

# A GUIDE TO FOURIER TRANSFORMATION ON MEASUREMENT

**T. Nakanishi, D. Ito and K. Kariya**

Department of Electrical and Electronic Engineering  
Faculty of Science and Engineering, RITSUMEIKAN University  
Noji-higashi Kusatsu Shiga 525-8577, Japan

*Abstract: The strict theory of Fourier transformation algorithms is required in the scientific education. However, the intuition sense is also required in the measurement, because the measurement objects have many projective spaces and an appropriate measurement should be carried out on an appropriate projective space. The intuition sense will be requested in the education where not only conceptual lectures but practical examples and applications are given. One of the applications is shown on this report; that is the image processing method. These measurement methods are employed on the industrial site especially to measure something of moving objects, such as the size of the objects. An automobile flow measurement is one of the examples of this method. A vehicle size on a video picture plane appears as brightness distribution, which is one of the projective spaces of the measurement object. Further more, the spatial frequency spectrum of brightness distribution is also a projective space of the object. An appropriate projective space should be selected for the practical measurement.*

*Keywords: spatial frequency, image processing, non-contact measurement*

## 1 INTRODUCTION

Sometimes indirect methods are preferably used though basic measurements are achieved in the direct methods. The remote sensing technique is a typical example of indirect measurements. The main scheme of an indirect measurement is a decision of an appropriate transducer. On a computer-aided measurement, for example, force is changed to electric quantity by the strain gages, brightness is changed by some opt-electronic elements and others. This method is called the lower order transformation. On the other hand, almost all physical quantity will be changed to electric quantity by higher order transformation.

In higher level measurement, electric quantity must be changed to appropriate dimension though the electric quantity is directly used in the usual measurement such as time domain. The amplitude probability density distribution analysis or Fourier transformation is examples of the higher level measurement; the former is to change the time domain to amplitude domain and the latter is to change time domain to frequency domain. Both of them are mapping of projective spaces from a measurement object. The most of measurement are called as the problems of the construction that mapping processes, because the numerical expression of object information is done by the combinations of some steps of mapping processes. The spatial frequency spectrum made by Fourier transformation is the projective space of a measurement object. These transformations are necessary to pick up the required information from a lot of unnecessary information or noises. An appropriate projective space should be selected in the practical measurement. One of the frequency domain methods is shown in this report.

## 2 FOURIER SERIES

An arbitrary periodic function is transformed to the trigonometrically series on some mathematical restrictions and this transformation is called Fourier transformation. This series consists of an infinite number of the bases expressed by sinusoidal functions, which are called the harmonics. In other words, a periodic function is made up from a set of harmonics, and instantaneous values of periodic function are mapped to amplitude values of each sinusoidal function, as the linear transformation is. This transformation which was studied for the analysis of partial differential equations is applicable to detection of information from the measurement object because the measurement are analysis of the construction of mapping processes.

In case of frequency domain measurement, that is harmonic analysis method, the harmonics correspond to a projective space of the input signal from a measurement object, as shown in Fig. 1. The rectangular waveform on the front end in this figure is the input signal from the sensing detector. A

set of sinusoidal functions corresponds to the information of an object, and also amplitude distribution, which is observed from right hand side, does. This amplitude distribution is expressed on frequency domain.

So we can measure the object either on time domain view or frequency domain view. We must chose either of them to achieve appropriate result due to accuracy, efficiency or other factors.

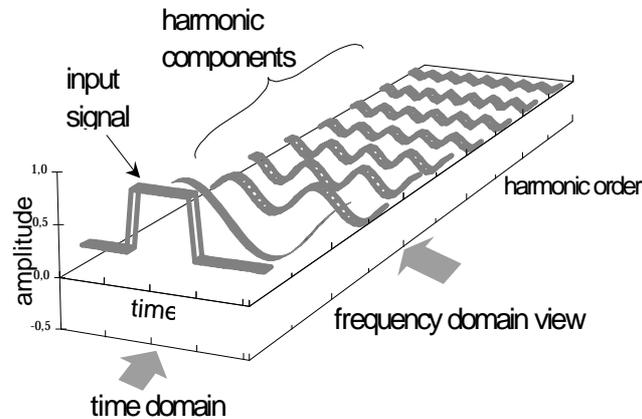


Figure 1. The input signal and its harmonic components.

