XVI IMEKO World Congress
Measurement - Supports Science - Improves Technology - Protects Environment ... and Provides Employment - Now and in the Future
Vienna, AUSTRIA, 2000, September 25-28

THE TRANSFORMATION OF THE ROUNDNESS PROFILE

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Abstract: Reference (especially two- and three-point) methods could not be applied to the accurate measurement of roundness profiles because of various drawbacks. Theoretical and experimental studies have proved that the only way to eliminate these drawbacks is to mathematically transform the profile measured by a reference method into the real profile. The transformation, whose fundamentals are presented and discussed in this work, enabled the development of an original computer program. The mathematical model of the transformation was statistically tested by means of a specially developed computer measuring system, and the results provided basis for the construction of measuring systems applicable to accurate measurements of roundness profiles directly in production.
Keywords: roundness profile, reference method, transformation

1 INTRODUCTION

Industrial practice applies two groups of methods to measuring roundness profiles: the reference and non-reference methods. In the majority of cases nowadays measuring instruments are based on the latter ones. Since the measuring basis of the non-reference methods is the axis of the measured object, it is necessary that such instruments have a rotary table or a rotary spindle. Although the instruments assure high measuring accuracy, reaching even 0.1 μm, they require labour-consuming activities connected with the centring and alignment of the measured objects, and therefore, their application is limited mainly to laboratories. The instruments cannot be used in industrial conditions, at least, because of little measuring capacity. Modern industry needs instruments that can be used directly on the machine tool or the production test stand (see Ref. [1]). The requirements concern mainly the quick check of roundness profiles in the series production, and this type of check involves applying other, mainly reference, measuring methods.

From the above outline it appears that the contemporary engineering industry expects research centres to adapt reference methods to accurate measurements of roundness deviations. Hence, we can observe the renaissance of reference methods, whose applications, until now, have been limited to approximate measurements only.

2 CHARACTERISTICS OF REFERENCE MEASURING METHODS

In the reference methods used for measuring roundness deviations, one can distinguish between points of support (base points) and points of measurement. Their position with regard to the assumed co-ordinate system is determined by the angular parameters, that is the angles a and b. As shown in Fig. 1, the angle $2a$ is the angle between the tangents up to the supports of the measured object, whereas the angle b is the angle between the direction of measurement (i.e. direction of the displacement of the contact tip of the measuring instrument) and the co-ordinate axis X.

A characteristic feature of reference methods is that the measured deviation ΔF does dot coincides with roundness deviation ΔR. That is why the basic problem of their application is the necessity to obtain information on the dominant type of the form error (oval, trilobing, etc.), for which, by means of an appropriate coefficient, called the coefficient of detectability K_n, it is possible to estimate the roundness deviation ΔR

$$\Delta R = \frac{\Delta F}{K_n} \tag{1}$$

The coefficient of detectability for each harmonic is defined by

$$K_n(a, b) = \frac{F_n}{R_n} \tag{2}$$

where R_n and F_n stand for amplitudes of the n-th harmonic of the real and the measured profiles respectively, that is

$$R(j) = \sum_{n=1}^{\infty} R_n \cos(jn + j_{Rn}) , \; F(j) = \sum_{n=1}^{\infty} F_n \cos(jn + j_{Rn}) . \qquad (3)$$

