Dependence of the Flow Velocity Field in Critical Nozzles on the Pressure Ratio

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Abstract: RTA (Recovery Temperature Anemometry), which estimates the flow velocity field based on the recovery temperature measured by a very thin thermocouple wire without disturbing the flow field, is introduced in the paper. Measurements of flow fields in critical nozzles by RTA reveal very interesting phenomena such as interaction of oblique shock system fixed on the nozzle geometry and a strong shock moving along the nozzle axis depending on the pressure ratio, which is considered to have a relationship with the premature unchoking phenomenon. Possibility to estimate the flow velocity distribution in the boundary layer is also discussed.

Keywords: Critical flow nozzle, Flow velocity field, Prandtl number, Recovery temperature, Boundary layer, Shock interaction, premature unchoking phenomenon

1. Introduction

Critical nozzle is one of the most accurate devices to measure gas flowrate on virtue of its insensitivity to the downstream condition. It is widely used in the fields all around the world [1] especially in order to transfer the values defined by the gas flow standards at the highest accuracy. Lots of measurements carried out by the standard facilities have made the performance of critical nozzles satisfying for most applications.

However there still remain a couple of major problems to be solved, one of which is the premature unchoking phenomenon (PUP) where critical nozzle does not choke even a lower back pressure ratio (downstream pressure/upstream pressure) than the theoretical back pressure ratio to choke is applied on it. PUP is observed almost inevitably in all nozzles of the common shape at the low Reynolds number, typically lower than several 10^4. When PUP occurs, the flowrate is defected in the order of 0.1 % in the beginning then gets larger when the Reynolds number is lowered. However, in many cases, even at these Reynolds number where PUP is expected, critical nozzles still can choke if the pressure ratio is kept at lower than 0.5, therefore, flow facilities can generate the exact flowrate by keeping the pressure ratio low enough even at the low Reynolds number, however, this is very energy consuming. One big problem is that PUP is unperceivable along with a common procedure of flow facilities as well as it has various behaviors depending on the Reynolds number, nozzle shape, operating condition, and so on by unknown reasons, therefore, it is not possible to set down general criteria to avoid PUP except for keeping the back pressure ratio lower than 0.5 when the Reynolds number is small.

For example, when a critical nozzle is sucking the air from the atmosphere, the maximum throat diameter where PUP occurs will be typically about 10 mm when the nozzle has the ISO 9300 toroidal throat geometry. To investigate PUP, the conventional measurements using Pitot tube, hot wire, and so on does not suit because the nozzle size is small. In addition, since the flowrate defect caused by PUP is very small, a model of the nozzle that has, for example, a window to enable optical observations will destroy the flow field specific to PUP. Under these conditions, RTA, Recovery Temperature Anemometry, was proposed to investigate the transonic flow velocity field with the minimum disturbance introducing into the flow field [2]. It measures the recovery temperature in the flow field by a very thin thermocouple wire of typically 10~50 μm diameter, and then converts into flow velocity based on the recovery factor. When the recovery factor is not unity, the thicknesses of the thermal and the velocity boundary layers differ from each other, which results in a wall temperature different from the stagnation, residue of which is a function of the flow velocity. Since temperature sensor in a flow field will indicate the temperatures of itself controlled by its wall temperature, a simple measurement of temperature over a flow field will yield its flow velocity field.

The sensitivity of a thermocouple wire is concentrated exactly at the contact point that makes the space resolution of RTA extremely high, which is completely different from the conventional method such as hot/cold wire anemometry. The space resolution of RTA is almost the same as the wire diameter, that is 10~50 μm in this case. This resolution is so high that the measuring points can be easily submerged even in the boundary layer generated on the nozzle wall whose thickness sometimes gets more than 0.1 mm.

The flow velocity fields measured by RTA coincides very well with 1D and 2D theoretical predictions by assuming the most common recovery factor, that is, that of the laminar boundary layer on flat plate, which is the square root of the Prandtl number. It is surprising that the RTA results does not depend on the wire direction against the flow, therefore the recovery factor employed is quite versatile for thin wires.