### 3 APPRICATION ON TRAFFIC FLOW MEASUREMENT

On the industrial site, non-contacting method, such as image processing method is required to measure the geometrical length of a moving object. Without stopping a rotating object or a running object, direct measuring instruments may not be used. The method for measure the length is reported here, which uses spatial frequency analysis of video picture plane and this method are applied to automobile traffic flow measurement.

In the present condition of the world, the number of automobiles has been increasing every year. It has become important to analyze traffic jam in recent years. If it can solve the problem like traffic jam, it will be removed pollutants and lighten the burden imposed on drivers.

As the examples of traffic flow measurement for flow survey, there are the methods of using loop-coil sensor or the ultrasonic wave sensor. These methods have some advantage that construction is large-scale facilities, expenses pile up and be fixed measurement points. By way of another method, though there is the method by manpower, it is difficult to measure such as distinction of the type of an automobile, and it is inferior in accuracy in long time measurement.

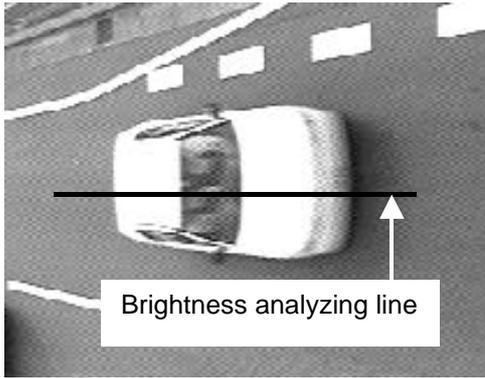
We consider real-time processing of traffic flow measurement using a TV camera system that is a general purpose use. The system is using a handy TV camera, a personal computer and an image processor system.

An image processor takes in the picture from TV camera and memorizes the video picture. By this system construction, we tried to measure vehicle length with non-contacting method. Utilizing this system, we considered automatic measurement of a mixing rate of a large-size vehicle. This rate is one of the factors that influence the state of traffic flow.

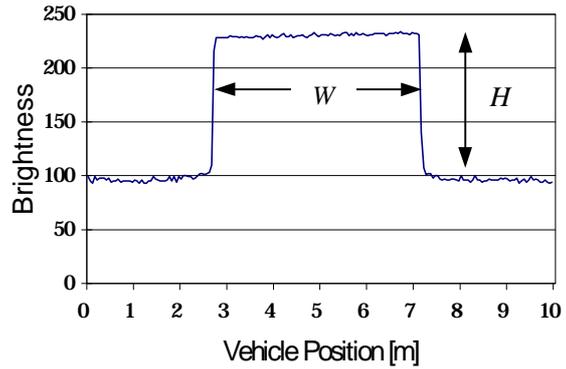
The general method to measure the geometrical length of an object using video plane is that the method of using edge detection of an object on video plane. On the case, it is difficult to decide the threshold level of brightness distribution. Another method to measure of using video plane is using the template corresponding method. This method takes enough time to decide appropriate template on a computer because measurement objects have many patterns. Therefore, we apply the method using spatial frequency analysis of video picture plane. This method has advantage of noise margin by employment of integral operation to calculate Fourier coefficients.

### 4 VEHICLE LENGTH MEASUREMENT

There is much information on a video picture plane. If analysis employs all brightness of video picture plane, amount of information is too much and it is difficult to practice real-time measurement. We employed only finite number of pixel points. It set up a brightness analyzing line on vehicle moving direction of video picture plane. It is shown in Fig. 2. Proposed method is realized short time computing by means of limited information on analyzing line.



