

# Uncertainty Analysis for Multiphase Flow: A Case Study for Horizontal Air-Water Flow Experiments

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## Abstract

This work describes the procedure used to define the measurements uncertainties of experiments in horizontal air-water two-phase flow performed in order to identify influences due to pipe diameter. These experiments were performed with 4 different pipe diameters, always using the same test section length, therefore varying the length-by-diameter (L/D) ratio. Several parameters were measured, such as fluid flow, pressure, temperature and pressure drop; other parameters were calculated, such as the superficial velocities of each fluid, as well as their respective properties. The main parameters studied were the flow patterns for different velocity configurations and the two-phase pressure drop to be used for model improvement, thus the importance of uncertainties analysis. The sources of uncertainty were defined, detailed, systematically studied and quantified. The flow maps with its uncertainties could show the regions a pattern can be defined, and the plots of two-phase pressure drop variation with diameter validated the homogeneous model as a good approach to calculate pressure drop

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## 1. Introduction

Correctly quantifying the superficial velocities of each fluid, their respective properties, as well as the pressures involved in multiphase flow experiments are very important for the qualification of the results. The uncertainty definition of each measured or calculated result should show the boundaries within which the comparison between results, whether by repetition of the same experiment or in comparison with others, can be performed with a certain degree of confidence. However, in general, a few studies describing the methodology for calculating uncertainties in multiphase experiments are available, although this kind of studies usually shows the uncertainty calculated in their presented results.

An example of a study that sought to identify and quantify the sources of uncertainty arising from multiphase processes, focusing the analysis on energy market demands, especially in the oil and gas industry, is the "Characterization of Confidence in Multiphase Flow Predictions" [1]. In that study the authors emphasize the importance of uncertainties quantifying for the design of multiphase systems, as well as to analyze a

systematic method of propagation of uncertainty in deterministic multiphase models and to briefly review some of the tools available to perform each step. They also notes that the uncertainties in multiphase models are not always considered in the available studies, and observes that important variables such as fluid properties and flow characteristics are often not considered as a source of uncertainty.

The work of Pereyra *et al.* [2], also focusing on the oil and gas industry, present a methodology for comparing the results of prediction models of flow patterns, using as an example the one proposed by Barnea, D. [3], with results obtained experimentally and available in databases. In it the confidence level of the models relative to the experimental is quantified by the success fraction of the prediction in relation to the realized one.

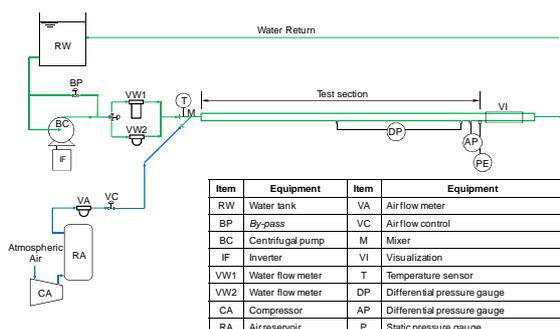
In the definition of the uncertainties involved in the present study results, a description of the Brazilian version of the "Guide to the expression of uncertainty in measurement: evaluation of measurement data" is given by describing that "the purpose of a measurement is to determine the value of the measurand, that is, the value of the

particular quantity to be measured." ([4], p.4) and that "... In general, the result of a measurement is only an approximation or estimate of the value of the measurand and thus is complete only when accompanied by a statement of the uncertainty of that estimate" ([4], p.4). Thus, it is necessary to define what are the measurands whose values are necessary in the quantification of some physical quantity, and their respective standard uncertainties, to finally determine the expanded uncertainty of that physical quantity under study.

This work aims to show the uncertainty calculations for the velocity and pressure results of horizontal air-water two-phase flow experiments. The experiments have been made to study the influence of pipe diameter [5] and this article intends to analyse the uncertainty of the results applying a simple, but systematic, methodology presented by Brazilian guide to the expression of uncertainty in measurements [4].

## 2. Experimental facilities and procedures

The experiments were performed at the Experimental Laboratory of Petroleum (LabPetro) of the Center for Petroleum Studies (CEPETRO) at University of Campinas (UNICAMP), and at the Flow Measurement Laboratory (LV) of the Center for Mechanical, Electrical and Fluid Flow Metrology (CTMetro) at Technological Research Institute (IPT). Four setups were built, each one with different pipe inner diameters: 27.75 mm, 53.55 mm, 82.25 mm and 106.47 mm. All of them were built with a fixed test section length of 12 m, resulting in experiments with 432, 224, 146 and 113 diameters of length (L/D), respectively. Pressure drop were measured with the upstream tap at 9.0 m from the test section inlet and 2.5 m apart from the downstream one. A visualization section was installed at the end of the test section. Tap water and compressed air were the working fluids. A schematic draw of experimental setups is shown in Figure 1.



