Uncertainty estimation of a liquid flow standard system with small flow rates

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Abstract

A liquid flow standard system has been used to calibrate liquid volume of a fuel-oil flow meter at small flow rates between 50 L/h and 700 L/h. However, the system has not been used to calibrate volume flow rate because the system runs at the standing-start-and-finish mode. To calibrate the volume flow rate of the fuel-oil flow meter, a flow diverter was installed and its performance was estimated in terms of measurement uncertainty. Diversion timing errors were corrected by linear curve fitting between measured and corrected elapsed time. Uncertainty contributions of the diversion timing errors amounted more than 60 % of the total uncertainty levels. The expanded uncertainty of volume flow rate was estimated to be more than 0.55 % in (50 ~ 700) L/h (k = 2.26) when the collected weight of liquid was about 10 kg. The expanded uncertainty became larger as the collected liquid weight was reduced.

Introduction

There are many concerns of calibrating a fuel-oil flow meter below 1,000 L/h in applications for automobile or aeronautical industry. The fuel-oil flow meter can estimate liquid volume by measuring elapsed time of steady flows in a pipe at a certain flow rate. Because the elapsed time can be measured in order of 10^{-3} s, the accuracy of flow rate measurement becomes important in determining the measurement uncertainty of the fuel-oil flow meter. KRISS has a liquid flow rate standard system (hereafter, LFSS), which can cover flow rates between 50 L/h and 700 L/h. The purpose of the LFSS is to determine the K-factor of the fuel-oil flow meter in units of [pulse/L]. Nevertheless, the LFSS has not been used to calibrate volume flow rate because it requires a flow diverter to run the LFSS in the flying-start-and-finish mode [1].

There are three types of flow diverters, i.e., a swivel, a rotary, and a linear type diverter [2, 3, 4]. The swivel type diverter is a traditional one which has a hinge to divert the flow path from one side to the other side in a LFSS. The rotary type diverter has an advantage to reduce timing errors by rotating the flow diverter in one direction. The linear type diverter has a simpler design compared with the swivel or the rotary type. Hence, the operation of the linear type becomes easier and the diversion timing errors might be between those of the swivel and the rotary type. A disadvantage of the linear diverter is that adjustment of the location of an optical sensor should be precise enough to balance the amount of liquid into and out of a weighing tank during flow diversion. The shape of a flow nozzle used in the diverter is also important. Some parameters to obtain a good shape of flow profile at the outlet of the nozzle are the area ratio between the inlet and the outlet, the divergence angle of the nozzle and the aspect ratio between the width and the depth of the rectangular shape of the nozzle outlet.

In the present study, a new LFSS was built to provide a service for calibrating volume flow rate as well as liquid volume. Tap water was used as a fluid medium for testing purposes before using light oil for calibration service. The LFSS was designed to measure flow rate with the flying-start-and-finish mode by means of a linear diverter. Diversion timing errors were estimated by introducing the Jones-Hibbert method [5, 6]. Covariance between liquid density and its buoyancy correction factor was also calculated because the volume flow rate is obtained by dividing mass flow rate with liquid density. After that, the measurement uncertainty of the LFSS was estimated as a function of collected weight and elapsed time

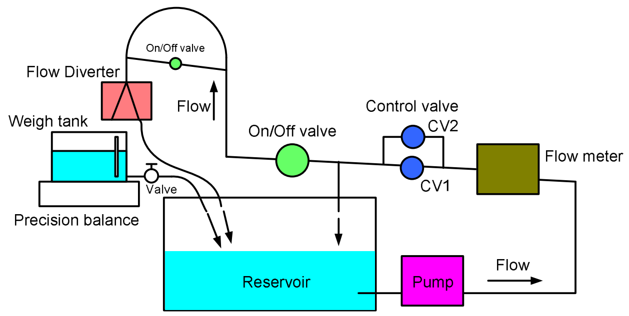

Figure 1. Schematic diagram of liquid flow standard system

of liquid by flow diversion.

