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MODELING OF DEFECTS OF FLEXIBLE ELEMENTS OF BELT DRIVES*

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Abstract: The defects of flexible element (belt) have been modeled in this work. The dependency of bearing housing vibration velocity of dynamic test bench has been got. It is defined that the quantity of defects influences the parameters of the vibration of mechanical system.
Keywords: defect, vibration, belt.

1 INTRODUCTION

The most characteristic defects of flexible elements of belt drives are fractures of a compression layer which, developing during exploitation, reaches the cord, and exfoliation. These defects, as the performed tests [1] show, are the consequence of damages occurred in the compression layer, due to fatigue. Literary sources show that interest in dynamics, stability [2] and control possibilities of such mechanical systems as band-saws, air cable lines, pipes with flowing fluids and belt drives, is sufficiently big and lasts for a rather long period of time. Modeling of defects of flexible elements of belt drives is performed in this work, a model and dynamics equation of transverse oscillations of a flexible element with a defect is formed and dependencies of vibration activity of a bearing point of a driven pulley of a dynamic stand on the quantity of defects are presented.

2 MODEL AND DYNAMICS EQUATION OF TRANSVERSE OSCILLATIONS OF A FLEXIBLE ELEMENT WITH A DEFECT

In this part we will evaluate an influence of defect occurrence to characteristics of stiffness of a flexible element, form a model and dynamics equation of transverse oscillations of a flexible element with a defect, evaluating some excitation mechanism occurring in a belt drive.

2.1 Modelling of a defect in a flexible element

According to elasticity test [3] of v-belts, pulling (tension) force transmitted by separate parts of a flexible element distributes itself as follows: 96% for cord material or cord threads, 3% for rubber filling and 1% for covering cloth. Similarly we can approximately affirm that position of cord material (threads) in a cross-section of the belt complies with a position of a neutral layer. However, in the course of production process of flexible elements, deviations from a cross-section form, position of load bearing layer and stiffness are unavoidable. We will focus a greater attention to stiffness of the belt in a longitudinal direction. Results of stiffness measuring presented in literary sourcies [4] show that in separate places of a closed contour of a flexible element, stiffness in a longitudinal direction may vary. This difference in case of v-belts with casing (piece production) occurs due to a reduction of a number of cord threads in appropriate contour places of a belt in a run of production. For example, stiffness of a v-belt in a longitudinal direction measured in a segment of 70mm decreases by 9% when a number of cord threads in this interval decreases by one unit. Such change of physical characteristics in a certain interval of a flexible element may be called local heterogeneity. In the run of modelling of transverse oscillations of an analyzed flexible element, in order to model a defect occurred, we will change a meaning of local stiffness in a certain interval of a belt perimeter in proportion with a quantity of occurred heterogeneity (fractions in a compression layer of a belt, break of cord threads, etc.). Thus, common stiffness of a flexible element in a longitudinal direction, at the presence of belt drive of two pulleys and constant distance between the axis of shafts, is calculated as follows:

$$k_{il,total}=k_{il,v}+k_{il,ap}\,,$$
$$\frac{1}{k_{il,v}}=\frac{1}{k_1}+\frac{1}{k_2}+\frac{1}{k_3}+\ldots+\frac{1}{k_n}=\left(\frac{l}{EA}\right)_1+\left(\frac{l}{EA}\right)_2+\left(\frac{l}{EA}\right)_3+\ldots+\left(\frac{l}{EA}\right)_{n/2}, \qquad (1)$$
$$\frac{1}{k_{il,ap}}=\frac{1}{k_{1+n/2}}+\frac{1}{k_{2+n/2}}+\frac{1}{k_{3+n/2}}+\ldots+\frac{1}{k_n}=\left(\frac{l}{EA}\right)_{1+n/2}+\left(\frac{l}{EA}\right)_{2+n/2}+\left(\frac{l}{EA}\right)_{3+n/2}+\ldots+\left(\frac{l}{EA}\right)_n;$$

Here $k_1, k_2 \ldots k_n$ – meanings of belt stiffness accordingly measured in $i = 1, 2, \ldots n$ intervals; $k_{il,v}$, $k_{il,\ ap}$ – accordingly total rigidities of upper and lower belt branches; $k_{il,\ total}$ – total stiffness of the belt. E – modulus of belt elasticity, A – area of belt cross-section. Total stiffness of the belt is calculated following the precondition that its total perimeter is divided into n number equal l length intervals, the each of which were interpreted as a successively joint spring having stiffness $k_i = EA/l$.