**Figure 1:** Scheme of the experimental setup. FLOMEKO 2019, Lisbon, Portugal

Flow patterns classification was done by visual observation and statistical analysis of pressure signature on time (further details of this statistical analysis can be found in reference [5]). The test procedure was made in two steps: the first one with water single-phase flow, aiming to validate the pressure sensor; and the second were the tests with two-phase air-water flows. The first step was done daily before the two-phase flow tests to validate the results of pressure drop in comparison with established theoretical models and was named validation step. After validation, the tests with two-phase flow could get, and they were performed with a fixed water superficial velocity for which the air superficial velocity could be varied.

All the superficial velocities combinations were executed three times, in ascending, descending and ascending order, to neutralize hysteresis effects and evaluate the setups reproducibility, with an acquisition time of about 2 minutes, in addition to obtaining statistically relevant data.

A total of 397 experimental data points were obtained to investigate mainly pressure drop and build the flow maps. Each data point were repeated three times. Table 1 summarizes the ranges of air ( $j_2$ ) and water ( $j_1$ ) superficial velocities, besides with the total points and the flow patterns obtained for each experimental setup.

**Table 1:** Ranges of superficial velocities studied for each setup.

D (mm)	$j_1$ (m/s) (water)	$j_2$ (m/s) (air)	Flow Pattern	Nr. of Points
27.75	0.04 to 6.37	0.30 to 15.87	SS, WS, EB, S, B, A	120
53.55	0.05 to 2.12	0.25 to 4.32	WS, EB, S	70
82.25	0.02 to 0.76	0.11 to 1.94	SS, WS, EB, S	129
106.47	0.10 to 3.72	0.10 to 1.02	SS, WS, EB, S, B	78

## 3. Uncertainties

For each physical quantity measured, or calculated, on the experiments, the way to define its uncertainty had 11 steps:

1. Define physical quantity;
2. Define a physical model of the quantity to be measured;
3. Define a mathematical model of the quantity;
4. Identify the sources of uncertainty in the measurement;
5. Organizing the sources of uncertainty and look for cause and effect relations;
6. Quantify the variability of each source;
7. Reduce to standard uncertainty;

8. Determining combined standard uncertainty;
9. Look for correlated input quantities;
10. Define reliability;
11. Determining expanded uncertainty.

### 3.1 Physical quantities

The physical quantities are those that the experiments seek to determine. Fluid velocities were calculated, and pressures were measured. There are:

- superficial velocities of each fluid;
- two-phase and single phase (water) pressure drop.

To do so, the following quantities were measured:

- mass flow rate of each fluid;
- water temperature;
- static pressure of the mixture;
- single and two-phase pressure drop.

Thus, the superficial velocities of each fluid were calculated from: the measurements of flow, static pressure, water temperature and area of the pipe section. The velocity uncertainty of each fluid will be composed of these variables.

The uncertainties of the single and two-phase pressure drop measurements were compounded by the calibration results of the instruments used, or the uncertainty reported by the manufacturer, and the reproducibility of each result.

### 3.2 Physical models

The respective physical models to be considered for each physical quantity are:

- superficial velocity of each fluid:
  - steady state flow;
  - the fluids are considered incompressible;
  - variations on the density of each fluid does not influence the flow measurement;
  - the pipe has constant section area along the entire test line;
  - the measured temperature and static pressure influences the calculated result of the air superficial velocity, as it influences the value of its density, although the variations do not characterize compressibility effects.
- two-phase pressure drop:
  - the constant relative roughness of the pipe;
  - the constant diameter of the pipe;
  - the gravitational acceleration is constant;
  - constant static pressure in the point of measurement for each experiment;