**Figure 2.** Setting up the vehicle length Measurement line

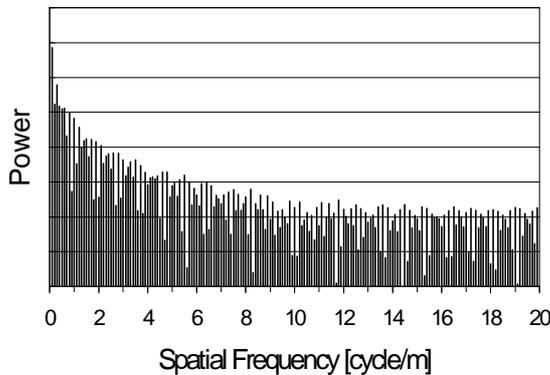


**Figure 3.** An example of brightness distribution

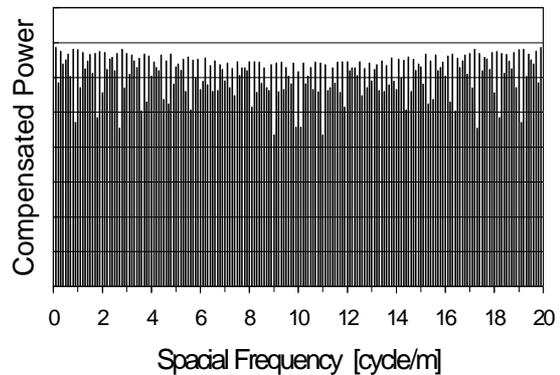
The example of brightness distribution when a vehicle exist on the analyzing line is shown on Fig. 3. The vehicle length on video plane is  $W$ . The brightness difference between vehicle and background brightness is  $H$ . And  $f$  is spatial frequency which unit is explained [cycle/meter]. Fourier transformation concerning this brightness distribution to get power spectrum is calculated, and the power spectrum envelope is shown in following.

$$P(f) = \left| \frac{H}{pf} \sin(pWf) \right|^2 \tag{1}$$

This power spectrum of brightness distribution includes decrement term  $1/f^2$ . The periodic feature of the power spectrum depends on vehicle length  $W$  in eq. 1. Furthermore, the spectrum is not affected by vehicle position because the information is not included in eq.1.



**Figure 4.** Power spectrum of brightness distribution



**Figure 5.** Compensated power spectrum

Fig. 4 shows the power spectrum of brightness distribution for the typical vehicle model which length is 4.3 [m]. The horizontal axis shows the spatial frequency, and the vertical axis shows the relative power to frequency component. We tried doing 2nd Fourier transformation to clarify the component depending on vehicle length  $W$ , that is, periodic feature. But we could not get the periodic term at 2nd Fourier transformation because the decrement term  $1/f^2$  was involved in the original power spectrum. So, we removed the decrement term from eq. 1. The original power spectrum multiplied by  $f^2$  is compensated. The compensated power spectrum is shown in following, and Fig. 5 shows the compensated power spectrum.

$$P_c(f) = P(f) \times f^2 = \frac{H^2}{2p^2} \{1 - \cos(2pWf)\} \tag{2}$$

This compensated spectrum has no decrement term, and the envelope of this spectrum is composed of the cosine function component.

The 2nd Fourier transformation is done by integral calculation of only cosine transformation because eq. 2 is an even function. Fig. 6 is obtained by the 2nd Fourier transformation. The periodic feature that is the component depending on vehicle length  $W$  appeared clearly as large power components. This 2nd Fourier analysis clarifies the periodic feature.

So the vehicle length is measured by the way of as mentioned above, we consider realizing the automatic measurement. To realize this, the timing to calculate a twice Fourier transformation is important. The vehicle length analysis should be operated only when recognize the vehicle.