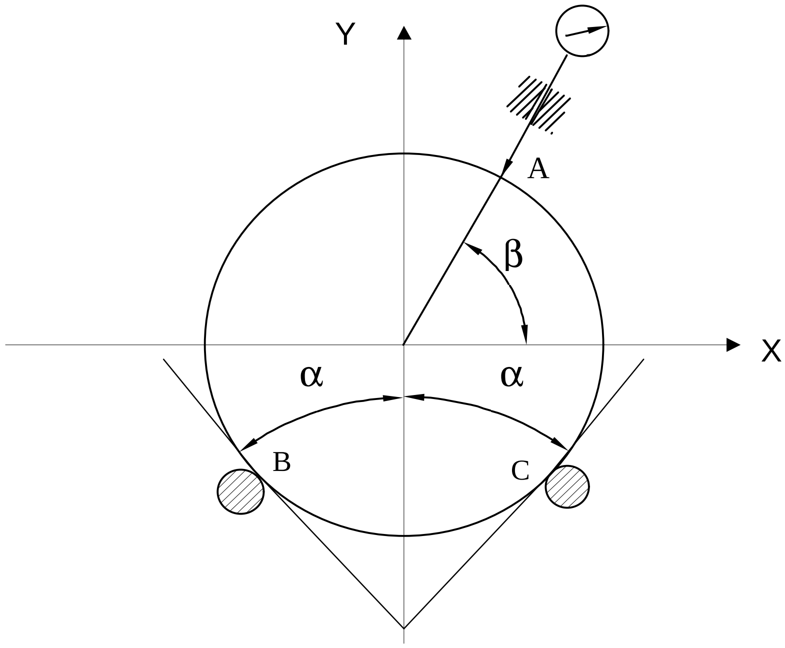

Figure 1. The principle of measurement of roundness profiles by reference methods.

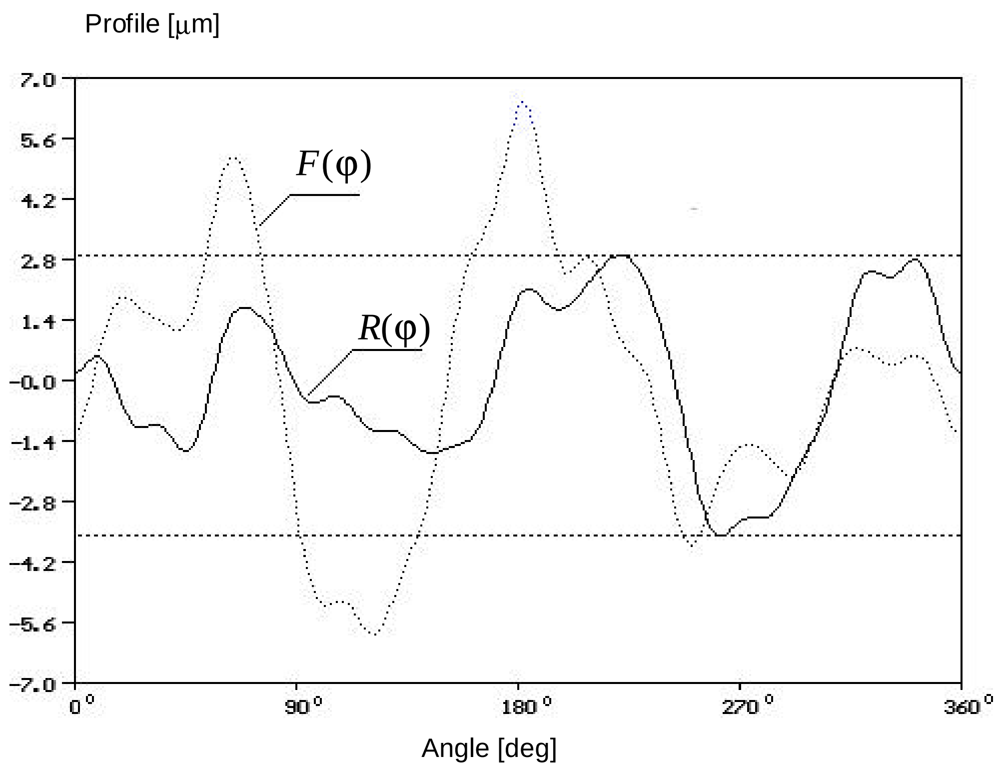

Figure 2. Comparison of the profile $F(j)$ measured by a reference method with the real profile $R(j)$ measured by a non-reference method.

Theoretical studies provide evidence that this coefficient for particular types of reference methods is the function of the method parameters, i.e. the angles a and b, as well as the number n determining, for regular profiles, the so-called n-lobing, and, for irregular profiles, the number of the subsequent harmonic of the expansion of the real profile in a Fourier trigonometric series.