### 3.3 Mathematical models

The mathematical models directly influence the velocities results. For superficial velocities, air density and pipe areas, the mathematical models are:

$$j_1 = \frac{W_1}{\rho_1 A} \quad (1)$$

$$j_2 = \frac{W_2}{\rho_2 A} = \frac{W_2}{\frac{P_{est} A}{RT}} = \frac{W_2 RT}{P_{est} A} \quad (2)$$

$$\rho_2 = \frac{P_{atm} + P_{est}}{RT} \quad (3)$$

$$A = \frac{\pi D^2}{4} \quad (4)$$

### 3.4 Sources of uncertainty in the measurement

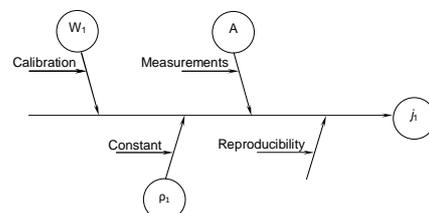
The following sources of uncertainty are considered:

- uncertainty of calibrated instruments: experimental standard deviation of the mean; or the worst uncertainty result of calibration certificate, when available;
- manufacturer specification uncertainties of the instruments, when calibration results are not available;
- uncertainty in temperature and static pressure measurements: for the gas component of the mixture, both the uncertainty of the calibration and the local fluctuation of the variables;
- reproducibility uncertainty: once the velocity and two-phase pressure drop will be presented in average values, it is the experimental standard deviation of the mean of the three measurements taken each time the setup repeats a given test;
- uncertainty in considering atmospheric pressure and water density constants for some experiments;
- uncertainty of the pipe diameter measurements.

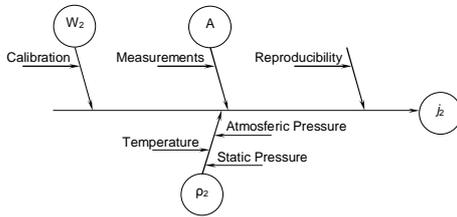
### 3.5 Organizing the sources

Organizing sources on cause and effect diagrams for better understanding their influence:

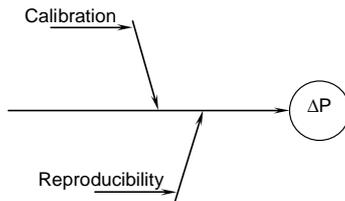
- superficial velocities:
  - water:



- air:



- pressure drop:



### 3.6 Quantifying and reduce each source

- uncertainty of water and air flow (considering infinite degrees of freedom and normal probability distribution):

$$u(W_i) = \frac{U(W_i)}{k} \quad (5)$$

- pressure drop, static pressure and temperature (considering infinite degrees of freedom and normal probability distribution):

$$u(\Delta P) = \frac{U(\Delta P)}{k} \quad (6)$$

$$u(P_{est}) = \frac{U(P_{est})}{k} \quad (7)$$

$$u_1(T) = \frac{U(T)}{k} \quad (8)$$

$$u_2(T) = \frac{\sigma_T}{\sqrt{n-1}} \quad (9)$$

The uncertainty of the static pressure fluctuation is not considered, since this fluctuation is expected for the different flow patterns, when in two-phase flow. In addition, fluctuating temperature during measurement is also a source of uncertainty in its measurement. Thus, for temperature:

$$u^2(T) = \sum_{z=1}^2 u_z^2(T) \quad (10)$$

- pipe diameter uncertainty:
  - method uncertainty (with triangular distribution and 50 degrees of freedom):

$$u_1(D) = \frac{\text{Assigned value}}{\sqrt{6}} \quad (11)$$

- uncertainty from the resolution of reference instrument (with rectangular distribution and infinite degrees of freedom):

$$u_2(D) = \frac{\text{Resolution}}{2\sqrt{3}} \quad (12)$$

- uncertainty of the reference instrument calibration (with normal distribution and infinite degrees of freedom):

$$u_3(D) = \frac{U(\text{Reference})}{2} \quad (13)$$

- uncertainty of the measurements (with normal distribution and degrees of freedom being the number of samples minus 1):

$$u_4(D) = s(\bar{D}) = \frac{s(D_j)}{\sqrt{n}} \quad (14)$$

The uncertainty values of pipe areas will be mathematically defined from the calculated uncertainties of each pipe diameter:

$$u^2(D) = \sum_{z=1}^4 u_z^2(D) \quad (15)$$

- water density uncertainty:
  - at LabPetro (for rectangular distribution and infinite degrees of freedom):

$$u(\rho_1) = \frac{\rho_{1max} - \rho_{1min}}{\sqrt{12}} \quad (16)$$

- at IPT (for triangular distribution and infinite degrees of freedom):

