XVI IMEKO World Congress
Measurement - Supports Science - Improves Technology - Protects Environment ... and Provides Employment - Now and in the Future
Vienna, AUSTRIA, 2000, September 25-28

IDENTIFICATION OF THE TECHNICAL OBJECT STATE WITH GIVEN RELIABILITY STRUCTURE

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Abstract:The reliability state of the technical object (system) depend on state of its elements and reliability structure of the system. We consider problem of identification of the technical object state. Problem is formulated as the bayesian classification issue.
Keywords: identification, Bayesian classification

1 INTRODUCTION

Some problems of the identification of the technical object state were discussed in [3], [4], [5], [6]. The identification of the system state belongs to the metrology problems class. In reliability theory, it is well known, the state of system depend on its reliability structure and depend on the states of the system components. It could happen that the real object state has been identified as a different state. In the case of the binary reliability states the first and second types errors are possible. The first type error is consist in the qualification of the functioning object as the failed one. The second type error means that the failed object is recognised as the up one. The errors of the first type cause the unfounded outages while the second type errors could be the cause of the accidents.

2 BAYESIAN MODEL OF IDENTIFICATION

We investigate object (system) consisting of n components. The state of component $k, \quad (k = 1,2,...,n)$ is represented by binary random variable S_k which takes value in set {0,1} where 0 is failed state ("down state") and 1 is working state {"up state"}. We suppose that the distributions $p_k(s_k) = P(S_k = s_k), \quad s_k \in \{0,1\}, \quad k = 1,2,...,n$ are given. Checking states of the system identifies the state S_k as $X_k \in \{0,1\}$. The random variables X_k, S_k are dependent. Conditional distributions of X_k given S_k are as follows

$$f_k(x_k \mid s_k) = P(X_k = x_k \mid S_k = s_k), \quad x_k \in \{0,1\}, \quad s_k \in \{0,1\} k = 1,2,...,n.$$

The measure of the first type error of identification of the k component state is defined as conditional probability $a_k = f_k(0 \mid 1) = P(X_k = 0 \mid S_k = 1), \quad k = 1,2,...,n.$

This error appears when the "up state" (1) of the component is qualified as "down state "(0). The second type error appears when the state of the component is "down"(0) but it is recognised as "up state"(1). Conditional probability $b_k = f_k(1 \mid 0) = P(X_k = 1 \mid S_k = 0), \quad k = 1,2,...,n$

is the measure of the second type identification error of the k component state.

We assume that the random variables $(S_1, X_1),...,(S_n, X_n)$ are independent. Then

$$P(S_1 = s_1, X_1 = x_1,..., S_n = s_n, X_n = x_n) =$$

$$= p_1(s_1) f_1(x_1 \mid s_1)... p_n(s_n) f_n(x_n \mid s_n), \quad s = (s_1, x_1,..., s_n x_n) \in \{0,1\}^{2n}.$$

Hence, the joint distribution of the random vector $S = (S_1, S_2,..., S_n)$ is given by

$$P(S_1 = s_1,..., S_n = s_n) = p_1(s_1)... p_n(s_n), \quad s = (s_1, s_2,..., s_n) \in \{0,1\}^n.$$

It means that random variables $S_1, S_2,..., S_n$ are independent.

Suppose that the reliability structure of the object is monotone, i.e. the system structure function $f:\{0,1\}^n \to \{0,1\}$ is non-decreasing in each argument and $f(0,...,0)=0$; $f(1,...,1)=1$.

Reliability of the object is

$$R = E(f(S_1,...,S_n)) = P(f(S_1,S_2,...,S_n)=1) = P((S_1,...,S_n)\in f^{-1}(1)). \quad (1)$$

Unreliability of the object is given by

$$Q = P(f(S_1,...,S_n)=1) = P((S_1,...,S_n)\in f^{-1}(0)) = 1-R.$$

The vector

$$p = (p(0), p(1)), \quad \text{where} \quad p(0)=Q,\ p(1)=R \quad (2)$$

is treated as a prior distribution in the identification problem.

The identification of the states of the system components consist in recognising the states of components on the base of observation $x=(x_1,...,x_n)\in \aleph = \{0,1\}^n$ which is value of the random vector $X=(X_1,X_2,...,X_n)$ with independent components. The set, $\aleph=\{0,1\}^n$ is called the observation space. Identification of the object state means recognising the state of the object on the base of the observation $x=(x_1,...,x_n)\in\aleph$. Formally, the identification consist in finding the proper function $I:\aleph\to\{0,1\}$, which is equivalent to the partition $\{\aleph_0,\aleph_1\}$ of the observation space $\aleph$; $\aleph_0 = I(0), \aleph_1 = I(1)$.

From the assumption mentioned earlier, it follows that

$$P(X\in f^{-1}(j), S\in f^{-1}(i)) =$$
$$= \sum_{(x_1,...,x_n)\in f^{-1}(j)} \sum_{(s_1,...,s_n)\in f^{-1}(i)} p_1(s_1)...p_n(s_n) f_1(x_1|s_1)... f_n(x_n,s_n).$$

Identification of the object state depends on the structure function f. The value $f(j|i) = P(f(X)=j|f(S)=i) = P(X\in f^{-1}(j)|S\in f^{-1}(i)),\quad i,j\in S=\{0,1\}$ is the probability that the state of the system is recognised as j if in fact the state is i. Note that

$$f(j|i) = \frac{P(X\in f^{-1}(j), S\in f^{-1}(i))}{P(S\in f^{-1}(i))}$$
$$= \frac{\sum_{(s_1,...,s_n)\in f^{-1}(i)} \sum_{(x_1,...,x_n)\in f^{-1}(j)} p_1(s_1)...p_n(s_n) f_1(x_1|s_1)... f_n(x_n|s_n)}{\sum_{(s_1,...,s_n)\in f^{-1}(i)} p_1(s_1)...p_n(s_n)}.$$

