

# Model Study on the volume value of the gas discharged by high precision bell prover

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## Abstract

This paper is based on the 0.1 class 2000L standard bell prover of gas flow which is the provincial highest measurement standard established by our institute. The research subject of this device is from the provincial science and technology special project of hebei province. In this paper, the concept of volume coefficient is proposed for bell provers with different accuracy levels. The common mathematical model is analyzed and a new Fourier fitting mathematical model is proposed. The mathematical model of the inner volume of the bell prover is given by the definite integral principle. The actual volume value of the gas discharged from the bell prover is derived by analyzing the starting and stopping process of the bell. That is the volume of the bell corresponding to the distance translated downward by the height of the bell falling which is measured by the displacement measuring mechanism. It is pointed out that in calibrating high accuracy grade bell prover, the effective use segment and its lower volume segment should be calibrated as well.

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## 1. Introduction

Bell prover gas flow standard device (hereinafter referred to as "bell prover") is a kind of metering standard equipment which is used to calibrate, calibrate and test the gas flow meter with gas as the medium. Many countries have studied and established it as a reference device for low-pressure gas flow. The main factor affecting the uncertainty of flow measurement of bell prover is the internal volume value. According to JJG165-2005 regulation, for devices with a volume of 500L or more, dimensional measurement is recommended. Measuring instruments of bell prover's diameter include: diameter ruler<sup>[1,2]</sup>, thickness meter, inside diameter micrometer<sup>[2]</sup>, laser tracker<sup>[2]</sup>, special device developed and so on, the height measuring instruments are: altimeter or steel ruler, encoder, laser interferometer<sup>[2]</sup>, grating ruler<sup>[3]</sup> and so on.

Diameter ruler, thickness gauge, steel ruler, as the conventional measuring instrument, it is mainly used for low accuracy level for the

calibration of bell. Laser tracker, laser interferometer, grating ruler and special device are convenient to measure the diameter of many heights, mainly used for the segmented calibration of bell prover with high accuracy. After calibrating the volume of the shield, the specific relationship between the inner diameter and the height of the shield was obtained. After calibrating the volume of the bell, the specific relationship between the inner diameter and the height of the bell is obtained, which is called the volume coefficient of the bell, with  $r = f(h)$ . The volume coefficient model is different between whole section calibration and section calibration

When the whole section of the bell is calibrated, the structure of the bell is considered to be an ideal cylinder. According to the verification rules, the inner radii of the upper, middle and lower sections are measured, the average radius  $\bar{r}_i$  ( $i$  is the number of measurements) was used as the measurement result take, and the volume coefficient is a constant, with  $f(h) = \bar{r}_i$ . The displacement measuring mechanism installed on the bell prover measures that the bell is lowered

from position  $h_{\text{begin}}$  to position  $h_{\text{end}}$ . Drop height difference is  $\Delta h = h_{\text{end}} - h_{\text{begin}}$ . The volume of gas discharged by the bell falling is the value of the internal volume of the bell within the range of this height difference, which is  $V = \pi f(h)^2 \cdot \Delta h$ . Because  $f(h)$  is the constant, the volume value of the gas discharged is only related to the height difference and has nothing to do with the starting and stopping position of the bell.

However, for high precision bell prover, the influence of processing accuracy and deformation during transportation on the inner volume of the bell prover should be considered. In this case, the radius values corresponding to different heights of the bell are inconsistent, so it is necessary to calibrate the bell piecewise to obtain the radius values in different height difference sections, and the volume coefficient is a piecewise function, as shown in formula (1):

$$f(h) = \begin{cases} \bar{r}_{1i}, & h_0 \leq h < h_1 \\ \bar{r}_{2i}, & h_1 \leq h < h_2 \\ \vdots & \\ \bar{r}_{ni}, & h_{n-1} \leq h < h_n \end{cases} \quad (1)$$

Where,  $h$  is the height of the bell,  $n$  is the number of segments, and  $i$  is the number of measurements within each segment.

By this time,  $V = \pi f(h)^2 \cdot (h_{\text{end}} - h_{\text{begin}})$ , It can be seen that the descending position of the bell should be considered when the bell is used in sections

In this method, the radius values of different heights of the bell are refined to a certain extent, but the radius values in each segment are still a constant, and the radius values of any heights are not given fundamentally. On this basis, a new calibration method is proposed, and a volume coefficient model in the form of continuous smooth function is established. The objective is to obtain the actual value of the volume of gas discharged at any height when the bell descends.

## 2. Establishment of the bell volume coefficient model

Taking the 0.1 class 2000L bell prover of Hebei Institute of Metrology Supervision and Inspection as an example, using Photography to measure the inner radius of the bell. The cross-section radius of 160 different heights at 11 mm intervals in the range of 1800 mm of axial height is

obtained and the two-dimensional array of heights and radius is formed, which is  $(h_i, r_i)$ ,  $i = 1, 2, \dots, 160$ . Matlab software was used for curve fitting of two-dimensional array [6]. After analysis and comparison of fitting error, it was determined to adopt eight-time Fourier fitting curve, that is, to obtain the function of inner diameter about standard height. Its mathematical model is shown in formula (2):

$$r = f(h) = a_0 + \sum_{k=1}^8 [a_k \cos(k\omega h) + b_k \sin(k\omega h)] \quad (2)$$

Where  $a_0, a_k, b_k$  are the Fourier coefficient,  $\omega$  is fundamental frequency. These parameters can be obtained by curve fitting. Its function curve and plot diagram of measured data are shown in figure 1.

