END-SHORTING EFFECT ON ELECTROMAGNETIC FLOWMETERS

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Abstract

This study analyzes an end-shorting effect on electromagnetic flowmeters with short distance between connecting flanges around about 5D. The Bevir 's formulation is extended to conducting pipe wall. Through the evaluation of the Green function satisfying the corresponding boundary conditions, presice formula is obtained.

Introduction

Electro-magnetic flowmeters have been one of the standard flowmeters to measure liquid flowrate in industry. Theoretical background has been confirmed in the literatures; standard exhaustive text books were written by Shercliff[1] and Schmmartz[2].

Conventional electromagnetic flowmeters have almost uniform magnetic field, and a pair of point electrodes are installed on a circular pipe, where output flow signal is proportional to the flowrate when the velocity profileis axisymmetric. To increse the SNR, almost all conventional and commercially available flowmeters adopt low frequency rectangular wave and additional higher frequency waves to drive the field with well designed coils. Due to the progress in designing the driving coil, magnetic flowmeters have been reduced in length down to 5D[8]; since finiteness of the meter turns out the dependence of the flow signal on liquid conductivities, a compensation method was implemented.

After the introduction of the weight vector by Bevir[3], characteristics of the electromagnetic flowmeters were analysed rigourously through this concept for the various setups including variations of driving magnetic fields[4], [5], [6] and meter size finiteness[7].

This paper describes an additional strict treatment of the effect of the connected pipelines of a conductive wall, which brings the dependency on liquid conductivity for the meters of short meter lengths. The Bevir's weight vector is extended to analyze the finiteness of the meter.

Theory of Electro-magnetic Flowmeter

The electro-magnetic flowmeter has a long history from Faraday, and the theoretical basis have been founded in literatures. We extend the formulation by Bevir[3] to the flowmeter of finite length. The formulatin and the weight vectors were already been extended to include displacement current $\partial \boldsymbol{D}/\partial t$ to treat the high frequency field[9],[10]. Operational principle of electromagnetic flowmeters are given by Maxwell's equations, providing that permeability of the fluid is equal to μ_o of vacuum and the flow velocity is much smaller than the speed of light. The condition for immediate dielectric relaxation[1],[11], that is negligible displacement current, is $\omega\epsilon/\sigma \ll 1$, where ω denotes field excitation angular frequency.

We adopt scalar and vector potentials, U and $\boldsymbol{A}$ of Coulomb gauge:

$$\boldsymbol{E} = -\operatorname{grad} U - \frac{\partial \boldsymbol{A}}{\partial t}, \tag{1}$$

$$\boldsymbol{B} = \operatorname{rot} \boldsymbol{A}, \tag{2}$$

$$\operatorname{div} \boldsymbol{A} = 0 \tag{3}$$

In order to extend Bevir's formulation[3] to include the vector potential and displacement current, let U, $\boldsymbol{j}$ and $\boldsymbol{D}$ be a potential and current and electric flux densities when driving unit alternating current $e^{-\mathrm{i}\omega t}$ is fed between the electrodes S_1 and S_2 without the fluid motion and field excitation; let U^{m}, $\boldsymbol{j}^{\mathrm{m}}$ and $\boldsymbol{D}^{\mathrm{m}}$ be the potential and current and electrical densities induced by the fluid motion under the field excitation.

$$\boldsymbol{j} = \sigma \boldsymbol{E} = -\sigma \operatorname{grad} U \tag{4}$$

$$\boldsymbol{D} = \epsilon \boldsymbol{E} = -\epsilon \operatorname{grad} U \tag{5}$$

$$\begin{aligned}\boldsymbol{j}^{\mathrm{m}} &= \sigma(\boldsymbol{E}^{\mathrm{m}} + \boldsymbol{v} \times \boldsymbol{B}) \\ &= \sigma\left(-\operatorname{grad} U^{\mathrm{m}} - \frac{\partial \boldsymbol{A}}{\partial t} + \boldsymbol{v} \times \boldsymbol{B}\right)\end{aligned} \tag{6}$$

$$\begin{aligned}\boldsymbol{D}^{\mathrm{m}} &= \epsilon \boldsymbol{E}^{\mathrm{m}} + (\epsilon - \epsilon_o)\boldsymbol{v} \times \boldsymbol{B} \\ &= \epsilon\left(-\operatorname{grad} U^{\mathrm{m}} - \frac{\partial \boldsymbol{A}}{\partial t}\right) + (\epsilon - \epsilon_o)\boldsymbol{v} \times \boldsymbol{B}\end{aligned} \tag{7}$$

With these notations, Bevir's formulation is extended as follows.