In reference measuring methods, the quantity that is actually measured is the deviation F, being the function of the angles a, b, and φ, where φ is the angle of the rotation of the object. Hence, the profile $F(j)$ for the defined parameters of reference methods (the angles α and β) differs considerably from the real profile $R(j)$ (see Fig. 2).

Until now, reference methods have been applied to the assessment of roundness profiles with the known dominant types of form errors only by means of the so-called roundness deviation. It has been impossible, however, to represent the real roundness profile $R(j)$ in a complex and graphical way.

3 EVALUATION OF THE INVESTIGATIONS INTO REFERENCE METHODS BY VARIOUS RESEARCH ESTABLISHMENTS

Particular interest in reference methods could be observed in the years 1970-90. Intensive theoretical studies carried out simultaneously by various research establishments aimed at the complex analysis of the three-point method. Experimental work focused on the comparison of results of measurements of out-of-roundness by reference and non-reference methods (see Refs. [2,3,4,5,6,7]).

A major drawback was that the comparison of reference methods with non-reference ones was not comprehensive. Random comparison did not provide enough information to state whether or not it was possible to replace non-reference methods with reference ones.

No solutions have been found to eliminate basic disadvantages of reference methods. The main factor limiting broader application of these methods is that the measurement of the roundness deviation in the unknown case of the profile is not possible unless the profile is a regular one. It should be noted, however, that this is true only for an approximate measurement or a measurement specified by the standardisation ISO documents (see Ref. [8]). In the case of the known roundness profile, the coefficient of detectability for the dominant harmonic (being one of the terms of a series) is usually taken into account. Other harmonics, i.e. those that may include various coefficients of detectability, are not analysed, as they may have some influence on the accuracy of the measurement of the deviation. It should be mentioned, however, that practical application of the coefficient of detectability is quite complicated and time-consuming and may result in a number of errors.

All the above discussed drawbacks cause that reference methods used in a traditional way cannot be applied to accurate measurements of form profiles even if the assessment is made by means of the roundness deviation. Further information can be found in Ref. [9], which deals with statistical testing of the experimental method error. The obtained accuracy of the method in the relative form, for particular samples ranging 22%-63%, confirms the non-applicability of traditional reference methods to accurate assessment of roundness profiles.

As shown in the theoretical analysis concerning the errors of reference methods, the measuring accuracy of the roundness deviation is affected mainly by the irregularity of the measured roundness profile. However, in practice, when measuring an unknown roundness profile by a reference method, the obtained measured profile $F(j)$ differs from the real profile, and it is not possible to numerically define the essential coefficients referring to particular harmonics of the analysed profile. Hence, the only method to eliminate this error is the development of a mathematical transformation of the measured profile $F(j)$ into the real (transformed) profile $R_p(j)$. The mathematical model of this transformation enabled the development of a computer program, which appears quite useful in carrying out computerised measurements of roundness profiles. The knowledge of their runs makes it possible to perform a complex analysis and assessment of the real transformed roundness profile (see Ref. [10]).

The developed mathematical model of this transformation was formulated on the basis of a traditional form of the Fourier series and it was mainly applied to the assessment of roundness profiles determined by the number of harmonics ranging mainly $n \in [2,10]$, and sometimes $n \in [2,15]$.

In many research centres, the main objective was to develop such a transformation. The effect was, however, that it concerned only one type of reference methods, and was not of a universal character. In addition, there was no information about the accuracy of the mapping of the measured roundness profiles (see Refs. [10, 11]). Ref. [12] deals with a reference measuring system applied to measurements of big machine parts. The system operates on the basis of such a transformation of profiles, but no detailed description is provided.

