{7 l/R}$.

According to ***the second algorithm*** of diameter reconstruction while using the diffraction angles in the middle region for which ΔD_{II} = const, one can assume from the physical point of view that the error *DD* value is the following: $\Delta D_{II} \sim \sqrt[3]{l^2 D}$. It was established that proportional constant value is 0,1. As a result , one may receive the following expression for the error value:

$$\Delta D_{II} = 0.1\sqrt[3]{l^2 D_0} \tag{8}$$

In Fig. 5 there is a plot of function (8) depending on the diameter reconstruction error ΔD. The difference of approximated function ΔD_{II} from the real error value ΔD, for example, at R = 60 µm, λ = 0.633 µm is about 0.005 µm. The angle range, in which this approximation function is valid, is determined by inequality $0.2q_{cr}^{(2)} \le q_{II} \le 0.5q_{cr}^{(2)}$. One can see that the methodical error of given algorithm is negligible small and, in our opinion, the main contribution to error will give ones of minima angular position determination.

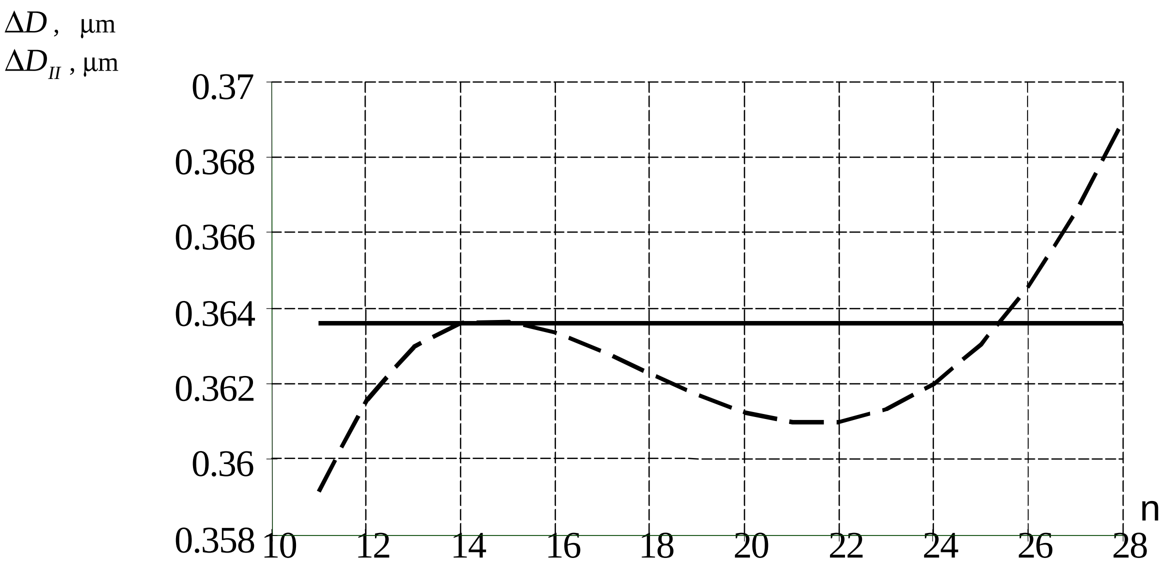

Figure 5. Error of diameter reconstruction ΔD *(n)* versus diffraction number *n* in the middle angular region while using the standard algorithm applied to plane screen (dashed plot).Approximation function ΔD_{II} is shown by solid plot.

Th-e third algorithm of cylinder diameter reconstruction is based on using its diffraction pattern under large observation angles (region III). Cylinder spectrum analysis allows to approximate error *DD* by the quadratic function, namely:

$$\Delta D_{III} = D_0 q^2 / 14, \tag{9}$$

where proposed angle range for reconstruction *D* is equal to $q_{cr}^{(1)} \le q_{III} \le q_{cr}^{(2)}$.

In Fig. 6 the plot of function (9) and dependence of error reconstruction by the third algorithm are presented. It is evident, that the dependence (9) sufficiently exactly describes the error in given range.

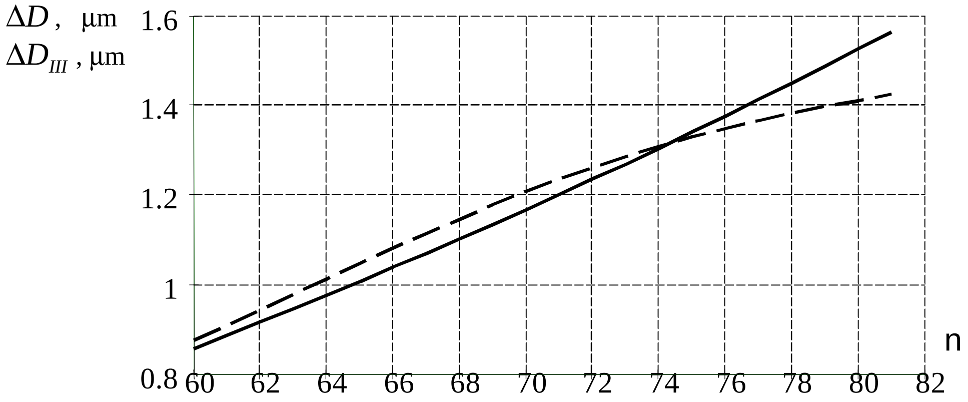

Figure 6. Error of diameter reconstruction ΔD *(n)* versus diffraction number *n* in the large angular region while using the standard algorithm applied to plane screen (dashed plot). Approximation function ΔD_{III} *(n)* is shown by solid plot.

The calculated maximal function ΛD_{III} deviation from the real error ΔD in this range is about ~ 0.1 μm. It should be, however, noted, that given calculations are based on equation (2), which true for small observation angles.

5 CONCLUSION

The obtained results of light diffraction by circular metal cylinder with regard to the influence of the wave reflected from its surface are presented in this work. The field calculation was made by stationary phase method under cylinder radius $R >> l$. It is shown that the cylinder diffraction pattern is appreciably different from the pattern for the plane screen (plane analogue of cylinder) with the same size $D = 2R$, for angles q ³ $q_{cr.} = \sqrt[3]{l / R}$. As a result of analysis of the diffraction minima behaviour the equation for their angular positions (case of small observation angles) was found. The three algorithms of cylinder diameter reconstruction using its diffraction pattern in various angle ranges were proposed. These algorithms allow theoretically to increase appreciably (tens of times) the accuracy of the dimensional measurements for metal circular cylinder, that requires, however, the experimental testing.

The obtained results could be applied to optical dimension inspection, optics and laser metrology.

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