Let us consider the following surface integral,

$$\iint_S \left[U^{\mathrm{m}}\left(\boldsymbol{j} + \frac{\partial \boldsymbol{D}}{\partial t}\right) - U\left(\boldsymbol{j}^{\mathrm{m}} + \frac{\partial \boldsymbol{D}^{\mathrm{m}}}{\partial t}\right)\right] \cdot d\boldsymbol{S}$$

where integration surface S consists of electrode surfaces S_1, S_2 and outer pipe surface S_3. When the flow conduit pipe is grounded and has higher conductivity, the integral surface can be reduced to S_1 and S_2, and potentials U^{m} and U are constant there. Since signal voltage amplifier has much higher input impedance, the surface integral reduces to the terms with the applied virtual current:

$$\iint_{S_1}\left(\boldsymbol{j} + \frac{\partial \boldsymbol{D}}{\partial t}\right)\cdot d\boldsymbol{S} = -\iint_{S_2}\left(\boldsymbol{j} + \frac{\partial \boldsymbol{D}}{\partial t}\right)\cdot d\boldsymbol{S} = e^{-\mathrm{i}\omega t} \tag{8}$$

Consequently, the surface integral is equal to the flow signal:

$$U_o = \iint_S \left[U^{\rm m} \left(\boldsymbol{j} + \frac{\partial \boldsymbol{D}}{\partial t} \right) - U \left(\boldsymbol{j}^{\rm m} + \frac{\partial \boldsymbol{D}^{\rm m}}{\partial t} \right) \right] \cdot d\boldsymbol{S}$$
$$= (U_1 - U_2) e^{-{\rm i}\omega t} \quad (9)$$

By using Gauss integral formula, it is converted to following volume integral,

$$U_o = \iiint_V \left[({\rm grad}\, U^{\rm m}) \cdot \left(\boldsymbol{j} + \frac{\partial \boldsymbol{D}}{\partial t} \right) - ({\rm grad}\, U) \cdot \left(\boldsymbol{j}^{\rm m} + \frac{\partial \boldsymbol{D}^{\rm m}}{\partial t} \right) \right] d\tau \quad (10)$$

where vector identity div rot $\equiv 0$ is applied to give div rot $\boldsymbol{H} = {\rm div}\,(\boldsymbol{j} + \partial \boldsymbol{D}/\partial t) = 0$. Let us take the same frequency ω for driving magnetic flux density $\boldsymbol{B}$ and virtual current density $\boldsymbol{j}$, and hence all field variables have time dependence $e^{-{\rm i}\omega t}$, and partial derivative with respect to time t can be replaced by $-{\rm i}\omega$. Substituting Eqs.(4),...,(7) in the above volume integral, and some manipulations give the output signals between electrodes in the following separated formulas:

$$U_o = U_{\rm A} + U_{\rm v} \quad (11)$$

$$U_{\rm A} = - \iiint_V \left(\boldsymbol{j} + \frac{\partial \boldsymbol{D}}{\partial t} \right) \cdot \frac{\partial \boldsymbol{A}}{\partial t}\, d\tau \quad (12)$$

$$U_{\rm v} = \frac{\sigma - {\rm i}\omega(\epsilon - \epsilon_o)}{\sigma - {\rm i}\omega\epsilon} \iiint_V \left(\boldsymbol{j} + \frac{\partial \boldsymbol{D}}{\partial t} \right) \cdot (\boldsymbol{v} \times \boldsymbol{B})\, d\tau \quad (13)$$

$U_{\rm A}$ depends on eddy currents induced by vector potential of the driving field, and $U_{\rm v}$, on flow velocity field. The latter component coincides the output flow signal of the conventional electromagnetic flowmeter[11].

If the fluid has the enough conductivity and displacement current can be ignored as in usual operation, the flowmeter signals are reduced to,

$$U_{\rm A} = - \iiint_V \boldsymbol{j} \cdot \frac{\partial \boldsymbol{A}}{\partial t}\, d\tau \quad (14)$$

$$U_{\rm v} = \iiint_V \boldsymbol{j} \cdot (\boldsymbol{v} \times \boldsymbol{B})\, d\tau = \iiint_V (\boldsymbol{B} \times \boldsymbol{j}) \cdot \boldsymbol{v}\, d\tau \quad (15)$$

Conventional electromagnetic flowmeters have been designed to cancel the first component $U_{\rm A}$ by using the symmetric configuration of driving coils and electrodes. We have proposed the use of the eddy current dependent component for the auto calibration.