Experimental Method

An experimental setup for the LFSS is displayed as shown in Fig. 1. A pipeline with diameter of 20 mm was used as a main test line. A pump (Laing E6 vario-25/180 G) was used to induce water from a reservoir to the main test line with a flow rate between 50 L/h and 700 L/h. Two needle valves with diameters of 19.1 mm (3/4") and 6.4 mm (1/4") were attached as a control unit to adjust the flow rate in the main test line. A bypass line was also constructed to maintain the flow rate in the pipe with a stable condition. An air vent valve was installed to remove air bubbles from the main test line. A ball valve (Kitz C-1 3/4" UTE) was located to operate the LFSS in the standing-start-and-finish mode. A linear flow diverter (Jeongsang Engineering Inc.) was placed to operate the LFSS in the flying-start-and-finish mode. Both the ball valve and the flow diverter were operated by pneumatic power to make the system cost-effective than the existing flow standard systems. A weighing tank which was a rectangular box with 450 mm × 280 m × 200 mm (height × width × depth), was located on top of a precision balance (Mettler Toledo 64000) with a measuring capacity up to 64 kg.
In measuring the flow rate in the main test line, water pressure and water temperature in the pipe were monitored by using a pressure transducer (Sensys PSHD0030PGPG) and a thermometer (Fluke 2180A). An electro-magnetic flow meter (E+H Promag W53H08) was used as a reference flow meter as timing-errors of the flow diverter were to be found [5]. The flow meter produced pulse signals at a rate of 5000 pulse/s when the flow rate was1000 L/h.

W [g]	*t* [s] @ 10A			*t* [s] @ 20A		
	50 L/h	100 L/h	200 L/h	150 L/h	400 L/h	700 L/h
350	24	12	6	6	3	2
700	48	24	12	12	6	3
1300	96	48	24	24	12	6
2700	180	96	48	48	24	12
5400	360	180	96	96	48	24
10000	720	360	180	180	96	48

Table 1. Experimental conditions for elapsed time as a function of collection weight at a certain flow rate

To determine *K*-factor of the flow meter in units of [pulse/L], a counter/timer (Agilent 53131A) was connected to an optical switch of the flow diverter. The counter/timer measured elapsed time of liquid being collected into the weigh tank [1, 5]. In-house software (LabVIEW 2010) was programmed to measure flow quantities such as the liquid temperature, the liquid pressure, the collected weight, the elapsed time and the pulse output of the flow meter. Tap water was used as a fluid medium to facilitate testing of the LFSS. We planned to use light oil after all the evaluation of the LFSS was completed.
Volume flow rate was measured according to the following formula [1].

$$Q = \frac{\varepsilon W}{\rho t} \tag{1}$$

Here, Q is the volume flow rate [L/h], W is the collection weight [kg], ρ is the water density [kg/m^3] and t is the elapsed time [s]. ε is called the buoyancy correction factor and has the following definition [1].

$$\varepsilon = \frac{1-\rho_a/\rho_p}{1-\rho_a/\rho} \tag{2}$$

Here, ρ_a is air density [kg/m^3] and ρ_p is dead weight density [kg/m^3]. Type A uncertainty of Q can be obtained by repetitive measurements of volume flow rate.

$$u_A(Q) = \left(\frac{1}{n(n-1)} \sum_{i=1}^{n} (Q_i - \overline{Q})^2 \right)^{1/2} \tag{3}$$

Here, Q_i is the i-th measurement value of Q [L/h], $\overline{Q}$ is the averaged value of Q [L/h] and n is the number of measurements. Type B uncertainty of Q can be established as follows

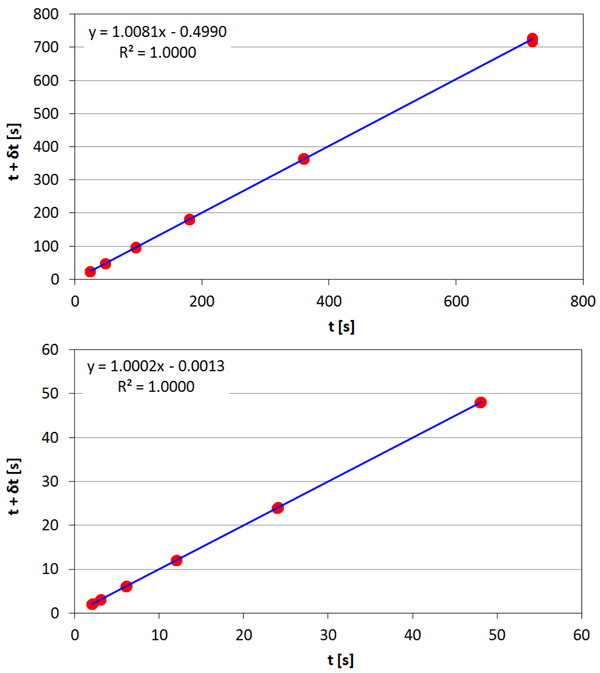

Figure 2. Correction of elapsed time of the flow diverter (top) 50 L/h (bottom) 700 L/h