2.2 Model of transverse oscillations of a defective flexible element of a belt drive and its calculations

In order to describe transverse oscillations of a belt branch being in no contact with pulleys mathematically, we will take an oscillation model of a string [5] moving with a constant speed through two points and having length adequate to a length of a part of the bench that has no contact with pulleys, as a base. Before forming the mentioned model, we will make the following preconditions:

- we do not evaluate stiffness of a belt branch of no contact with the pulleys in a transversal direction
- the speed of the belt moving in a longitudinal direction is constant
- belt weight is constant in a unit of length
- neglecting of gravity forces for determination of a position of the belt branch
- small amplitudes of transverse oscillations in comparison with length of the belt branch
- flexible element of a unitary measure
- constant run-on and run-off the pulley points of a flexible element.

During the operation of a belt drive, its flexible element directly contacts the pulleys. Deviations frequently occurring in belt drives are eccentric pulley position in shaft respect. Due to this reason, an eccentric pulley movement appears. With the presence of eccentric movement, the pulley gets a possibility to with respect to the shaft's in two perpendicular directions: one – along a straight line connecting the centres of the pulleys (in case of two pulley drive), the other – perpendicular to this straight line. In this case we find out that after occurrence of an eccentric pulley movement, a flexible element is parametrically being excited by force. Taking into consideration the facts decribed above, a dynamic model of a flexible element with a defect of a belt drive is going to look like it is shown in the figure 1.

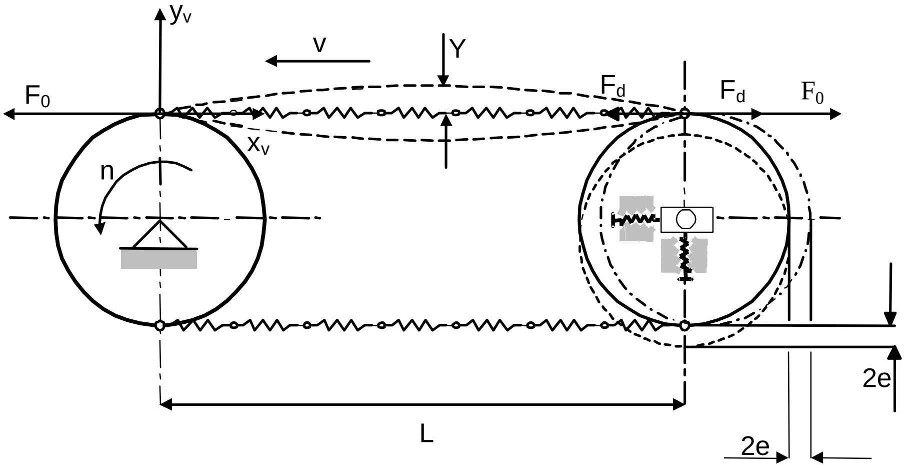

Figure.1 Model of transverse oscillations of a flexible element with a defect of a belt drive