Formulation of End-shorting

Let us consider the short electromagnetic flowmeter with the length of $2L$ and the pipe diameter of $2a$ inserted in the pipeline with sufficiently conductive wall.

When we can neglect the displacement current, virtual current distribution $\boldsymbol{j}$ is described with static electrical scalar potential ϕ, which has been obtained by solving the following Neumann problem in the cylindrical coordinates:

$$\Delta \phi(\boldsymbol{x}) = 0 \quad (16)$$

$$\left. \frac{\partial \phi}{\partial \rho} \right|_{\rho = a} = -\frac{\tilde{I}}{\sigma} \frac{1}{\rho} \delta(\varphi \pm \varphi_0) \delta(z), \quad |z| \le L \quad (17)$$

$$\phi(a, \varphi, z) = 0, \quad |z| > L \quad (18)$$

where $\tilde{I}$ is the amplitude of virtual current. When a pair of point electrode is put at $\boldsymbol{x}_0$ $(\rho_0 = a, \varphi_0 = \pm\pi/2, z_0 = 0)$ the scalar potential and the virtual current is obtained through the solution of the corresponding Green function for the following Neumann problem.

$$\Delta G(\boldsymbol{x}; \boldsymbol{x}_0) = -\, \delta(\boldsymbol{x} - \boldsymbol{x}_0) \quad (19)$$

$$\left. \frac{\partial G}{\partial \rho_0} \right|_{\rho_0 = a} = 0, \quad |z_0| \le L \quad (20)$$

$$G|_{\rho_0 = a} = 0, \quad |z_0| > L \quad (21)$$

An exact form of this Green function reads

$$G(\boldsymbol{x}; \boldsymbol{x}_0) = \sum_{m=0}^{\infty} \frac{\epsilon_m}{2\pi^2} \cos\, m(\varphi - \varphi_0) \int_0^{\infty} \left[K_m(\lambda\rho_0) - \frac{K'_m(\lambda a)}{I'_m(\lambda a)} \right] I_m(\lambda\rho) \cos \lambda z\, d\lambda \quad (22)$$

$$\bar{G}(\boldsymbol{x}; \boldsymbol{x}_0) = \sum_{m=0}^{\infty} \frac{\epsilon_m}{2\pi^2} \cos\, m(\varphi - \varphi_0) \int_0^{\infty} \left[K_m(\lambda\rho_0) - \frac{K_m(\lambda a)}{I_m(\lambda a)} \right] I_m(\lambda\rho) \cos \lambda z\, d\lambda \quad (23)$$

where $\epsilon_m = 2 - \delta_{0m}$ denotes Neumann factor, and $I_m(\cdot)$ and $K_m(\cdot)$, the modified Bessel functions of first and second kinds, respectively.

Poetntial Field and Virtual Current

Since we have seperate Green functions corresponding to the flowmeter, $|z| \le L$, and the pipelines, $|z| > L$, the Bevir's original formulation cannot be applied directly. Hence, we adopt Green's Theorem which is equivalent to the Bevir's formulation.

We can evaluate the potential field $\phi(\boldsymbol{x})$with the following volume integration of the above Green functions:

$$\phi(\boldsymbol{x}) = \iiint_V d\boldsymbol{x}_0 G(\boldsymbol{x}; \boldsymbol{x_0}) \sigma {\rm div}_0 (\boldsymbol{v} \times \boldsymbol{B})$$
$$= \iiint_V d\boldsymbol{x}_0\, {\rm grad}_0\, G(\boldsymbol{x}; \boldsymbol{x_0}) \cdot \sigma (\boldsymbol{v} \times \boldsymbol{B})$$
$$= \iiint_V d\boldsymbol{x}_0 [\boldsymbol{B} \times \sigma\, {\rm grad}_0\, G(\boldsymbol{x}; \boldsymbol{x_0})] \cdot \boldsymbol{v} \quad (24)$$

