DYNAMIC CHARACTERISTICS OF A HYDRAULIC WHEATSTONE BRIDGE MASS FLOWMETER

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A hydraulic Wheatstone bridge represents a potential solution for a direct mass flow meter with a linear measuring characteristic. In order to investigate the dynamic performance of the flowmeter, a dynamic physical-mathematical model was built. Since the flowmeter's characteristic is linear, it was found to be unaffected by the square root errors, which is the case with standard differential pressure flowmeters. On the other hand, the flowmeter's frequency characteristic exhibits a typical resonance. The theoretical results were also confirmed experimentally. Experimental studies of the water flow pulsation effects on the flowmeter were conducted in a test rig with a diaphragm-type flow pulsator.

Introduction

The hydraulic Wheatstone bridge is one of the earliest methods used for direct mass flow measurements (see, e.g., the patent [1]). It represents a linearized differential pressure flowmeter, which is achieved with a design solution in analogy with an electrical Wheatstone bridge. A typical configuration of a hydraulic Wheatstone bridge is shown in Fig. 1. It consists of four, precisely matched flow restrictors (e.g., orifices with pressure loss coefficients K_i; see the definition of K_i in the next section) arranged in a bridge network and a pump that provides a constant recirculating volumetric flow q_{vr} through a connecting conduit. The measured mass flow q_m affects the flow and the pressure distributions in the bridge. In the case of an ideal flowmeter configuration with identical flow restrictors of constant K_i = K and a constant volumetric recirculating flow, the dependence of the differential pressure across the bridge Δp_{13} or across the connecting conduit Δp_{24} on the measured mass flow rate q_m can be written as [2]:

$$\Delta p_{13} = \begin{cases} K q_{vr} q_m & \text{for } q_m \le \rho q_{vr} \\ \dfrac{K}{2\rho}\left(q_m^2 + \rho^2 q_{vr}^2\right) & \text{for } q_m > \rho q_{vr}, \end{cases}$$
$$\Delta p_{24} = \begin{cases} \dfrac{K}{2\rho}\left(q_m^2 + \rho^2 q_{vr}^2\right) & \text{for } q_m \le \rho q_{vr} \\ K q_{vr} q_m & \text{for } q_m > \rho q_{vr}. \end{cases} \quad (1)$$

If the flowmeter uses the differential pressure across the bridge Δp_{13} when $q_m \le \rho q_{vr}$ (the flow conditions are indicated with dashed arrows in Figure 1) and the differential pressure across the connecting conduit Δp_{24} when $q_m > \rho q_{vr}$ (the flow conditions are indicated in Figure 1 with full arrows), the ideal characteristic of the flowmeter can be written as:

$$\Delta p_{id} = K q_{vr} q_m . \quad (2)$$

Due to its linear measuring characteristics it has been successfully developed as a commercial device, which is primarily used for emission and fuel consumption measurements in the automotive industry and fuel flow measurements in oil industry. It is claimed to give a true mass flow rate, unaffected by density, temperature or viscosity changes.

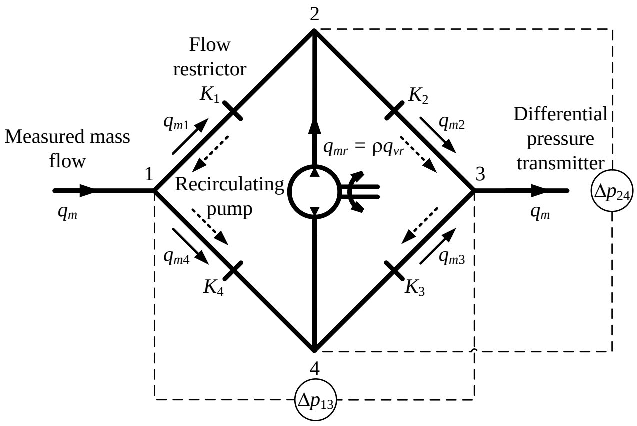

Figure 1. Schematic view of the hydraulic Wheatstone bridge mass flowmeter

There are not many accessible research studies that investigate the dynamic performance of hydraulic Wheatstone bridge mass flowmeters. A conference paper [3] presented a dynamic modelling approach to simulate the dynamic performance of a Wheatstone bridge using state space equations and the bond-graph method. The derived mathematical model considers the effects of fluid capacity and inertia. In [4] the dynamic performance of the hydraulic Wheatstone bridge was studied experimentally. The experimental results predict the suitability of the flowmeter for measuring the high frequency flow pulsations of the fuel in order to determine the performance of an internal combustion engine.