$$u(\rho_1) = \frac{\rho_{1max} - \rho_{1min}}{\sqrt{24}} \quad (17)$$

- atmospheric pressure uncertainty:
  - for LabPetro tests the atmospheric pressure was verified on a specific website that measures it on that region [6] (for rectangular distribution and infinite degrees of freedom):

$$u(P_{atm}) = \frac{P_{atmmax} - P_{atmmin}}{\sqrt{12}} \quad (18)$$

- at IPT, with the uncertainty of barometric pressure gauge (normal distribution) and the local fluctuation (rectangular distribution) for infinite degrees of freedom:

$$u_1(P_{atm}) = \frac{U(P_{atm})}{k} \quad (19)$$

$$u_2(P_{atm}) = \frac{P_{atmmax} - P_{atmmin}}{\sqrt{12}} \quad (20)$$

And so:

$$u^2(P_{atm}) = \sum_{z=1}^2 u_z^2(P_{atm}) \quad (21)$$

- reproducibility of values, by the experimental standard deviation of the mean (with normal distribution and degrees of freedom corresponding to the number of sample minus 1), for pressure drop and superficial velocity:

$$u(R\Delta P) = s(\overline{\Delta P}) = \frac{s(\Delta P_z)}{\sqrt{n}} \quad (22)$$

$$u(Rj_i) = s(\bar{j}_i) = \frac{s(j_i)}{\sqrt{n}} \quad (23)$$

### 3.7 Determining combined standard uncertainty

- Combined standard uncertainty of area:

$$u_c(A) = \sqrt{[c_A \cdot u(D)]^2} = \sqrt{\left[\frac{\partial A}{\partial D} \cdot u(D)\right]^2} \quad (24)$$

- Combined standard uncertainty of air density

$$u_c(\rho_2) = \sqrt{\left[\frac{\partial \rho_2}{\partial P_{atm}} \cdot u(P_{atm})\right]^2 + \left[\frac{\partial \rho_2}{\partial P_{est}} \cdot u(P_{est})\right]^2 + \left[\frac{\partial \rho_2}{\partial T} \cdot u(T)\right]^2} \quad (25)$$

- Combined standard uncertainty of air superficial velocity

$$u_c(j_2) = \sqrt{\left[\frac{\partial j_2}{\partial W_2} \cdot u(W_2)\right]^2 + \left[\frac{\partial j_2}{\partial A} \cdot u_c(A)\right]^2 + \left[\frac{\partial j_2}{\partial \rho_2} \cdot u_c(\rho_2)\right]^2 + u(Rj_2)^2} \quad (26)$$

- Combined standard uncertainty of water superficial velocity

$$u_c(j_1) = \sqrt{\left[\frac{\partial j_1}{\partial W_1} \cdot u(W_1)\right]^2 + \left[\frac{\partial j_1}{\partial A} \cdot u_c(A)\right]^2 + \left[\frac{\partial j_1}{\partial \rho_1} \cdot u(\rho_1)\right]^2 + u(Rj_1)^2} \quad (27)$$

- Combined standard uncertainty of pressure drop

$$u_c(\Delta P) = \sqrt{u(\Delta P)^2 + u(R\Delta P)^2} \quad (28)$$

### 3.8 Correlated input quantities

It was considered in the present work that the physical quantities are independent. Thus, they are not correlated with each other.

### 3.9 Reliability

The expanded uncertainty of measurement,  $U$ , is stated as the combined standard uncertainty multiplied by the coverage factor  $k$ , which for a distribution  $t$  with  $\nu_{eff}$  (effective degrees of freedom) corresponds to a coverage probability of approximately 95%. The expanded uncertainty will be:

$$U = k \cdot u_c \quad (29)$$

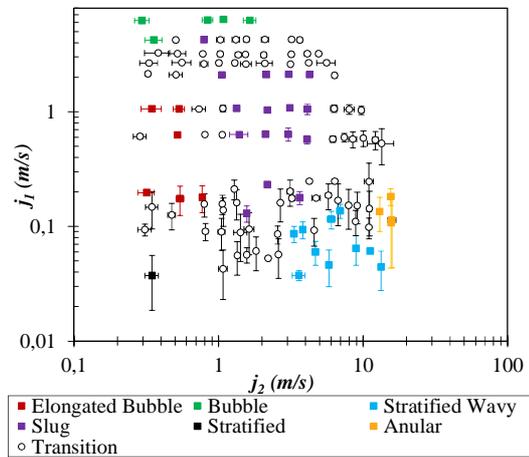
- Coverage factor  
In accordance with [4].