[7, 8].

$$u_B(Q)=\sqrt{\left(\frac{\varepsilon}{\rho t}u(W)\right)^2+\left(-\frac{\varepsilon W}{\rho t^2}u(t)\right)^2+\left(\frac{W}{\rho t}u(\varepsilon)\right)^2+\left(-\frac{\varepsilon W}{\rho^2 t}u(\rho)\right)^2-\frac{2\varepsilon W^2}{\rho^3 t^2}r(\varepsilon,\rho)u(\varepsilon)u(\rho)} \quad (4)$$

Here, $u_B(Q)$ indicates propagation of uncertainty factors such as W, t, ε and ρ to the type B uncertainty. $r(\varepsilon,\rho)$ represents correlation coefficient between ε and ρ [7, 8]. If the ratio of ρ_a to ρ, i.e., ρ_a/ρ, is sufficiently small in comparison with 1, ε can be approximated by Taylor expansions [9].

$$\varepsilon \cong \left(1-\frac{\rho_a}{\rho_p}\right)\left(1+\frac{\rho_a}{\rho}\right)=1-\frac{\rho_a}{\rho_p}+\frac{\rho_a}{\rho}-\frac{\rho_a^2}{\rho_p\rho} \quad (5)$$

Therefore, the standard uncertainty of ε can be expanded as follows.

$$u(\varepsilon)\cong\sqrt{\left(\left(-\frac{1}{\rho_p}+\frac{1}{\rho}-\frac{2\rho_a}{\rho_p\rho}\right)u(\rho_a)\right)^2+\left(\left(\frac{\rho_a}{\rho_p^2}+\frac{\rho_a^2}{\rho_p^2\rho}\right)u(\rho_p)\right)^2+\left(\left(-\frac{\rho_a}{\rho^2}+\frac{\rho_a^2}{\rho_p\rho^2}\right)u(\rho)\right)^2} \quad (6)$$

All the uncertainties such as $u(W)$, $u(t)$, $u(\varepsilon)$ and $u(\rho)$ can be obtained by combining both type A and type B uncertainties [7, 8]. The expanded uncertainty of $U(Q)$ can be derived by

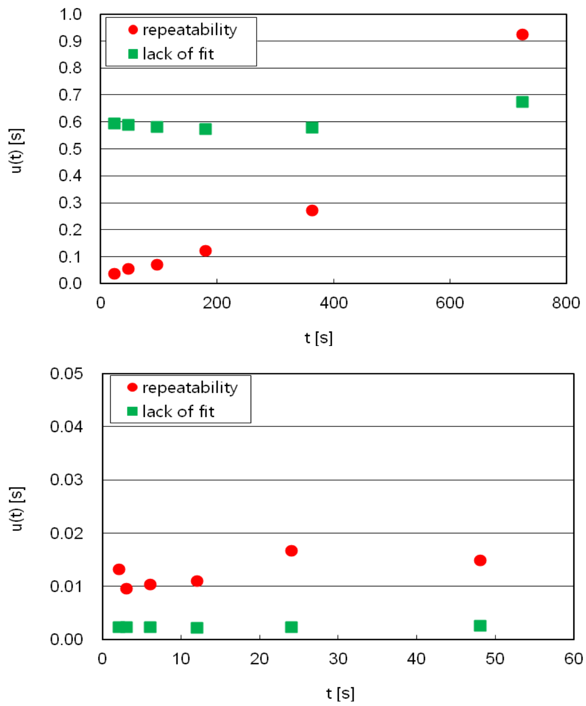

Figure 3. Uncertainty factors in determining timing errors (top) 50 L/h (bottom) 700 L/h

multiplying $u(Q)$ and a coverage factor k with confidence level of 95 %.

$$U(Q)=k\sqrt{u_A(Q)^2+u_B(Q)^2} \quad (7)$$

To determine k, Student-t distribution with effective degrees of freedom ν_{eff} should be obtained by using the Welch-Satterthwaite formula [7, 8].