The purpose of this paper is to introduce new findings on the dynamic properties of the Wheatstone bridge mass flowmeter. Section 2 presents a dynamic physical-mathematical model of the flowmeter that was built by upgrading the steady-state flow model with the effects of fluid compressibility and inertia. This mathematical dynamic model was presented by the authors of this paper in [5]. Section 3 presents the developed experimental test facility with an integrated water-flow pulsator for an experimental investigation of the water flow pulsation effects on flowmeters. In Section 4 the results of the theoretical and experimental analyses are discussed and compared.

Physical-mathematical dynamic model

In order to examine the dynamic behaviour of the hydraulic Wheatstone measuring bridge, a dynamic model of the flowmeter was built on the basis of reference [2], but with a (more correct) nonlinear, square-root dependence between the flow rate and the pressure difference across the flow restrictors. The main modelling assumptions are that the local pressure losses across the flow restrictors are considered to be much greater than the other local and line pressure losses in the bridge conduits, that the inner diameter of the bridge conduits is assumed to be constant and that the hydraulic bridge is placed in the horizontal plane.

As shown schematically in Fig. 2, the dynamic model takes into account the fluid dynamics modelled by the lumped fluid capacitances C_i added on the nodes 1, 2, 3 and 4 of the bridge network and the lumped fluid inertia L_5 associated with the flow in the bridge's outlet duct.

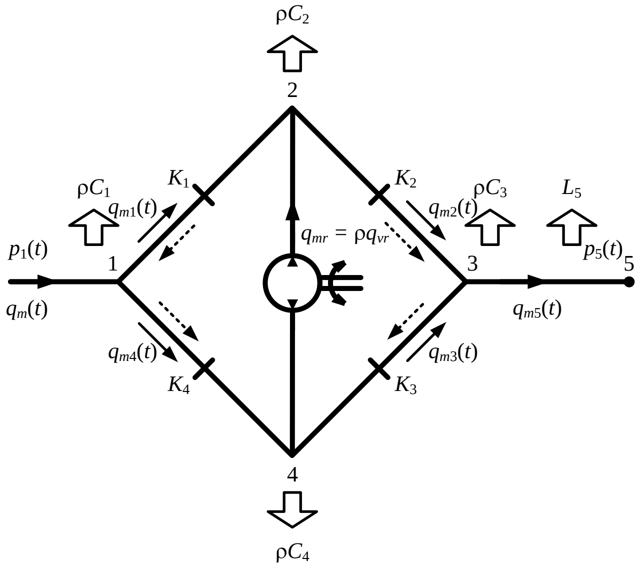

Figure 2. Schematic view of the dynamic model of the hydraulic Wheatstone bridge

The fluid capacitance C is introduced into the model as the relation between the volume flow rate change Δq_v and the pressure time derivative $\mathrm{d}p/\mathrm{d}t$ in a rigid pipe segment [6]:

$$\Delta q_v = C\frac{\mathrm{d}p}{\mathrm{d}t}, \tag{3}$$

where C depends on the bulk modulus $\beta = c^2\rho$ (where c is the fluid speed of sound) and the fluid volume V in the pipe segment:

$$C = \frac{V}{\beta} = \frac{V}{c^2\rho}. \tag{4}$$

The fluid inertia L is introduced into the model as the relation between the pressure change Δp and the volume flow rate time derivative $\mathrm{d}q_v/\mathrm{d}t$ in the pipe segment [6]:

$$\Delta p = L\frac{\mathrm{d}q_v}{\mathrm{d}t}, \tag{5}$$

where L depends on the length of the pipe section l and its internal cross-sectional area A:

$$L = \frac{\rho l}{A}. \tag{6}$$

The equations below are written for the proposed direction of the fluid flow, which is indicated in Figure 1 by the full arrows. The law of the conservation of mass can be written for all four nodes (1, 2, 3 and 4) of the bridge network as:

$$\begin{aligned} &q_m = q_{m1} + q_{m4} + \rho C_1\frac{\mathrm{d}p_1}{\mathrm{d}t},\quad q_{m1} + \rho q_{vr} = q_{m2} + \rho C_2\frac{\mathrm{d}p_2}{\mathrm{d}t},\\ &q_{m2} + q_{m3} = q_{m5} + \rho C_3\frac{\mathrm{d}p_3}{\mathrm{d}t},\ q_{m4} = q_{m3} + \rho q_{vr} + \rho C_4\frac{\mathrm{d}p_4}{\mathrm{d}t}, \end{aligned} \tag{7}$$

where q_m is the measured mass flow rate, q_{vr} is the recirculating volume flow rate, ρ is the fluid density and q_{mi} is the mass flow rate in a particular branch conduit. The law of the conservation of energy is written in the form of the non-recoverable pressure losses of the turbulent flow, which are defined by the pressure loss coefficients K_i of the flow restrictors:

$$\Delta p_i = \frac{K_i|q_m|q_m}{\rho}, \tag{8}$$

where Δp_i equals:

$$\begin{aligned} &\Delta p_1 = p_1 - p_2,\ \Delta p_2 = p_2 - p_3,\\ &\Delta p_3 = p_4 - p_3,\ \Delta p_4 = p_1 - p_4. \end{aligned} \tag{9}$$

By adding the inertia effect of the fluid in the outlet duct the energy equation can be written as:

$$p_3 - p_5 = \frac{L_5}{\rho}\frac{\mathrm{d}q_{m5}}{\mathrm{d}t}. \tag{10}$$

The mass flow rates in Eqs. (7) can be replaced with the pressure losses across the flow restrictors by using Eq. (8). So, the physical-mathematical model under discussion is generally fulfilled with five, first-order, nonlinear differential equations. The equations are solved with the fourth-order Runge-Kutta fixed-step method for the defined (in general, time dependent) boundary conditions $p_5(t)$, $p_1(t)$ or $q_m(t)$ and $q_{vr}(t)$, where for the initial conditions the steady state is considered.

The results presented in Section 4 are obtained for the following input data. At the inlet of the hydraulic bridge a sinusoidal source of pulsations $q_m(t) = q_m(1 + \varepsilon_P\sin(2\pi f_P t))$ is assumed, where q_m is the time-mean value of the mass flow rate, ε_P is the relative amplitude and f_P is the pulsation frequency. At the outlet of the hydraulic bridge a constant pressure value $p_5(t) = p_5$ is assumed and the recirculating pump is considered to be the source of the constant volumetric recirculating flow $q_{vr}(t) = q_{vr}$. In addition to the above assumptions, the orifice pressure loss coefficient $K_i = K$ is assumed to be independent of the flow rate. Referring to the actual configuration of the Wheatstone bridge flowmeter discussed in the next section, we take into account the experimentally evaluated sensitivity of the Wheatstone bridge $K = 0.84$ cm^{-4}, the capacitances $C_1 = 2.1 \cdot 10^{-13}$ m^{5}N^{-1} (volume of approximately 0.4 dm^3), $C_2 = 5.1 \cdot 10^{-13}$ m^{5}N^{-1} (volume of approximately 1 dm^3), $C_3 = 3.2 \cdot 10^{-13}$ m^{5}N^{-1} (volume of approximately 0.6 dm^3) and $C_4 = 2.1 \cdot 10^{-13}$ m^{5}N^{-1} (volume of approximately 0.4 dm^3). The water density is 998 kg/m^3, the speed of sound is 1429 m/s. The water inertia L_5 is evaluated to be about $7.5 \cdot 10^5$ kg m^{-4} (approximate outlet duct length from the bridge to the expansion chamber is 0.34 m).

Measurement system

The experimental study of the hydraulic Wheatstone bridge mass flowmeter was carried out in the water flow rig that was designed for pulsating flow tests on different flowmeters (described in detail in [7]). The measurement system is schematically shown in Fig. 3.