- Effective degrees of freedom  
From the Welch-Satterthwaite equation:

$$\nu_{ef} = \frac{u_c^4(y)}{\sum_{i=1}^N \frac{u_i^4(y)}{\nu_i}} \quad (30)$$

## 4. Results

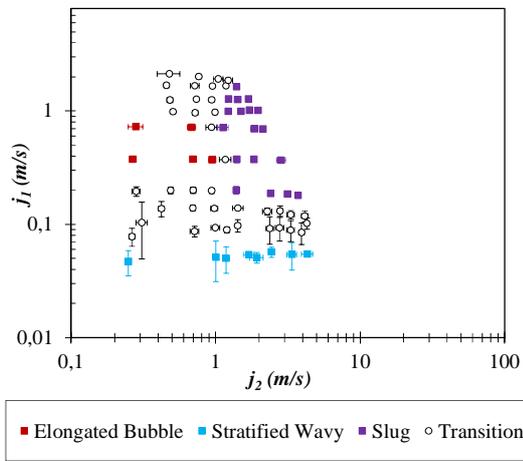
Flow pattern maps for each different setup show regions of different flow patterns for the experiments. Figure 2 to Figure 5 shows the maps, with their respective uncertainties for each velocity.



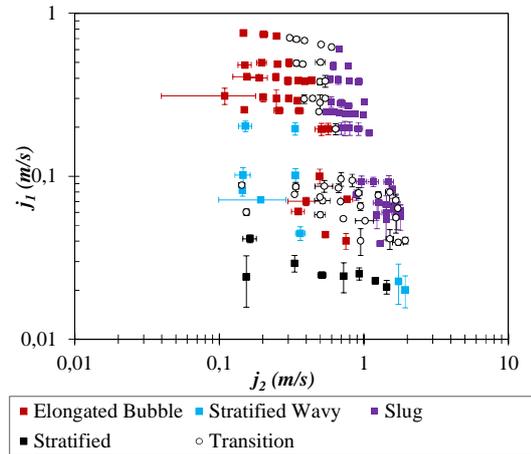
**Figure 2:** Flow pattern map for 27.75 mm inner diameter.

Also, the uncertainties for one and two-phase pressure drop were calculated. From the measured two-phase pressure drop, its variation with diameter, for homogeneous model, was proposed in the form shown in Equation 31. For each setup velocities and fluid properties, shown in the graphs from Figure 6 to Figure 8, the pressure drop was calculated, in addition to the uncertainty of the calculation, and the results were plotted together.

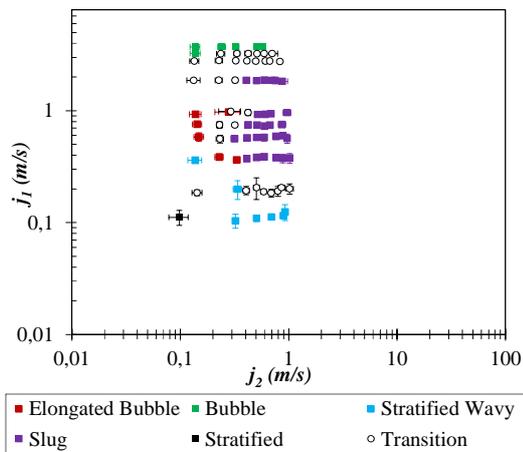
$$-\left(\frac{dP}{dx}\right) = \frac{2}{d} f \rho j^2 = (0.092 \mu_m^{0.2} \rho_m^{0.8} j^{1.8}) d^{-1.2} \quad (31)$$



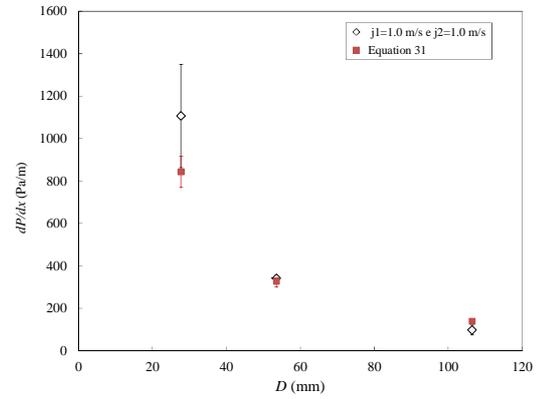
**Figure 3:** Flow pattern map for 53.55 mm inner diameter.