It is obvious that

$$p(i)f(j|i) = P(X\in f^{-1}(j), S\in f^{-1}(i) = \quad (3)$$
$$= \sum_{(x_1,...,x_n)\in f^{-1}(j)} \sum_{(s_1,...,s_n)\in f^{-1}(i)} p_1(s_1)...p_n(s_n) f_1(x_1|s_1)... f_n(x_n|s_n).$$

Let A be an action (decision) space. A function $L: S\times A\to\Re$ is a *loss function.* In our case $A = X = \{0,1\}$. The number $l_{ij} = L(i,j)$ represents the loss when the state of the object is i and that state is recognised as j. The function L can be written in the matrix form $L = [l_{ij} : i,j\in\{0,1\}]$. We assume that $l_{ij}=1$ for $i\neq j$ and $l_{ij}=0$ for $i=j$.

For any identification I, the *expected loss,* or *risk* is defined by the formula $R_i(I) = \sum_{j=1}^{k} l_{ij} p_{ij}(I)$, where $p_{ij}(I) = f(j|i)$. The function $r(p,I) = \sum_{i=1}^{k} p(i) R_i(I)$, where $p(i) = P(f(S)=i)$ is called the *Bayes risk* with respect to a prior distribution $p=(p(0),p(1))$.

The I^* is called a *Bayes rule* if $r(p, I^*) = \min_I r(p, I)$. From the Bayes formula a posterior probability of the state i if the vector x is given, is specified by the equation

$$p(i \mid x) = \frac{p(i) f(f(x) \mid i)}{\sum_i p(i) f(f(x) \mid i)}, \quad i \in \{0,1\}. \qquad (4)$$

The function $r(j \mid x) = \sum_i l_{ij} p(i \mid x), \quad j \in \{0,1\}, \quad x \in \aleph$

is called a *posterior Bayes risk* for the state (action) $j.$. The value of this function represents the average loss in the case of identifying the state j on the base on the observation $X = x$. From the classical decision theory (see [1], [2], [7], [8]) it follows that the Bayes rule minimises a posterior risk. For the loss given by (), the posterior risk is as follows

$$r(j \mid x) = \sum_i l_{ij} p(i \mid x) = \sum_{i \neq j} p(i \mid x) = 1 - p(j \mid x).$$

Then, in our case, the function I^* defined by the partition $\{\aleph_0^*, \aleph_1^*\}$, where $\aleph_i^* = \{x : p(i) f(f(x) \mid i) \geq p(j) f(f(x) \mid j) : j \neq i\}$ is the best Bayes identification.

3 EXAMPLE

Suppose that reliability structure of 3 - components system is given by

$$f(y_1, y_2, y_3) = y_1(y_2 + y_3 - y_2 y_3), \quad y_1, y_2, y_3 \in \{0,1\}.$$

Note that,

$f^{-1}(0) = \{(0,0,0), (0,1,0), (0,0,1), (0,1,1), (1,0,0)\}$ and $f^{-1}(1) = \{(1,0,1), (1,1,0), (1,1,1)\}$.

Suppose that

$$p_1(0) = 0.1,\ p_1(1) = 0.9,\ f_1(0 \mid 0) = 0.9,\ f_1(1 \mid 0) = 0.1,\ f_1(0 \mid 1) = 0.2,\ f_1(1 \mid 1) = 0.8,$$
$$p_2(0) = 0.2,\ p_2(1) = 0.8,\ f_2(0 \mid 0) = 0.8,\ f_2(1 \mid 0) = 0.2,\ f_2(0 \mid 1) = 0.2,\ f_2(1 \mid 1) = 0.8,$$
$$p_3(0) = 0.2,\ p_3(1) = 0.8,\ f_3(0 \mid 0) = 0.8,\ f_3(1 \mid 0) = 0.2,\ f_3(0 \mid 1) = 0.2,\ f_3(1 \mid 1) = 0.8.$$

From (1) and (2) we obtain

$$p(0) = Q = 0.136, \quad p(1) = R = 0.864.$$

From (3) we have

$$p(0) f(0 \mid 0) = 0.102976, \quad p(0) f(1 \mid 1) = 0.033024,$$
$$p(1) f(0 \mid 0) = 0.228096, \quad p(1) f(1 \mid 1) = 0.635904.$$

From the Bayes formula (4) we get the posterior distributions

$$\forall x \in f^{-1}(0), \quad p(0 \mid x) = 0.311,\ p(1 \mid x) = 0.689$$
$$\forall x \in f^{-1}(1), \quad p(0 \mid x) = 0.0494,\ p(1 \mid x) = 0.9506$$

As we see, in this case the states of the system which are recognised as failed states are in fact failed with probability 0.311 and they are working states with probability 0.689. The object states which are recognised as working states are in fact failed with probability 0.0494 and they are working states with probability 0.9506.

In our example, the best identification I^* is defined by the partition

$$\aleph_0^* = \{x : p(0)\, f(f(x)\,|\,0) \geq p(1)\, f(f(x)\,|\,1)\} = 0$$

$$\aleph_1 = \{x : p(1)\, f(f(x)\,|\,1) \geq p(0)\, f(f(x)\,|\,0)\} = \{x : f(x) = 0 \wedge f(x) = 1\}.$$

Then, in this case the best identification is the function given by $I^*=1$ for all $x \in \aleph$.

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