In the paper, some measurement results by RTA on the flow fields of critical nozzles of several shapes at various pressure ratios are shown, which clearly show an oblique shock system fixed on the nozzle shape, a moving strong shock whose location depends on the pressure ratio, their interactions, development of boundary layer on the throat, and other interesting phenomena.

2. RTA (Recovery Temperature Anemometry)

Deceleration of flow in the boundary layer towards the wall is accompanied by increase of temperature. In most gases, the recovery factor is smaller than the unity, that is, the temperature boundary layer is thicker than the velocity one, therefore, the temperature can not reach the stagnation temperature on the wall as depicted in Fig. 1.

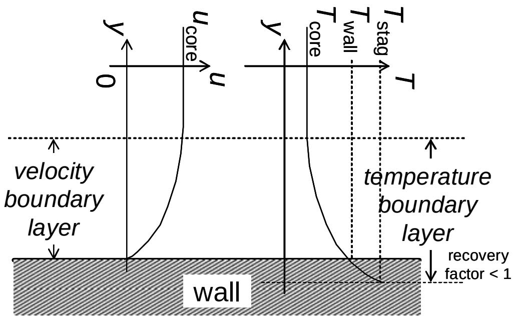


Fig. 1 The wall temperature

Introducing the recover factor f that is a function of the Prandtl number P_r of the flowing gas, the wall temperature T_wall is given by

$$T_\mathrm{wall} = \left\{ 1 + f\left(P_\mathrm{r}\right) \frac{\kappa - 1}{2} M_\mathrm{core}{}^{2} \right\} T_\mathrm{stat} \tag{1}$$

where κ, M_core, T_stat are the specific heat, the local Mach number of the core flow and its static temperature, respectively. Using the relationship between the static temperature and the Mach number in adiabatic flow, which is given by Eq. (1) with f=1, the core flow velocity is given by

$$u_{core} = \sqrt{\frac{2\kappa R}{\kappa - 1} \frac{\Delta T_{drop}}{1 - f}} \tag{2}$$

where R is the gas constant, $\Delta T_{\mathrm{drop}} = T_{\mathrm{stag}} - T_{\mathrm{wall}}$, and T_{stag} is the stagnation temperature. Since a temperature sensor indicates the temperature of itself that are to be equilibrium with the surroundings, a temperature sensor in a flow will simply measure T_{wall} itself.

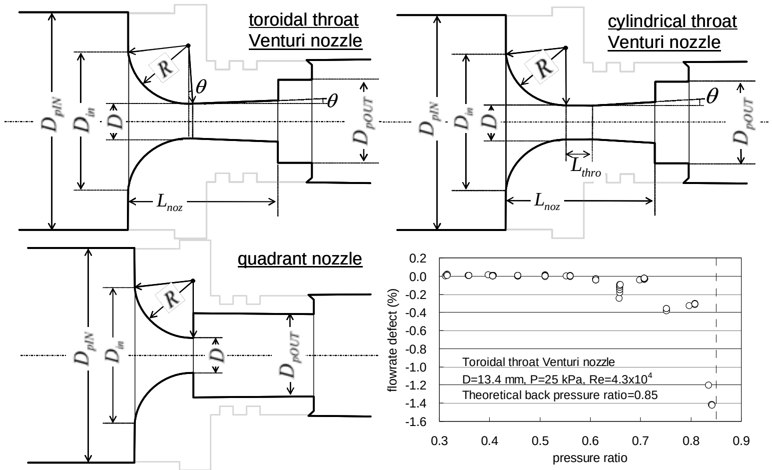
The recovery factor should depend on how the flow is decelerated in the boundary layer. However, it will be shown later that the most basic recovery factor, that is for the laminar boundary layer on flat plate, the square root of P_{r}, is accurately applicable to RTA using very thin thermocouple wire as shown later.

$$f = \sqrt{P_{\mathrm{r}}} \qquad (3)$$

In case of the air where P_{r}=0.71, κ=1.401, R=287.07 J/kg·K, the flow velocity is given by