A steady flow of water is produced by a variable speed controlled centrifugal pump (Grundfos, CRN4-120), where the pump intake is fed from a reservoir. The reference value of the mass flow rate is measured with a Coriolis mass flowmeter (Foxboro, CFS10 + CFT10). The water temperature is measured with an RTD sensor to calculate the density of the water. The flow pulsations with adjustable frequencies and amplitudes are generated by the pulsator, in which an electrodynamic shaker (LDS, V406) drives a diaphragm to create sinusoidal flow pulsations within the flow measurement system. The operation of the shaker is monitored with the aid of a signal from an ICP accelerometer (DeltaTron, 4507B004), which senses the motion of the diaphragm. The developed diaphragm water-flow pulsator is able to generate reproducible water flow pulsations with defined properties, such as the frequency (up to 1 kHz) and the amplitude of the pulsating flow. To reduce the flow pulsation effects on the reference Coriolis flowmeter and to shorten the wavelength of the pulsating flow, two expansion chambers are integrated upstream and downstream of the pulsator.

The value of the pulsating mass flow rate was estimated from the differential pressure measurements across an orifice (inner diameter $d = 12.3$ mm), which was integrated into the water flow rig (inner diameter $D = 25$ mm) downstream of the pulsator. The temporal inertia effects on the flow rate measured using the orifice plate flowmeter under pulsating flow conditions were evaluated in preliminary experiments using the method described in [8]. The temporal inertia term value of the used orifice was estimated to be about 0.7% of the convective inertia term value at the highest frequencies of pulsations generated in the experiments, and was therefore neglected. Two piezoelectric transducers (Kistler, 7261) were used to measure the dynamic pressure difference across the orifice. The piezoelectric pressure transducers have, considering the dynamics of the connecting tubing, a resonance at about 3.6 kHz (see e.g., [9]). Due to the fact that the piezoelectric measurement system does not measure the static pressure component, the signal of the differential pressure across the orifice Δp during the generated flow pulsations was constructed with the help of digital signal processing, where the dynamic pressure component measured with the pressure transducers was added to the mean differential pressure across an orifice determined from the measurements of the average mass flow with the Coriolis mass flowmeter.

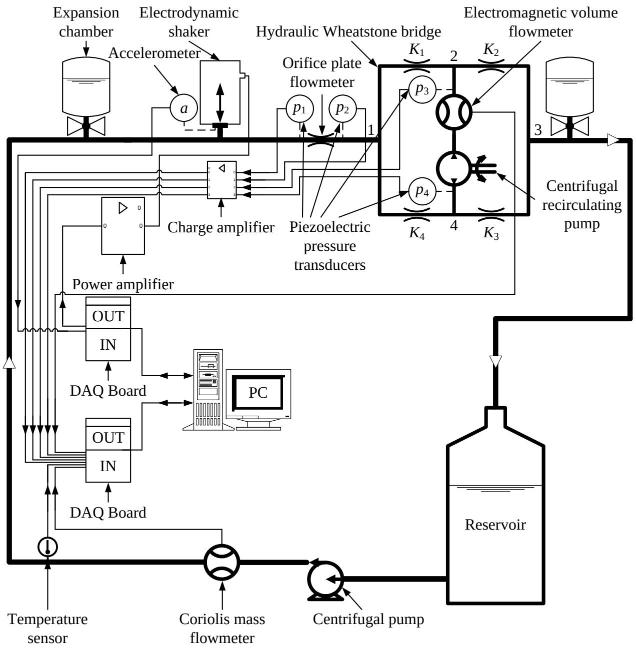

Figure 3. Schematic view of the measurement system

The Wheatstone bridge under test is made of branch conduits with inner diameters of $D = 24$ mm, where four orifices with inner diameters $d = 12$ mm are used as the flow restrictors. In the bridge's connecting conduit a recirculating flow was generated by the centrifugal recirculating pump (Grundfos, CRN2-110) and measured with an electromagnetic flowmeter (Foxboro, IMT25). The measurements were carried out for the flow conditions $q_m > \rho q_{vr}$, therefore, only the pressure difference across the connecting conduit Δp_{24} was measured with two piezoelectric transducers (Kistler, 7261), using the same digital signal processing algorithm as with the reference orifice plate flowmeter. The signals from all four pressure transducers are amplified using a charge amplifier (Dewetron, DAQ-Charge). The accelerometer's electrical output signal and all the other output signals are connected to two different data-acquisition (DAQ) boards, i.e., the National Instruments NI USB-9233 and PCI-6031E DAQ boards, respectively. The controller of the measurement system and the user interface are realized in the LabVIEW programming environment.