Experimental Results

Apart from the standard procedure for timing-errors of a flow diverter, elapsed time can be corrected by using δt as follows [1, 5].

$$\delta t=\left(\frac{\varepsilon WK}{\rho N}-1\right)t \quad (8)$$

Here, N is the number of pulses [pulse] and K is the K-factor of the flow meter [pulse/L]. Because the corrected elapsed time is $t+\delta t$, the proportionality factor $(\varepsilon WK)/(\rho N)$ can be determined by linear regression. From Fig. 2, the proportionality factors are 1.0081 and 1.0002 for 50 L/h and 700 L/h, respectively. Then, the uncertainty of the elapsed time can be estimated as follows [6].

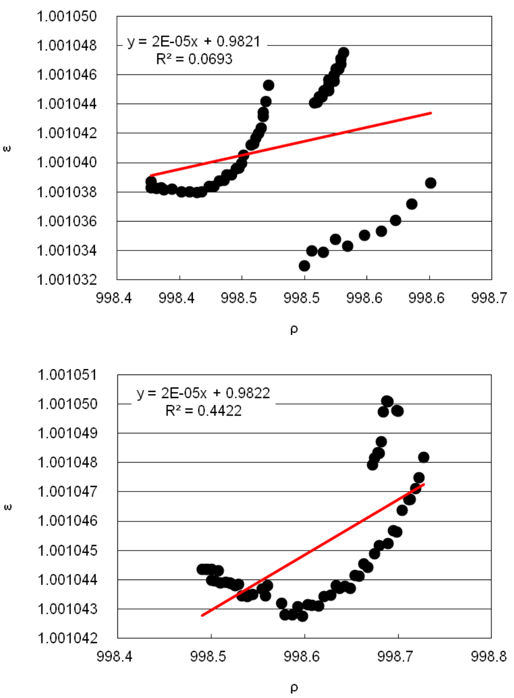

Figure 4. Buoyancy correction factor as a function of water density (top) 50 L/h (bottom) 700 L/h

$$u(t)=\frac{\rho N}{\varepsilon WK}\left(\frac{1}{m(m-1)}\sum_{i=1}^{m}\left((t+\delta t)_{i,t_0}-\overline{(t+\delta t)}_{t_0}\right)^2\right)^{1/2}+\frac{\rho N}{\varepsilon WK}s_{(t+\delta t)/t}\sqrt{\frac{1}{n}+\frac{\left((t+\delta t)_{t_0}-\overline{(t+\delta t)}\right)^2}{\left(\frac{\varepsilon WK}{\rho N}\right)^2\sum_{i=1}^{n}\left(t_i-\bar{t}\right)^2}} \quad (9)$$

Here, m is the number of repetition at a certain diversion time, n is the number of measuring points to construct a calibration curve, t_i is the i-th data of t, $\bar{t}$ is the averaged value of t, $(t+\delta t)_i$ is the i-th data of $t+\delta t$, $\overline{t+\delta t}$ is the averaged value of $t+\delta t$ and $s_{(t+\delta t)/t}$ is the standard error of the regression [6]. Note that the subscript t_0 indicates a certain diversion time for sampling the measurement data. The first term in Eqn. (9) indicates the standard error of estimates for the elapsed time by repetitive measurements at a certain flow rate. The second term indicates the effect of lack of fit to the calibration curve.

Covariance between ε and ρ can be calculated as follows [7, 8].

$$r(\varepsilon,\rho)=\frac{\sum_{i=1}^{n}\left(\varepsilon_i-\bar{\varepsilon}\right)\left(\rho_i-\bar{\rho}\right)}{\left(\sum_{i=1}^{n}\left(\varepsilon_i-\bar{\varepsilon}\right)^2\times\sum_{i=1}^{n}\left(\rho_i-\bar{\rho}\right)^2\right)^{1/2}} \quad (10)$$

From the experimental data displayed in Fig. 4, $r(\varepsilon,\rho)$ was calculated to be 0.26 at 50 L/h and 0.66 at 700 L/h.

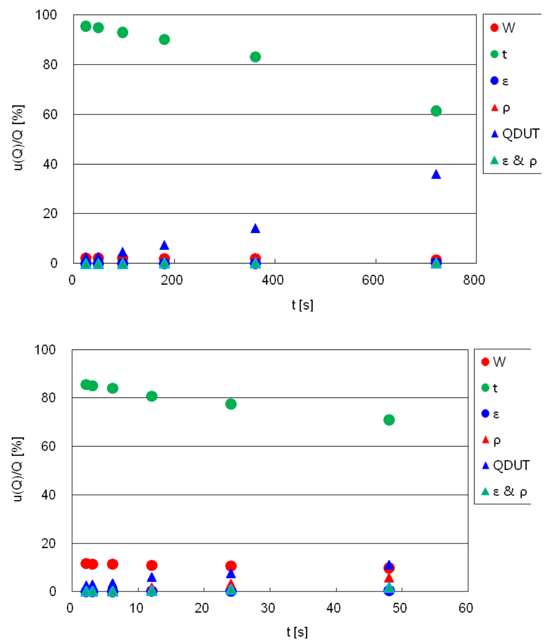

Figure 5. Uncertainty budget of liquid volume flow rate as a function of elapsed time (top) 50 L/h (bottom) 700 L/h