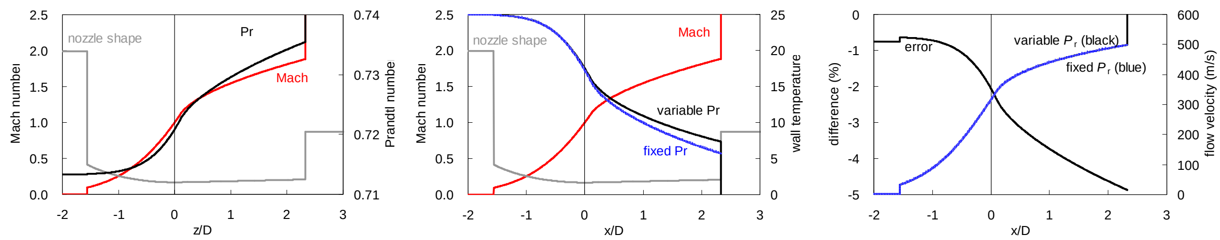
$$u_{\mathrm{core}} = 112.9\sqrt{\Delta T_{drop}} \; . \qquad (4)$$

ΔT_{drop} will be about 7.8 K at the critical point when the nozzle is sucking from the atmosphere at 25 ºC, therefore, a common thermocouple will resolve the flow velocity field in a critical nozzle.



Figs. 2 Nozzle geometries and premature unchoking phenomenon observed in one of them.

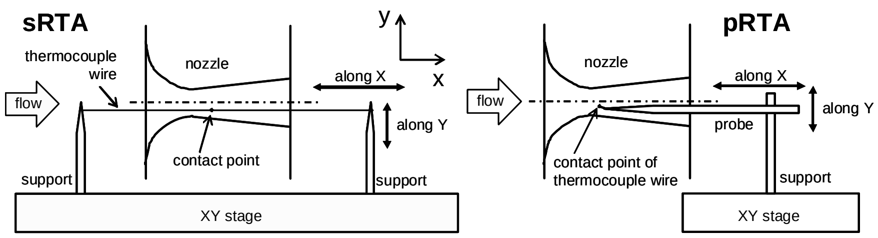
The geometries of the nozzles to be discussed in the paper are shown in Figs. 2 together with an example of the premature unchoking phenomenon observed in one of them [3], where the flowrate is defected at some pressure ratios larger than 0.5 although the nozzle should choke up to about 0.85 in theory when considering L_{noz}=4D and θ=3º.



Figs. 3 Variation of the Prandtl number in a toroidal throat Venturi critical nozzle (left), the wall temperature assuming a constant and variable Prandtl numbers (center) and the error in the estimation of the flow velocity by assuming a constant Prandtl number (right). x=0 at the throat.

The Prandtl number of the air is actually a function of pressure and temperature as shown in Figs. 3 (left) where the temperature and pressure distributions were calculated by assuming the 1D isentropic flow of the ideal gas. Although the accurate calculation of the flow requires the variable specific heat depending on the pressure and temperature, the variable compressibility factor and the variable viscosity, the effect of variable Prandtl number is estimated with the other physical properties kept constant. The center figure of Figs. 3 shows the wall temperatures calculated from Eq. (1) and the 1D isentropic Mach number distribution (the red line in the left figure) with a fixed and the variable Prandtl number. The fixed Prandtl number will introduce an error of a few degrees in the wall temperature when the flow velocity is high, but as shown in the right figure, it won't introduce serious qualitative difference into the flow velocity distribution. According to Eq. (4), the error is smaller than 5% in the critical nozzle as shown in the figure.