Differential equation describing transverse oscillations of a flexible element, evaluating parametrical forced excitation, will be as follows:

$$\frac{\partial^2 y}{\partial t^2} - 2v\frac{\partial^2 y}{\partial t \partial x} - \left(\frac{F_0}{r} + \frac{F_d}{r}\cos wt + \frac{F_n}{r} - v^2\right)\frac{\partial^2 y}{\partial x^2} + b\,\frac{\partial y}{\partial t} = 0; \qquad (2)$$

here - b - damping coefficient of transverse oscillations, $c^2 = F_0/r$ - wave propagation speed of a belt branch in longitudinal direction, x – co-ordinate, F_0 – primary tension force of the belt, r - belt weight in a length unit, v – movement speed of a flexible element in longitudinal direction, F_d – supplementary periodic component of a tension force occurred in the belt, due to parametrical excitation, F_n – supplementary component of a tension force occurred in the belt, due to a relative deformation of the belt. There is no precise analytical solution to the differential equation received, thus, we will use a *modified Runge Kuto* method [6] to solve it. Before reduction of degree of the differential equation and forming a joint system of differential equations, we change their partial derivative and derivative according to a co-ordinate with finite difference. Thus, (2) equations and limit conditions in *i* point will be as follows:

$$\ddot{y}_i - v\frac{(\dot{y}_{i+1} - \dot{y}_{i-1})}{h} - (\frac{F_0}{r} + \frac{F_d}{r}\cos wt + \frac{F_n}{r} - v^2)\frac{(y_{i+1} - 2y_i + y_{i-1})}{h^2} + b\dot{y}_i = 0; \quad (3)$$

$$y_1(t) = \dot{y}_1(t) = 0,$$
$$y_{m+1}(t) = e\sin wt,$$
$$\dot{y}_{m+1}(t) = ew\cos wt,$$
$$y_i(0) = 0,$$
$$\dot{y}_i(0) = v_0 \text{ , } kur\ i = 2,3,4,....m.$$

Here h – interval length; m – number of equal intervals, e – eccentricity of a driving pulley. The force F_n occurring, due to a branch bend of a moving belt in transversal direction and simultaneously due to a relative deformation occurring, is calculated according to the formula:

$$F_n = eEA = \left[\frac{L'}{L} - 1\right]EA; \quad (4)$$

In order to find length of a deformed flexible element, we will use the following expression:

$$L' = \sum_{i=1}^{m+1}\sqrt{\left(y_{i+1,j} - y_{i,j}\right)^2 + h^2} \;; \quad (5)$$

Here index i identifies an analyzed string point and index j – time moment during which the point is being analyzed; m – number of equal intervals into which the string is divided; h – interval length: $h = L/m$. Thus, in order to determine string deformation we need to know an oscillation form of a flexible element during a time moment j. We put L' meaning into (4) formula which then helps us to find the meaning of force F_n. Solving the formula (3), a local defect was introduced, i.e. we divided the whole belt according to perimeter into n intervals and then accordingly to the size of a modeled defect, we changed a stiffness meaning in a certain interval in a longitudinal direction. After changing of the stiffness value we recalculated total stiffness of the whole belt and accordingly adjusted the quantities of forces F_0 and F_d. Also the meaning of force F_n was calculated in each time moment j.

2.3 Calculation results of a model of transverse oscillations of a flexible element with a defect of a belt drive

Calculating system transverse oscillations of a flexible element, in which local heterogeneity appears, the following parameters were accepted: $F_0 = 195\ N$; $F_d = 24.38\ N$; $w = 398s^{-1}$; $e = 0.15\ mm$; $r = .11\ kg/m$; $L = 0.522\ m$; $b = 4.66$; $v = 17.1\ m/s$, for each interval of a flexible element $EA = 8*10^{+4}N$. During calculations we reduced in turn stiffness meaning of every interval by 22%. Such quantitative change of belt stiffness while pulling complies with fractions of a compression layer and a break of one cord thread [4]. Received calculation results of the model are shown below.