Uncertainty budgets can be drawn as a function of elapsed time in Fig. 5. The vertical coordinate shows the amount of contribution for each uncertainty factor when the total uncertainty scales to be 100 %. For example, the contribution of $u(t)$ was greater than 60 % of total combined uncertainty when the elapsed time was 720 s at 50 L/h. The repeatability of flow metering, $u_A(Q)$, became important in evaluating $u(Q)$ as the elapsed time was increased. The effect of $u_A(Q)$ was relatively unimportant at 700 L/h because the contribution of $u_A(Q)$ to $u(Q)$ was less than 11.2 % when the elapsed time was 48 s at 700 L/h. The effect of $u(W)$ was as large as $u_A(Q)$ in that it contributes to $u(t)$ less than 9.9 %.

The expanded uncertainty, $u(Q)$, is displayed in Fig. 6 and Table 2. Coverage factor k was found to be 2.26 because the effective degrees of freedom were 9. When the collection weight was targeted at 10000 g (i.e., 10 kg), $u(Q)$ was estimated as large as 0.09 % (k = 2.26) in the flow range of (150 ~ 700) L/h. On the contrary, $u(Q)$ was 0.55 % (k = 2.26) in (50 ~ 200) L/h. The main reason for the increase of uncertainty at low flow rates was that stability of small pipe flow was not as good as the stability of the large pipe flow. The stability affected on the repeatability of diversion time measurement. Therefore, some equipment such as a constant head tank was found to be necessary to stabilize the small pipe flow for achieving low

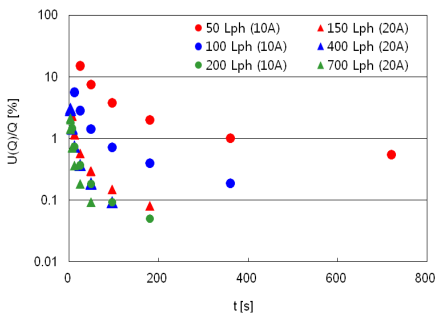

Figure 6. Expanded uncertainty of liquid volume flow rate

W [g]	*U*(*Q*)/*Q* [%] @ 10A			*U*(*Q*)/*Q* [%] @ 20A		
	50 L/h	100 L/h	200 L/h	150 L/h	400 L/h	700 L/h
350	15.20	5.68	1.47	2.33	2.92	2.01
700	7.51	2.81	0.73	1.14	1.44	1.42
1300	3.77	1.42	0.36	0.58	0.73	0.71
2700	2.02	0.72	0.19	0.29	0.37	0.36
5400	1.02	0.40	0.09	0.15	0.19	0.18
10000	**0.55**	**0.19**	**0.05**	**0.08**	**0.09**	**0.09**

Table 2. Expanded uncertainty as a function of collecton weight at a certain flow rate

measurement uncertainty at the low flow rate.

Conclusions

A liquid flow rate standard system was constructed and tested for providing a calibration service for fuel-oil flow meters in the flying-start-and-finish mode. As an initial step, tap water was used as a flowing medium before using light oil. Diversion time was pre-determined as a function of collection weight and volume flow rate. Ten consecutive measurements were made to determine the diversion timing errors when the collected weight and the volume flow rate were fixed. The diversion timing errors were obtained by linear curve fitting. Covariance between the water density and the buoyancy correction factors gave necessary information to calculate uncertainty factors due to dependence between the two parameters. As a result, the expanded uncertainty was estimated to be 0.09 % (k = 2.26) in (150 ~ 700) L/h and 0.55 % (k = 2.26) in (50 ~ 200) L/h. Increase of the uncertainty at 50 L/h was found to be caused by flow stability in the small pipe flow. This can be resolved by installing a constant head tank upstream of the main test line. Some more work regarding the constant head tank is still being undertaken.

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