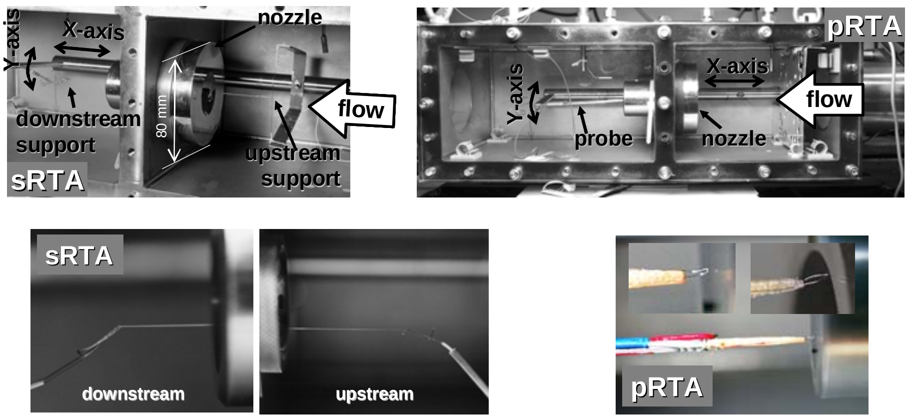
There are two ways for RTA to setup the thermocouple wire in a nozzle as shown in Figs. 4; one is to straddle and stretch the thermocouple wire and settle it along the flow which is named sRTA (streamwise RTA). The other is to put the contact point against the flow like a common probe which is named pRTA (probe RTA). As shown later, sRTA and pRTA produce the same result, therefore, RTA measures the magnitude of the flow velocity at the contact point regardless of the attack angle of the flow.



Figs. 4 Setups of a thermocouple wire for sRTA and pRTA.

sRTA introduces the minimum disturbance into the flow field, however, the wire can easily get vibration induced by the flow which will often result in cutoff of the wire when the pressure is high. On the other hand, pRTA is very robust but it will introduce relatively larger disturbance into the flow field owing to the existence of the support near the measuring point, however, it still holds the big advantage of the very small measuring volume, therefore, it will be applicable to measure the flow distribution at high pressure. The thermocouple wire also can be settled inclined against the flow because of its insensitivity on the attack angle that will enable measurements in a complicated space. In the measurements using K-type (chromel-alumel) thermocouple wires of 12.5, 25, or 50 mm diameter, there was no difference observed in the results depending on the wire diameter nor the attack angle.

The traverse employed for the RTA has a conventional horizontal X-axis as shown in Figs. 5. The X-axis has two arms of 80 mm length at the upstream and the downstream of the nozzle. In sRTA, the thermocouple wire is fixed on the tips of these arms whereas the probe in pRTA is settled on the downstream one. The traverse in the Y-direction is achieved by rotating the X-axis around its centerline, therefore, strictly speaking, the traverse is not linear but the curvature of the trajectory is large enough and in any case there is no effect if the flow is axi-symmetric. In the measurements shown below, K-type thermocouple wires were employed. The thin chromel and alumel wires settled in the flow were connected to thick ones of the same materials stuck on the arms, which were led to the inputs of a digital multimeter directly. The multimeter indicates the temperature by its own function with a resolution of 0.1 K. The



Figs. 5 The traverse and probes

measurement of the recovery temperature at one location took several 10 seconds, therefore, it took hours to obtain a full distribution over the flow field in a nozzle at one condition.

2. Measurement Results of the Flow Field in Critical Nozzles by RTA

2.1 Measurement along the Centerline of a Critical Nozzle

Fig. 6 is an example of the temperature drop distributions measured along the center line of a critical nozzle who has a toroidal throat with D=13.4 mm, L_{noz}=4D and θ=3°, kept at the upstream pressure of 25 kPa. The red line is the theoretical temperature drop calculated by Eq. (1) based on the theoretical 1D isentropic Mach number distribution that is shown in red in Figs. 3. The results by sRTA and pRTA coincide exactly from the nozzle inlet to slightly behind the throat (x=0) and they agree very well with the theoretical prediction in the same region. In the supersonic flow field, the measurements indicate that there are some structures existed that can not be predicted by the simple theory. The slight quantitative differences between sRTA and pRTA in this region are considered to be caused by the existence of the probe of pRTA, however, both have enough qualities to investigate the characteristics of the flow field.