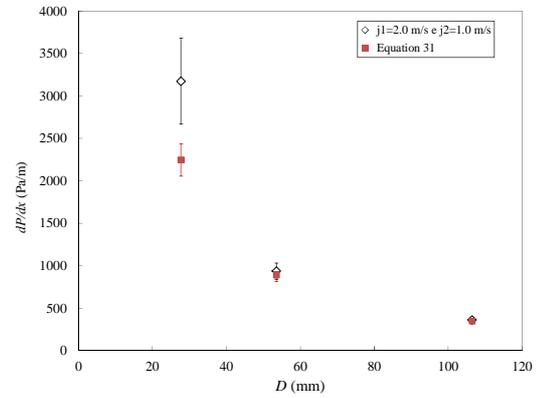
**Figure 4:** Flow pattern map for 82.25 mm inner diameter.



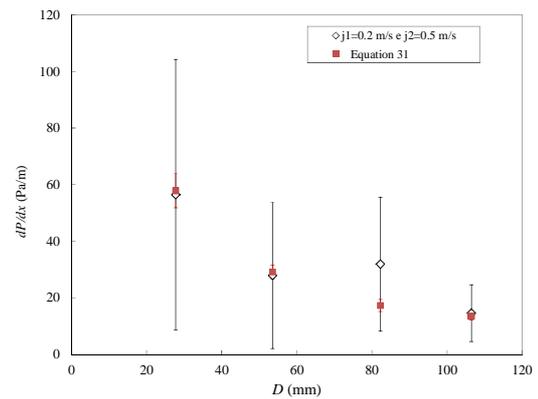
**Figure 5:** Flow pattern map for 106.47 mm inner diameter.



**Figure 6:** Two-phase measured pressure drop versus diameter, compared with calculated and its uncertainties, for  $j_1=1.0 \text{ m/s}$



**Figure 7:** Two-phase measured pressure drop versus diameter, compared with calculated and its uncertainties, for  $j_1=2.0 \text{ m/s}$



**Figure 8:** Two-phase measured pressure drop versus diameter, compared with calculated and its uncertainties, for  $j_1=0.2 \text{ m/s}$

## 5. Conclusion

From the analysis of the flow pattern maps proposed, it is possible to locate certain set of water-air velocities in a specified region or near other one. As the uncertainties in the maps are just

for superficial velocities, it can help understand regions of flow pattern transitions.

The variability of pressure drop results gave a better idea of how known are the process studied, or the experimental setup, and focus the conclusions for original study that aimed to understand the diameter influence on two-phase air-water flows. The two-phase pressure drop variation with diameter validated the homogeneous model as a good approach to calculate pressure drop when it is considered the uncertainties of both measured and calculated.

Consider the reproducibility of the results as a source of uncertainty, repeating each experiment in each setup three times, improves the quantification of the final uncertainties because it considers the possibility of new tests being performed, thus showing the capacity of the bench setups on reproduce the measured results.

### Acknowledgments

The authors would like to acknowledge IPT, ALFA Research Group and all of its contributors for the support and suggestions.

### Symbols and nomenclature

#### Latin Characters

$A$	Annular flow pattern; pipe section area
$B$	Bubble flow pattern
$D$	Pipe diameter
$EB$	Elongated bubble flow pattern
$f$	Friction factor
$j$	Superficial velocity
$J$	Mixture velocity
$k$	Coverage factor
$n$	Number of samples
$P_{atm}$	Atmospheric pressure
$P_{est}$	Static pressure
$R$	Gas constant (287,053 J.kg <sup>-1</sup> .K <sup>-1</sup> )
$S$	Slug flow pattern
$s$	Biased estimator of standard deviation
$SS$	Stratified smooth flow pattern
$T$	Temperature
$u$	Standard uncertainty
$U$	Expanded uncertainty
$u_c$	Combined standard uncertainty
$V_{eff}$	Degrees of freedom
$W$	Mass flow rate
$WS$	Wavy stratified flow pattern
$y$	Refers to a general physical quantity

#### Greek Characters

$\Delta P$	Pressure drop
$\mu_m$	Mixture viscosity
$\rho; \rho_m$	Density; mixture density
$\sigma$	Standard deviation

#### Superscripts and subscripts

1,2	1 refers do water; 2 refers to air
$i$	Refers to water or air (1 or 2)
$j$	Number of samples
$z$	Number of samples

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