Results

In order to examine the dynamic performance of the hydraulic Wheatstone bridge the simulations of the measuring time-mean value of the pulsating flow and the determination of the flowmeter's frequency characteristic were performed. The theoretical and experimental studies of the presented hydraulic Wheatstone bridge were carried out at a mean mass flow rate q_m of approximately 2000 kg/h (0.56 kg/s) and a volumetric recirculating flow of 1000 dm^3/h.

Time-mean value of the pulsating flow

In standard differential pressure flowmeters (orifice plates, nozzles etc.), the nonlinear relationship between the flow rate and the pressure difference, $q_m \propto \sqrt{\Delta p}$, is a possible source of the so-called square root error in the indicated time-mean value of the flow rate under pulsating flow conditions [10]. The square root error occurs when the averaging of the measured differential pressure is done before calculating the flow rate, which can be, e.g., the result of a slow frequency response of the differential pressure transmitter. The resulting relative square root error can be written as:

$$e_{SRE} = \frac{\sqrt{\overline{\Delta p(t)}} - \overline{\sqrt{\Delta p(t)}}}{\overline{\sqrt{\Delta p(t)}}} . \quad (11)$$

The calculated results in Fig. 4 show its dependence on the relative amplitude of the pressure pulsations $\varepsilon_{\Delta p}$, when the differential pressure varies over time according to $\Delta p(t) = \overline{\Delta p}\left(1 + \varepsilon_{\Delta p} \sin(2\pi f_P t)\right)$ due to the pulsating flow. It is evident that with an increase in the relative amplitude of the pulsation $\varepsilon_{\Delta p}$ from 0 to 1, the relative error reaches values up to about 11% and it is independent of the pulsation frequency f_P.

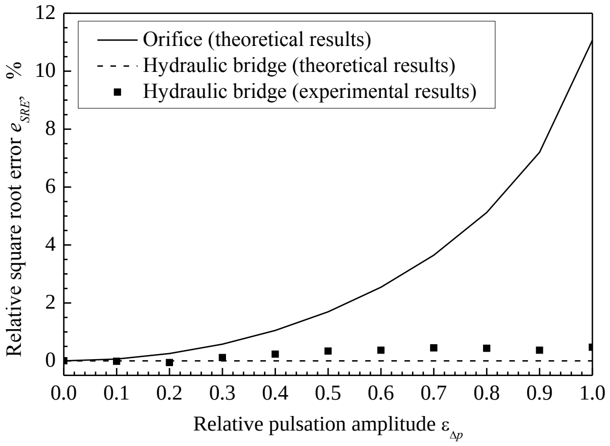

Figure 4. Variation of the relative square root error with the relative pressure pulsation amplitude

As might be expected for the linear hydraulic Wheatstone bridge, the simulations show that the errors in the determination of the time-mean value of the measured flow from the time-mean pressure differences were found to be practically zero. While the frequency characteristic of the hydraulic bridge was experimentally studied with the final implementation of the developed flow pulsator, the errors involved in measuring the time-mean value of the pulsating flow with the hydraulic bridge were determined in preliminary experiments using a pulsator consisting of two parallel conduits with the switching valve integrated into the main conduit [11]. The measurements were carried out at $f_P = 5$ Hz, where the relative amplitude of the differential pressure pulsations across the connecting conduit Δp_{24} was varied. The measurement results shown in Fig. 4 confirm that the square root errors of the hydraulic bridge do not exceed 0.5%. They probably occur due to the influence of the variation of the orifice pressure loss coefficient, which causes a certain degree of nonlinearity of the actual hydraulic Wheatstone bridge (see [5]).

Frequency characteristics

In order to examine the frequency characteristic of the hydraulic Wheatstone bridge the ratio between the relative amplitude of the mass flow pulsations measured with the hydraulic bridge and the relative amplitude of the actual pulsating mass flow was determined:

$$r = \frac{q_{m,HWB}^{RMS}}{q_{m,Orifice}^{RMS}} , \quad (12)$$

where $q_{m,HWB}^{RMS}$ and $q_{m,Orifice}^{RMS}$ are the root-mean-square values of the pulsating mass flow measured with the hydraulic Wheatstone bridge and the orifice plate flowmeter, respectively.

Fig. 5 presents the time variations of the fluctuating components of the mass flow rate measured with the orifice plate flowmeter and the Wheatstone bridge for the frequency of the pulsating flow $f_P = 120$ Hz. The flowmeter's response shows clear amplitude and phase dynamic errors.