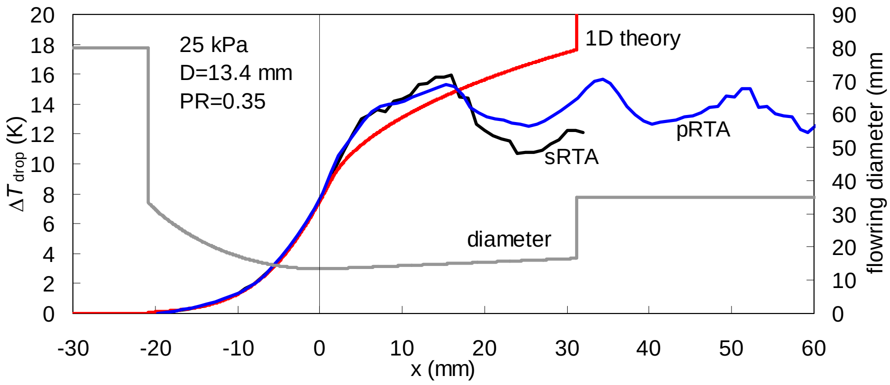


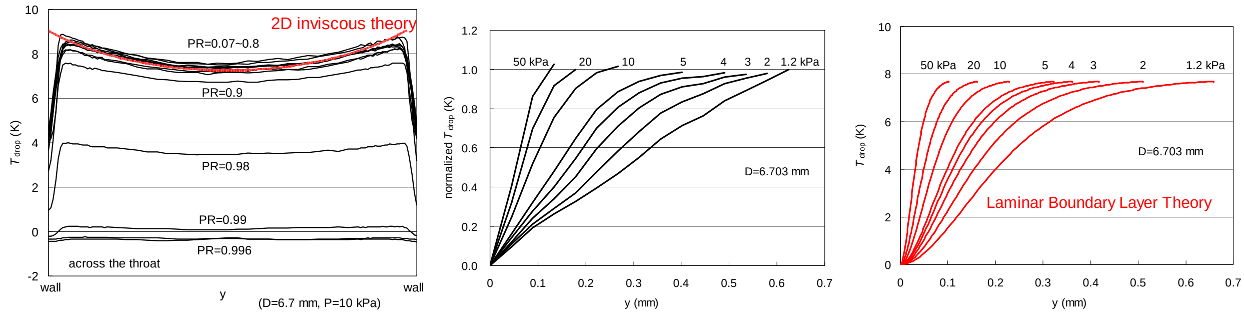
Fig. 6 Comparison of sRTA and pRTA results with 1D isentropic theory. Temperature drop ΔT_{drop} along the centerline of a toroidal throat critical nozzle were measured and calculated.

In some cases, straddled thermocouple wires had various shapes at their contact points, e.g., some were completely straight but some had their contact points clearly projected from its leg lines, but they all produced the same results. Preliminary measurements using inclined and perpendicular wires against the flow by sRTA also produced the same results, therefore, it is concluded that the recovery factor defined by Eq. (3) is applicable for any wire direction if the wire diameter is very small and also that RTA measures the magnitude of flow velocity at the contact point of the thermocouple wire regardless of the flow direction.

The small fluctuation in sRTA result in the supersonic region in Fig. 6 is considered to be an exact reflection of the fluctuating flow field because the pressure ratio (PR) of 0.35 is one of the most distinguished ones where the largest interaction of oblique and normal shocks takes place that introduces instability in the flow as shown later. On the other hand, in pRTA measurement, the probe is considered to have stabilized the flow field by its existence. The lower temperature in pRTA result also shows that the flow is accelerated by the existence of the probe.