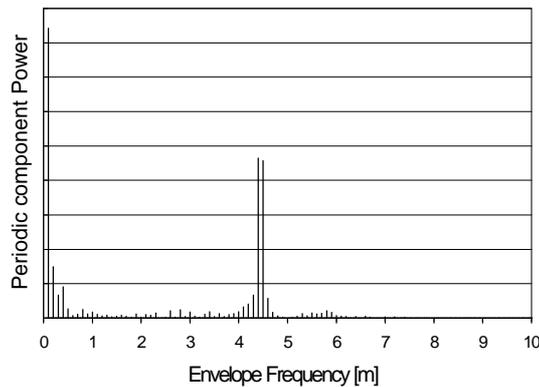


Figure 6. 2nd transformed power spectrum

## 5 VEHICLE RECOGNITION

Also it set up a brightness analyzing line to on the cross section direction of traffic flow to recognize the vehicle. This cross section line is shown in Fig. 7. Though the recognition is able by spatial frequency analysis, we employed the variance analyzing method that analyzes variance value of brightness distribution on analyzing line to achieve simple algorithm.

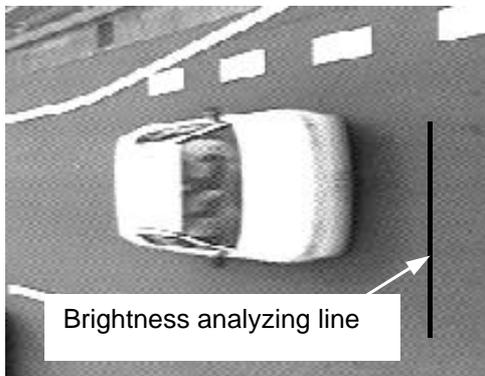


Figure 7. Setting up the vehicle recognition line

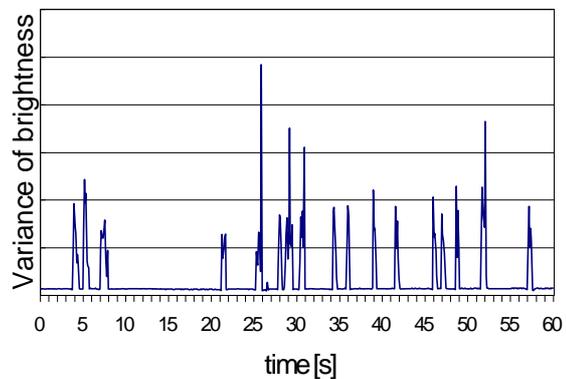


Figure 8. Variance of brightness distribution with elapsed time

Fig. 8 shows an example of the variance change of brightness distribution on the line with elapsed times. The horizontal axis shows the elapsed time, and the vertical axis shows the variance of brightness. It was analyzed automobile traffic flow on video picture for arbitrary 60 seconds, and 17 vehicles were passing. Though this traffic flow contains various vehicles about white vehicles, black and so on, the variance of brightness shows big change on a case where vehicle existing on the line compared with a case where vehicle does not exist on the line. For that reason, it is relatively easy to decide the threshold level. The vehicle length measurement operation, and count up the number of vehicle are triggered when variance of brightness exceed than the threshold level.

There are problems of the vehicle recognition that are no-count that not detects a vehicle and double-count that count plural vehicles against one vehicle. If variance change is a little, vehicle recognition is not done in spite of vehicle existence. To remove this error, the threshold level must be

settled to lower. This problem is solved by decision of the threshold level on practical experimentation.

Fig. 9 shows detail variance change between 6 to 9 seconds in Fig. 8. This is a case of one-vehicle passing between about 7 to 8 seconds. However, if the threshold level is not adequate to recognition, there is some possibility of doing double recognition. To avoid this double recognition, it set up the no recognition area on the time domain.

The problem is in case that the variance of brightness under the threshold levels exists between one vehicle passing. When the variance of brightness continues under the threshold level more than 0.2 seconds, we decided there is no vehicle on the line.

Results of experimental measurement by the above method, the accuracy of vehicle recognition is about 100 percent using the video picture during the daytime, though the accuracy decreases at twilight time. We have to further examine about decision of the threshold level.