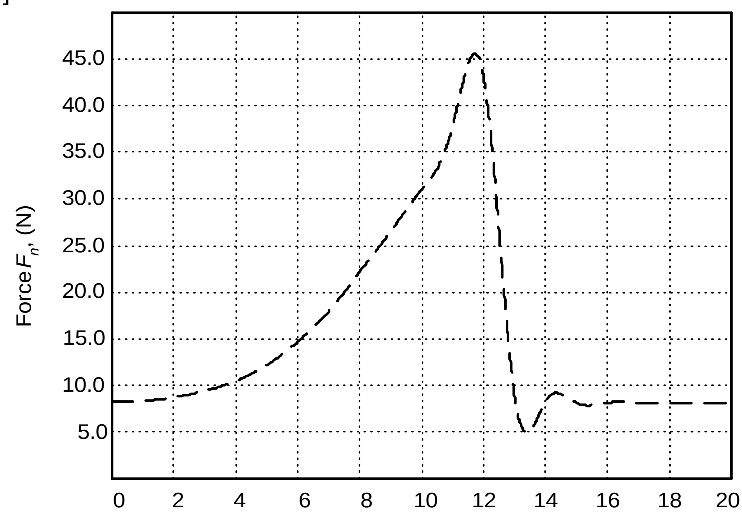

Figure 2. Dependence of force F_n occurring to belt deformation, on number of defective intervals

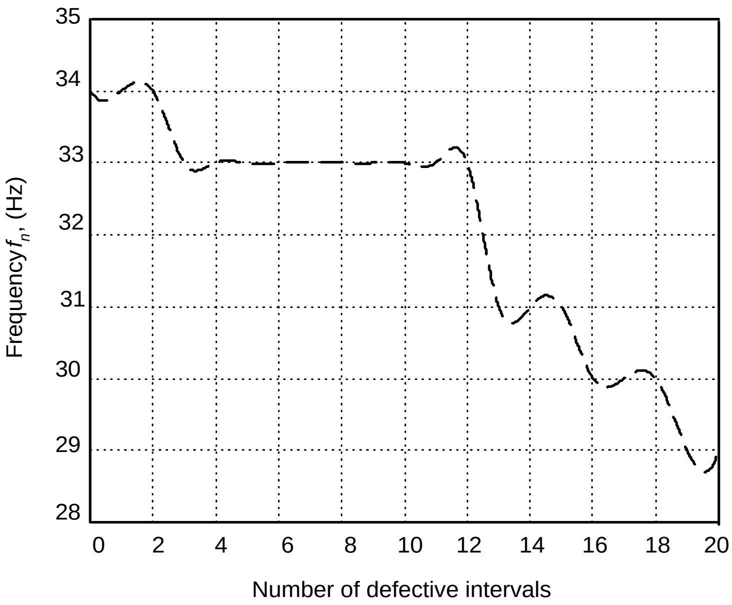

Figure 3. Dependence of natural frequency f_n of transverse oscillations of the belt on number of defective intervals

From received calculation results of the model of transverse oscillations of an analyzed flexible element with a defect is clear that size of local heterogeneity (modelled defect) and its quantative characteristics influence dynamic behaviour of the flexible element. We see from a graphical dependence Figure 3. that when a number of defective intervals is increasing in belt perimeter, its natural frequency f_n of oscillations is decreasing. Comparing natural frequency of a non-defective flexible element with frequency of a fully defective flexible element along its total perimeter, this reduction makes almost 15%. Excited parametrically and by force transverse oscillations of an analyzed flexible element are dying out in time. However, such behaviour is seen only till the number of defective intervals equals five. At this number of defective intervals the amplitude of transverse oscillations of a flexible element grows up – resonance appears. Because of this reason the force F_n occurring due to belt deformation makes a significant increase (approximately 12 times, Figure 2.). Such behaviour of a flexible element is observed till the number of defective intervals equals thirteen. Then amplitude meaning of transverse oscillations and size of force F_n spasmodically decreases and oscillations become dying out in time again.

3 EXPERIMENTAL TEST OF DEFECTIVE FLEXIBLE ELEMENTS OF BELT DRIVES

The purpose of this experiment was vibration activity analysis of correlation between a bearing point of a driving pulley of a dynamic stand and defect of the belt.