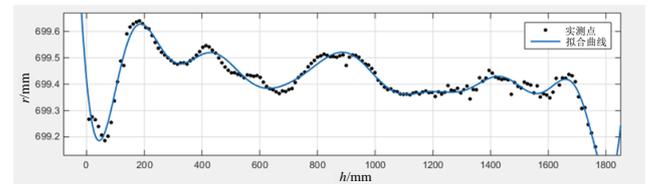


Figure 1 :comparison between fitting results and measured data

## 3. Calculation of the actual volume of gas discharged from the bell

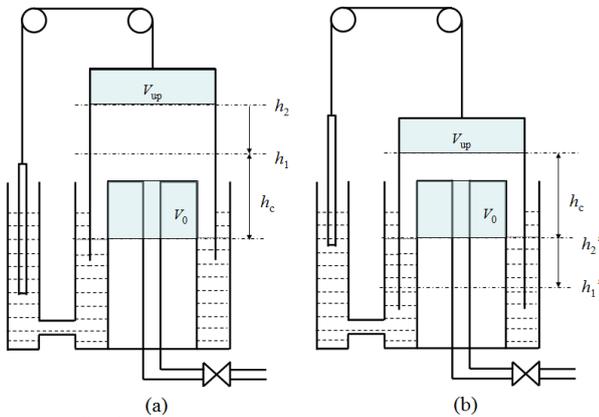
The above volume coefficient model is a continuous function, which can be calculated by definite integral to obtain the gas volume at any height of the bell, as shown in formula (3):

$$V = \int_{h_1}^{h_2} \pi r^2 dh = \int_{h_1}^{h_2} \pi [f(h)]^2 dh \quad (3)$$

Where,  $h_1$  is the height value measured by the displacement measuring mechanism before the bell descends,  $h_2$  is the height value measured by the displacement measuring mechanism after the bell descends. The actual value of the volume of gas discharged at any point in the bell is calculated.

As shown in FIG. 2, the bell device has a liquid level balancing mechanism, which is characterized by the liquid level remaining unchanged during the bell descending process. Let the difference between the indicating position of the displacement measuring mechanism and the internal liquid level be  $h_c$ . The bell descends to discharge gas, the actual height corresponding to the volume of this gas is  $h'_1$  to  $h'_2$ , however, the height measured by the displacement measuring mechanism is  $h_1$  to  $h_2$ . The bell descends from initial position  $h_1$  to final position  $h_2$ , the volume

of the gas in the bell at the initial position is  $V_1$ , and at the termination position is  $V_2$ . Where,  $h'_1 = h_1 - h_c$ ,  $h'_2 = h_2 - h_c$ .



**Figure. 2:** Diagram of the corresponding heights of the volume of gas discharged when the bell prover

$$V_1 = V_{up} + \pi \int_{h_1}^{h_2} f^2(h)dh + \pi \int_{h'_1}^{h_1} f^2(h)dh - V_0 \quad (4)$$

$$V_2 = V_{up} + \pi \int_{h'_2}^{h_2} f^2(h)dh - V_0 \quad (5)$$

During this descent, the volume of the gas discharged from the bell is:

$$\begin{aligned} V_D &= V_1 - V_2 \\ &= \pi \int_{h_1}^{h_2} f^2(h)dh + \pi \int_{h'_1}^{h_1} f^2(h)dh - \pi \int_{h'_2}^{h_2} f^2(h)dh \\ &= \pi \int_{h_1}^{h_2} f^2(h)dh + \pi \int_{h'_1}^{h'_2} f^2(h)dh + \pi \int_{h'_1}^{h_1} f^2(h)dh \\ &\quad - \pi \int_{h'_2}^{h_1} f^2(h)dh - \pi \int_{h_1}^{h_2} f^2(h)dh \\ &= \pi \int_{h'_1}^{h'_2} f^2(h)dh = \pi \int_{h_1-h_c}^{h_2-h_c} f^2(h)dh \quad (6) \end{aligned}$$

It can be seen from this that the actual value of the gas volume corresponding to the falling height of the bell measured by the displacement measuring mechanism is the inner volume corresponding to the downward translation  $h_c$  at this height.

#### 4. Discussion and Conclusion

For the low precision bell prover, its volume coefficient is a constant and has nothing to do with the starting and stopping position of the bell. The radius difference between the effective use segment of the bell and the volume segment below it can be ignored and only the radius value of the effective use segment can be calibrated<sup>[7]</sup>. For the high precision bell prover, it can be seen from formula (6) that the volume value of the discharged gas is closely related to the starting and stopping position of the bell and the difference of the indicating position of the

displacement measuring mechanism and the internal liquid level. Similarly, the volume coefficient in the form of piecewise function has the same problem. In bell prover calibration, besides calibrating the effective usage section, the lower section should be also calibrated to obtain the complete data group, so that the  $f(h)$  containing  $h_c$  can be fitted effectively.

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