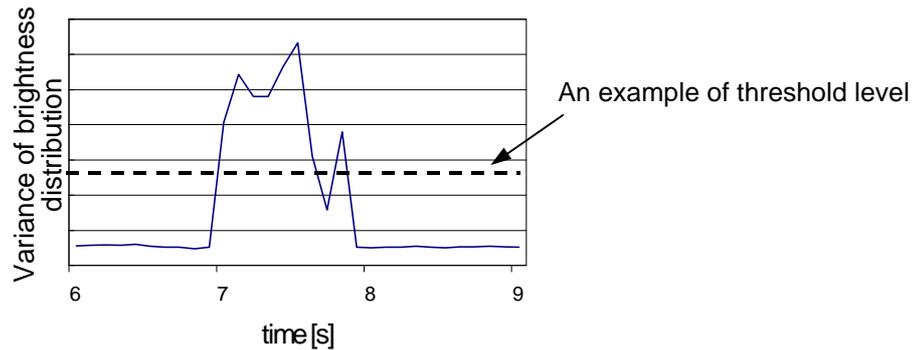


Figure 9. The expanded variance change of Fig. 8

## 6 EXPERIMENTAL RESULT

Fig. 10 shows the result of practical length measurement. We analyzed the video picture of traffic flow for 44 vehicles at the daytime. The some measurement results showed shorter length than practical length. In other words, the measured length showed a value different from vehicle length. Practical measurement is affected by the black part of a vehicle. Therefore, we have to consider effect of the black part of a vehicle.

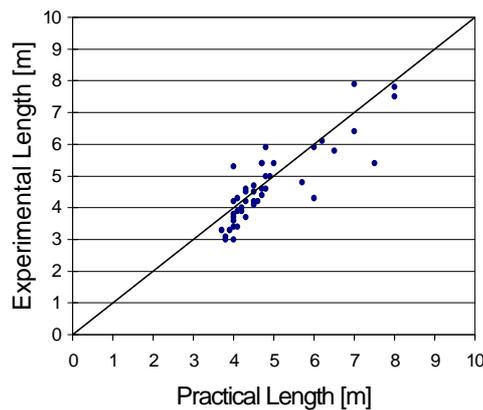


Figure 10. Result of vehicle length measurement

## 7 CONCLUSION

The total calculating time does not exceed practical vehicle interval on a traffic flow, though Fourier transformation requires many steps of calculation. In general, it is said that the max traffic quantity is about 2500 vehicles per hour. This means that a vehicle is passing per 1.5 seconds. Considering the above, it is nearly possible to measure traffic flow in real-time except some abnormal case. We have to consider corresponding to a case of a short interval between vehicles. It is necessary to build the high speed algorithm to improve the accuracy of a case of unusual short interval between vehicles.

## XVI IMEKO World Congress

Measurement - Supports Science - Improves Technology - Protects Environment ... and Provides Employment - Now and in the Future  
Vienna, AUSTRIA, 2000, September 25-28

The next problem to be solved is geometric distortion of video picture because vehicle length could not be measured accurately. It is necessary to compensate for a video picture to measure a vehicle length regardless of geometric distortion of video picture.

The advantage of this system using a portable camera is that it is possible to measure anywhere. It is a general purpose use in comparison with present measurement system that is in use. It is very effective to the traffic flow investigations.

In the experiment, the frequency domain measurement method was applied on the automobile traffic flow measurement to measure the size of a vehicle. The practical measurement was carried out using the video frame memory and numerical discrete Fourier transformation processing. The measurement errors were not large for the traffic flow investigations or same order instead of measuring various kinds of vehicles and measuring at twilight light condition.

**AUTHORS:** Daisuke ITO, Prof. Dr. Tsunehiko NAKANISHI, Prof. Dr. Komyo KARIYA, Department of Electrical and Electronic Engineering, Faculty of Science and Engineering, RITSUMEIKAN University, Noji-higashi Kusatsu Shiga 525-8577, JAPAN, Phone: +81-77-561-2674, Fax.: +81-77-561-2663, E-mail: [nakanish@se.ritsumei.ac.jp](mailto:nakanish@se.ritsumei.ac.jp)