3.1 Methodics of experiment

During experiment v-belts of A-140/55 type, manufacturer – firm “Rubena” (Czech Republic) of normal cross-section were tested in a free motion (without power transmission) regime. A number of rotations n of a driving pulley was equal to 1900 $\pm$ 30 r/min. Heterogeneity of v-belts was modelled, making an incision in their base up to a bearing layer and in one cord thread. Analogical defects were made in twenty belt intervals, in which total perimeter of the belt was divided. Exfoliation of v-belts was modelled, cutting the belt along the perimeter at the cord up to 10 cm. Four such incisions were made in the perimeter of the belt. Measurements of vibrations were performed after every separate defective interval. The belt prepared under these conditions was tested on a dynamic stand and every 0.5 $\pm$ 0.1h, 1.0 $\pm$ 0.2h bearing points at a driving pulley were measured in a horizontal and vertical direction of vibration. The identical test conditions were assured during experimental research.

3.2 The Experimental results

Results of experimental test are presented in a form of diagrams of vibration spectrum and as a changing of velocity of a bearing point in horizontal and vertical directions depending on a number of defective intervals in belt perimeter.

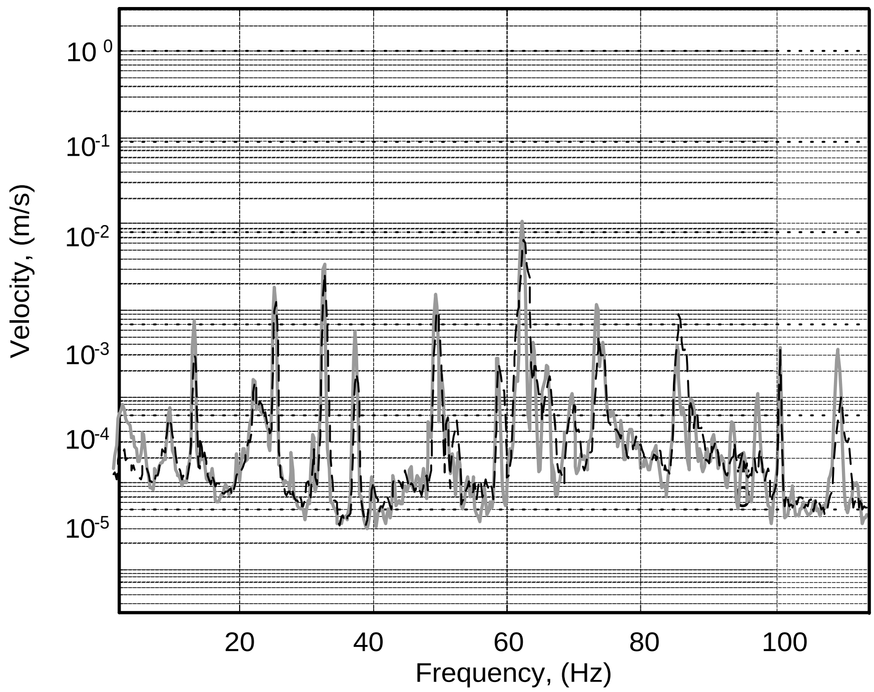

Figure 4. Spectrum of velocity of a bearing point in vertical direction
- - - non-defective belt, ——— belt with seven defective intervals