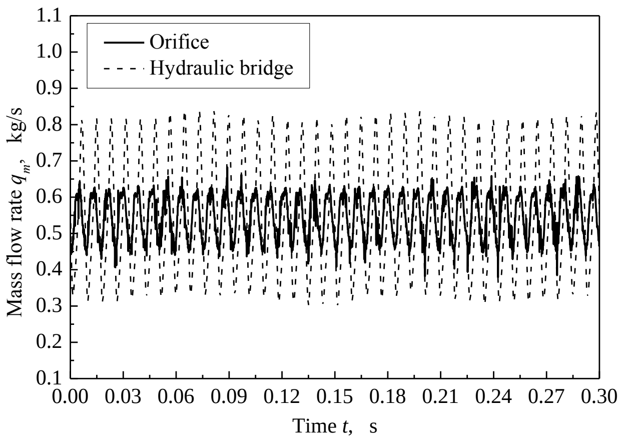

Figure 5. Measured time variation of the pulsating mass flow rate ($f_P = 120$ Hz)

Fig. 6 presents the normalized amplitude-frequency characteristic for pulsation frequencies from 10 Hz to 200 Hz, where the experimental results, obtained from the repeated measurements, are compared with the solution of the physical-mathematical model. The flowmeter has a typical resonance at about 164 Hz for water. The experimental and theoretical results show a similar trend for the frequency characteristic. The repeatability of the experimental results slightly deteriorates at the resonance frequency of the hydraulic bridge.

The simulations were also performed for another fluid (fuel with $\rho = 750$ kg/m^3 and $c = 1300$ m/s), which has a smaller bulk modulus $\beta = c^2\rho$ compared with water and so results in a smaller resonance frequency (dashed line). If the flowmeter configuration were to have a smaller fluid volume, which defines the fluid capacitance and the resulting compressibility effects, the resonance frequency would be higher.

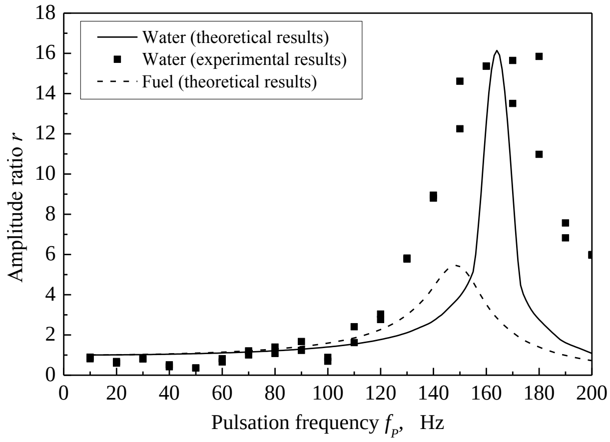

Figure 6. Amplitude frequency characteristics of the hydraulic Wheatstone bridge

Conclusions

This paper discusses the dynamic characteristics of a hydraulic Wheatstone bridge mass flowmeter. In order to investigate the flowmeter's dynamic performance a dynamic model was developed. This dynamic model takes into account the fluid dynamic characteristics affected by the lumped fluid capacitances of the volumes of the bridge segments and the inertia of the fluid in the hydraulic bridge's outlet duct. Since the ideal flowmeter's characteristic is linear, the measurements of the time-mean value of the pulsating flow were found to be unaffected by the square root errors, which is the case with standard differential pressure flowmeters. This is an important fact for the industrial application of the hydraulic bridge flowmeter, were differential pressure transmitters with a slow frequency response are commonly used. On the other hand, the flowmeter's frequency characteristic exhibited a typical resonance, the value of which depends on the fluid (the speed of sound and the density) and the dimensional properties (internal volumes) of the bridge. The results indicate that the flowmeter follows, nearly ideally, the pulsating flow rate only up to some defined values of the pulsation frequencies. The resonance frequency is expected to become higher for nearly incompressible fluids and smaller volumes of the flowmeter, while the fluid intertia effects can be minimized by shortening the hydraulic bridge's outlet pipe (e.g., by integrating the expansion chamber at the outlet of the flowmeter). The experimentally obtained results of the hydraulic Wheatstone bridge's dynamic performance are in good agreement with the mathematical solution.

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