2.2 Measurement over the Throat of a Critical Nozzle

Figs. 7 are the measurement results by sRTA across the throat of toroidal throat critical nozzles of 6.7 mm throat diameter at various pressure ratios (PR) or various upstream pressures. The axis Y is on the diameter of the nozzle throat. There are steep temperature increases observed near the wall. In the core flow, the nearer wall the location is, the faster the flow velocity is, as theories predict. At low PR where the flow is definitely choked, the distribution of the temperature drop agree very well with a precise inviscous 2D theoretical prediction [4]. The smaller temperature drop at the larger PR indicates that the nozzle is not any more choked and the flow velocity is lowered. The slightly higher temperature at PR=0.996 than the stagnation temperature is considered to be caused by the temperature rise that happens when stopping the flow, therefore, it is guessed that the stagnation temperature was possibly risen at this measurement. The steep increases of the temperature near walls has a clear dependence on the upstream pressure, in other words, the Reynolds number as shown in the center figure. The dependence is quite similar to the laminar boundary layer theoretical predictions [5] as shown in the right figure. The temperature drops shown in the center figure are normalized by the wall and core flow temperatures. Further detailed investigation needs improvements in the measuring setup and theory. It should be mentioned that the flowrate predicted by the combination of the 2D inviscous theory and the laminar boundary layer theory agrees very well with the measurements by a standard facility [6].



Figs.7 sRTA across the throat of toroidal throat critical nozzles. Lowest PR at right two figures.

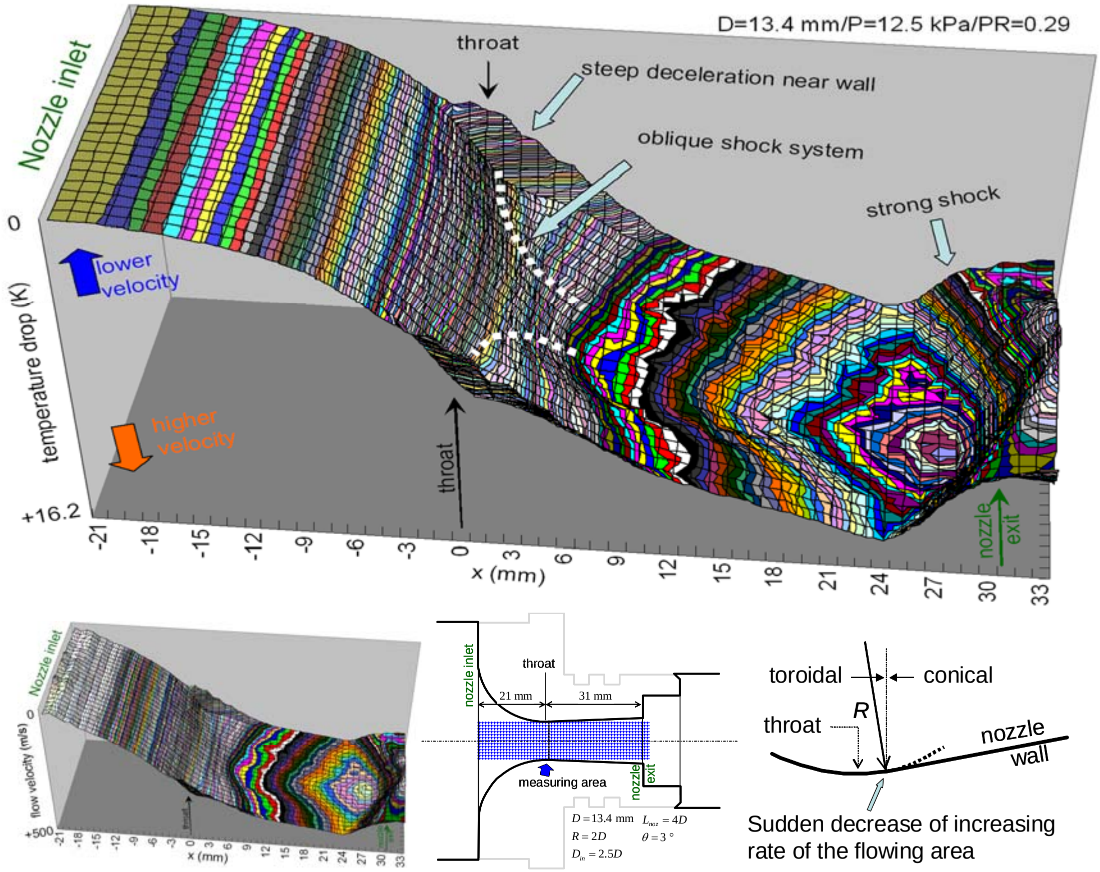
2.3 Measurement of the Flow Field in a Critical Nozzle

An example of the measured temperature distribution and the flow velocity field obtained by Eq. (4) are shown in Figs. 8 together with their measuring area and details of the nozzle geometry near the throat. For easy understandings, the vertical axes are reversed, thus the temperature drop or the flow velocity get larger towards the bottom in the figures.