4 CHARACTERISTICS OF THE MATHEMATICAL TRANSFORMATION

A typical diagram of the measurement of the roundness profile by a reference method is presented in figure 1. One can read that the value indicated by the inductive sensor depends on the value of the deviation in point A (where the measured object is in contact with the sensor) and in points B and C (where the object touches the supports). Let $R(j)$ stand for the real deviation of the object in a polar system related to the object centre. The angle j may identify itself with the angle of rotation of the object during the measurement. Therefore, the values of the profile in the points of contact with the sensor, the right support and the left support equal $R(j)$, $R(j - a - b)$, and $R(j + a - b - p)$ respectively. The relationship between the measured profile $F(j)$ and the real profile will be as follows:

$$F(j) = f\big(R(j), R(j - a - b), R(j + a - b - p)\big) \tag{4}$$

where f is a certain non-linear function dependent on the method parameters a and b. In practical applications, absolute changes in the value of the profile $R(j)$ due to the change in the angle φ are usually much smaller than the nominal value of the object radius. Hence, the function f can be linearized in the neighbourhood of nominal values of the profile, which gives

$$F(j) = F_0 + R(j) - R(j - a - b)\cdot\frac{\sin(a - b)}{\sin 2a} + R(j + a - b - p)\cdot\frac{\sin(a + b)}{\sin 2a} \tag{5}$$

A direct solution of the obtained functional equation formulated in the domain of the angle φ is difficult. The problem, however, can be simplified when the measured and the real profiles are presented in the form of a Fourier series.

Let $\hat{F}_n$ and $\hat{R}_n$ be the n-th components of the expansion of the profiles $F(j)$ and $R(j)$ in an exponential Fourier series for $n = -\infty, \ldots, -1, 0, 1, \ldots, \infty$, that is

$$R(j) = \sum_{n=-\infty}^{\infty} \hat{R}_n e^{inj}\ ,\ F(j) = \sum_{n=-\infty}^{\infty} \hat{F}_n e^{inj} \tag{6}$$

As the transformation determining Fourier coefficients is linear, from Eq. (5) and a known property of a Fourier series, which states that if $R(j)$ is a periodic function with a period of $2p$, and $\hat{R}_n$ are the coefficients of the expansion of the function $R(j)$ in a complex Fourier series, then $\hat{R}_n e^{-ing}$ are the coefficients of the expansion of the displaced function $R(j - g)$, it was found that

$$\begin{aligned}\hat{F}_n &= \hat{R}_n - \hat{R}_n e^{-in(a+b)}\cdot\frac{\sin(a - b)}{\sin 2a} + \hat{R}_n e^{-in(p-a+b)}\cdot\frac{\sin(a + b)}{\sin 2a}\\ &= \hat{R}_n\cdot\left(1 - e^{-in(a+b)}\cdot\frac{\sin(a - b)}{\sin 2a} + (-1)^n e^{-in(a-b)}\cdot\frac{\sin(a + b)}{\sin 2a}\right),\ n \neq 0\end{aligned} \tag{7}$$

or

$$\hat{F}_n = \hat{R}_n \cdot \hat{K}_n \tag{8}$$

$$\hat{K}_n = 1 - e^{-in(a+b)}\cdot\frac{\sin(a - b)}{\sin 2a} + (-1)^n e^{in(a-b)}\cdot\frac{\sin(a + b)}{\sin 2a} \tag{9}$$

The coefficient $\hat{K}_n$ will be called the complex coefficient of detectability for the n-th harmonic of the profile. From definition

$$\hat{K}_n = K_n e^{ij_{Kn}} \tag{10}$$

Let the number K_n be called a module of the complex coefficient of detectability or, simply, the coefficient of detectability. Using the coefficients of the expansion of profiles in a trigonometric Fourier series, equation (8) can be rewritten in the form

$$F_n = R_n \cdot K_n\ , \tag{11}$$

$$j_{Fn} = j_{Rn} + j_{Kn}. \tag{12}$$

From the obtained equations, the following algorithm of the transformation of the measured profile into the real profile can be formulated and it is as follows:

1. register the profile $F(j)$ and determine of the coefficients of the expansion in an exponential Fourier series $\hat{F}_n$,
2. on the basis of the complex coefficients of detectability $\hat{K}_n$, determine of the coefficients of a Fourier series $\hat{R}_n$ for the real profile;
3. on the basis of the coefficients of the expansion in a Fourier series, determine of the real profile $R(j)$.