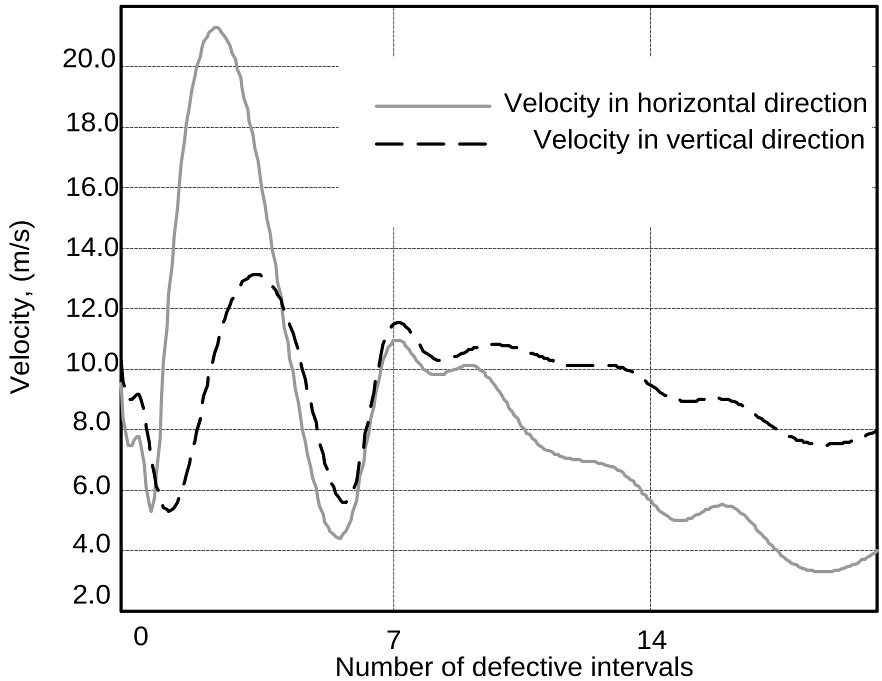

Figure 5. Dependency of velocity amplitude of a bearing point, excited by double belt run frequency of a driving pulley of a dynamic stand on the number of defective intervals

Received experimental test results show that physically modelled defects in a flexible element influence vibration activity of a bearing point of a dynamic stand. Increasing the number of defective intervals, meanings of velocity of a bearing point in both horizontal and vertical directions decrease (Figure 5.). However, when the number of defective intervals increases up to seven, velocity meanings increase in both directions. Increasing further the number of defective intervals, velocity of a driving pulley is gradually decreasing again. Looking at a spectrum dependency of velocity of a driving pulley of a dynamic stand (Figure 6.), we notice that velocity meanings increase at a double belt run frequency (24 Hz) and belt natural frequency (32 Hz), comparing a flexible element without defects with a defective one.

4. CONCLUSIONS

- Calculation results of a model of transverse oscillations of a defective flexible element of a belt drive showed that a modelled defect influences dynamic behaviour of a flexible element. Reaching an appropriate quantative and qualitative level of local heterogeneity, oscillation forms start to change, the meaning of force occurring due to deformation of a flexible element significantly increases. Further quantitatively developing local heterogeneity reaches a limit across which corresponding meanings and forms of oscillation amplitude and oscillation forms of a tested element become the same as of a non-defective flexible element. This is not to be applied to natural frequency of oscillations, which absolute change, comparing the state of a non-defective flexible element with the state of heterogeneity flexible element along its total perimeter, makes almost 15%.
- Experimental test showed that there is correlation between a defect in a flexible element and vibration activity of a bearing point of a driving pulley of a dynamic stand. Change dependency of vibroactivity of a bearing point on quantity of defect occurring is of a similar nature as in case of a model of dynamics of transverse oscillations of defective flexible element. During experiment, like in case of model calculations, vibration activity of a bearing point increases simultaneously to quantative defect development. Reaching certain quantative defect level, vibration activity of a bearing point starts its radical decrease. This phenomenon may be explained that, due to increase of heterogeneity in a flexible element, stiffness starts to change and natural frequency as well. In the run of increase of flexible element defect and change of natural frequency, it happens that natural frequency becomes equal to excitation frequency of some system element (in our case it is frequency of shaft rotations of a driving pulley) and this causes resonance. Due to increased oscillation amplitude, the meaning of force F_n also increases. Thus, we get that total tension force active in a flexible element increases and it causes bigger vibration activity of a bearing point. Disagreement received – after occurrence of several defective intervals, vibration activity of a bearing point increases in case of this experiment and additionally occurring force F_n in the belt in case of dynamic model – may be explained that size of physically modelled defect in separate flexible elements was not adequate to theoretically accepted one in case of calculation of transverse oscillations of the belt.
- Following calculations of the model of transverse oscillations of a defective flexible element of a belt drive and received experimental results, a certain qualitative and quantative level of occurring local heterogeneity (defect) noticeably changes dynamic behaviour of a flexible element. Thus, we think that there is a real possibility to decide about a technical state of a system flexible element using measurements of oscillations of bearing points.

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