In the subsonic region at the upstream of the throat, the flow is uniformly and gradually accelerated. Near the throat where the measuring area approaches to the nozzle wall, the steep temperature increases are observed.

Right after the throat, there are some structures projected from the walls. The structure is deceleration of flow and fixed on the nozzle geometry as shown later, thus it is considered to be an oblique compression wave originated by the discontinuity of the differential of the nozzle wall as shown in Figs. 8; the increasing rate of the flowing area behind the throat is suddenly decreased at the contact point, which will project a compression wave. Since the flow velocity is almost equal to the sound velocity right after the throat, the compression wave near the wall propagates at almost the right angle to the wall, but the farer from the wall it propagates, the smaller angle of the propagation it gets because the flow is accelerated towards the downstream.

At the exit of the nozzle, there is a sudden increase of the temperature, that is, a sudden decrease of the flow velocity, which is the strong shock. The strong shock is deformed by the oblique shock system to have its shape as a crinched pyramid.



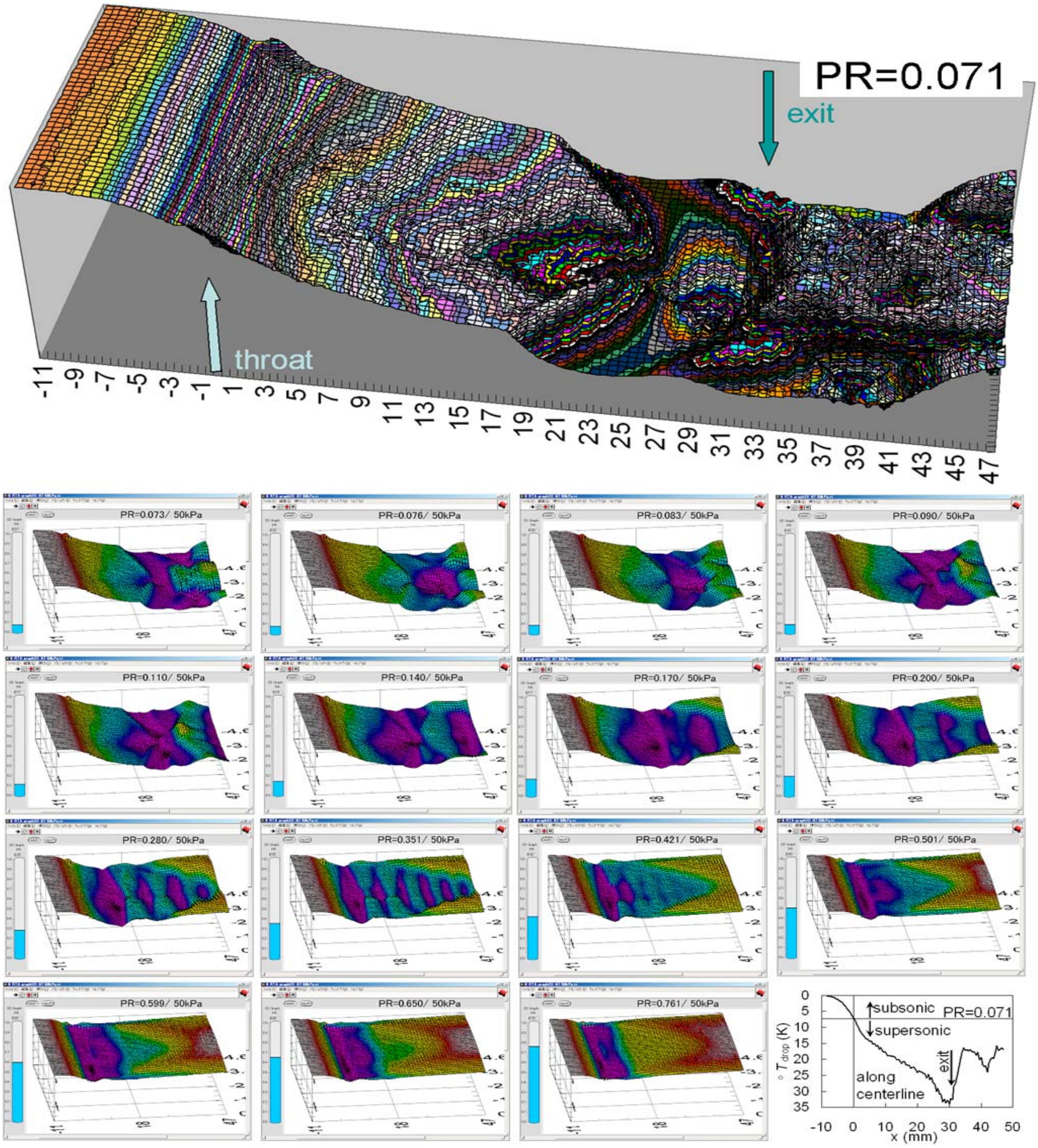
Figs. 8 An example of the measured recovery temperature distributions, flow velocity distribution, the measuring areas and the details of the nozzle geometry near the throat.