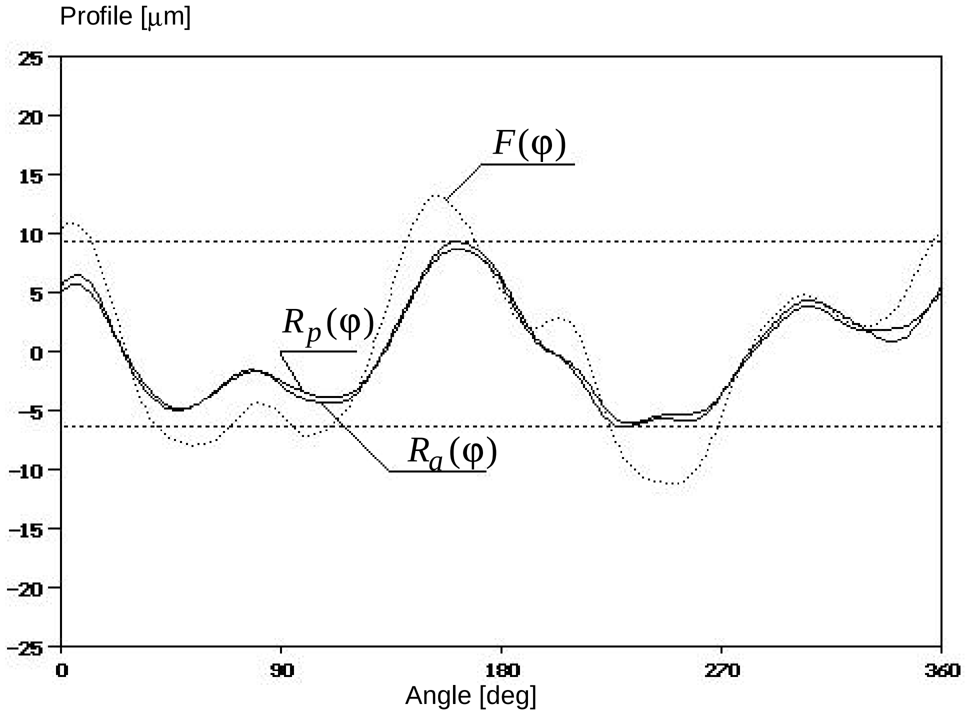

Figure 3. Comparison of the measured roundness profile $F(j)$, the transformed real profile $R_p(j)$ and the real roundness profile $R_a(j)$ obtained in the course of a non-reference measurement.

Examples of roundness profiles in the Cartesian co-ordinate system determined in the course of a transformation are shown in fig. 6. As we can see, the profile $F(j)$ measured by a reference method is completely different from the real profile $R_a(j)$ obtained in the course of a non-reference measurement. The real (transformed) profile $R_p(j)$ obtained in the course of a transformation, is almost the same as the real profile $R_a(j)$.

5 CONCLUSION

The conception of the mathematical transformation of the measured profile $F(j)$ by a reference method into the real (transformed) profile $R_p(j)$ enables the representation of reference methods for measurement of roundness profiles in the form of a universal mathematical model, being the basis of computerised reference measurements. It will allow the elimination of all the above mentioned drawbacks of the discussed methods. By obtaining the real transformed profile $R_p(j)$, it is possible:

- to measure the roundness profile for its unknown case,

- in the assessment of the roundness deviation, to take into account all harmonics of the profile in the assumed range,
- to eliminate the drawbacks of the calculations of the real roundness deviation for the established coefficients of detectability,
- to calculate and eliminate the method errors of a systematic character, and
- to perform the complex qualitative and quantitative analysis of the real transformed profile $R_p(j)$.

The presented new conception of the mathematical transformation enabled the development of a computer program, which became the basis for the construction of a computerised test stand equipped with original measuring instruments [4]. The stand can be used for measurements of surface waviness, which have not been performed by reference methods before. It also allows statistical testing of the developed mathematical model of the transformation of the profile $F(j)$ measured by a reference method into the real profile $R(j)$. Positive results of this testing made it possible to construct reference measuring instruments applicable mainly to accurate measurements of roundness profiles directly in production (see Ref. [5]).

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