Then, the flow signal is obtained as follows:

$$U_\mathrm{v} = \phi(a, \pi/2, 0) - \phi(a, -\pi/2, 0) \quad (25)$$

Through the linearity of the integration (24), we define the difference of the Green fucunction in the integrand as,

$$\boldsymbol{g} \equiv \mathrm{grad}_0\, G(a, \pi/2, 0; \rho_0, \phi_0, z_0) - \mathrm{grad}_0\, G(a, -\pi/2, 0; \rho_0, \phi_0, z_0) \quad (26)$$

and

$$\bar{\boldsymbol{g}} \equiv \mathrm{grad}_0\, \bar{G}(a, \pi/2, 0; \rho_0, \phi_0, z_0) - \mathrm{grad}_0\, \bar{G}(a, -\pi/2, 0; \rho_0, \phi_0, z_0) \quad (27)$$

These are considered the extensions of the virtual current distributions. Accordingly, the flow signal can be described as the following seperate volume integration:

$$U_\mathrm{v} = \int_0^L dz_0 \int_{-\pi}^{\pi} \rho_0\, d\varphi_0 \int_0^a d\rho_0 [\boldsymbol{B} \times \sigma \boldsymbol{g}] \cdot \boldsymbol{v} + \int_L^\infty dz_0 \int_{-\pi}^{\pi} \rho_0\, d\varphi_0 \int_0^a d\rho_0 [\boldsymbol{B} \times \sigma \bar{\boldsymbol{g}}] \cdot \boldsymbol{v} \quad (28)$$

Evaluation of End-shorting Effect

If $\boldsymbol{g}$ is used only for the whole volume integration, we have the standard output signal. Hence, in order to evaluate the end shorting effect, we focus the differnce of $\boldsymbol{g}$ and $b\bar{m}g$ in the region of conducting pipe lines, $|z| > L$. Each element of the difference is obtained as,

$$(\boldsymbol{g} - \bar{\boldsymbol{g}})_\rho = \sum_{m:\mathrm{odd}} \frac{(-1)^{(m-1)/2}}{\pi^2 a} \sin m\varphi_0 \int_0^\infty \frac{I'_m(\lambda\rho_0)}{I'_m(\lambda a)} \cos \lambda z_0 d\lambda \quad (29)$$

$$(\boldsymbol{g} - \bar{\boldsymbol{g}})_\varphi = \sum_{m:\mathrm{odd}} \frac{(-1)^{(m-1)/2}}{\pi^2 a} m \cos m\varphi_0 \int_0^\infty \frac{I'_m(\lambda\rho_0)}{I'_m(\lambda a)} \cos \lambda z_0 d\lambda \quad (30)$$

$$(\boldsymbol{g} - \bar{\boldsymbol{g}})_z = -\sum_{m:\mathrm{odd}} \frac{(-1)^{(m-1)/2}}{\pi^2 a} \sin m\varphi_0 \int_0^\infty \frac{I'_m(\lambda\rho_0)}{I'_m(\lambda a)} \sin \lambda z_0 d\lambda \quad (31)$$

Since the end-shorting effect comes from the region, $|z| > L$, we can estimate it through the above differences. We integrate them to obtain,

$$\delta U = \int_L^\infty dz_0 \int_{-\pi}^{\pi} \rho_0\, d\varphi_0 \int_0^a d\rho_0 [\boldsymbol{B} \times \sigma(\boldsymbol{g} - \bar{\boldsymbol{g}})] \cdot \boldsymbol{v} \quad (32)$$

Before detailed evaluations, we consider the sinmplified case same as in [4]. Let us suppose the uniform magnetic field, namely, $\boldsymbol{B} = [B\cos\varphi, -B\sin\varphi, 0]$ in cylindrical coordinates. Flow profile is also considered uniform other than the fully developed one. Then, the end-short effect δU is evaluated through the orthogonality of $\sin, \cos, I_m, K_m$ by change the order of integrations. Since magnetic field is uniform, orthgonality of trigonometric functions admits $m = 1$ only. After the evaluation of the each integration, we have the following estimate,

$$\delta U = -\frac{B}{\pi^2} \int_0^\infty \frac{I_1(\lambda a)}{\lambda I'_1(\lambda a)} \frac{\sin \lambda L}{\lambda} d\lambda \quad (33)$$

This negative effect converges to 0 as the meter length of L and the area of exciting magnetic field approach infinity. Since the magnetic field and the velocity profile have the distributions, this evaluation gives the supreme effect.

Conclusions

We have derived the presice formula to evaluate end-shorting effects on enectromagnetic flowmeters. Practical evaluation needs numerical treatments based on theoretical analyses. The effect of liquid conductivity is an open problem to clarify for the application.

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