As easily understood in Figs. 8 and also in Eq. (4), the information possessed by the flow velocity and the temperature distributions are exactly the same, but the compression effect by the square root makes the temperature distribution easier to understand the details than the velocity distribution, therefore, in the followings, only the temperature distributions are discussed.

2.4 Dependence of Flow Field of Toroidal Throat Venturi Nozzle on PR

Figs. 9 show the dependence of the flow field in a toroidal throat Venturi nozzle on the pressure ratio (PR). The nozzle has the throat diameter of 13.4 mm and the upstream pressure was kept at 50 kPa. Similar to Figs. 8 where the upstream pressure was 12.5 kPa, there is a large deceleration at the exit of the nozzle, however, as the last figure shows, the flow in the diffuser is still supersonic. Behind the exit, the flow velocity is changed periodically that forms the shock cells. The PR where the subsonic flow takes place in the diffuser is 0.5; when PR<0.5 the whole flow in the diffuser is supersonic. It can be clearly observed in the figure the moving shock whose location depends on PR, the oblique shock system whose location is fixed on the nozzle geometry and their interactions. At certain PRs, the shock interactions deform the flow field drastically that is considered to have a relationship to PUP.

All the flow fields from PR=0.30 to 0.60 at the upstream pressure of 25 kPa are gathered in Fig. 10. The vertical axis is not reversed in this case. The figure clearly shows the drastic deformation at around PR=0.35 where the strong shock is damped down by the large acceleration formed by the oblique shock system.



Figs. 9 Dependence of the flow fields of a toroidal throat Venturi nozzle on pressure ratio (PR)

2.5 Flow Field of Toroidal Throat Venturi Nozzle with too small inlet diameter

In ISO 9300, the minimum inlet diameter D_{in} is limited at 2.5D. According to calibrations of toroidal throat Venturi nozzle with various D_{in} beyond this limitation using the primary standard in Japan, the discharge coefficient is drastically affected when D_{in} =1.8. Fig. 12 shows its flow field where contraction of the flow from the inlet and deceleration near the throat wall are observed, which is considered to be the reason of the flowrate defect caused by the too small D_{in}.

In Fig. 12, very clear interaction between the oblique shock system and the strong shock are observed. The oblique shock system deforms the strong shock into crinched pyramid and consequently forms the shock cells. In the presentation, the deformation of the strong shock by the oblique shock when the pressure ratio is changed will be clearly shown by a movie.

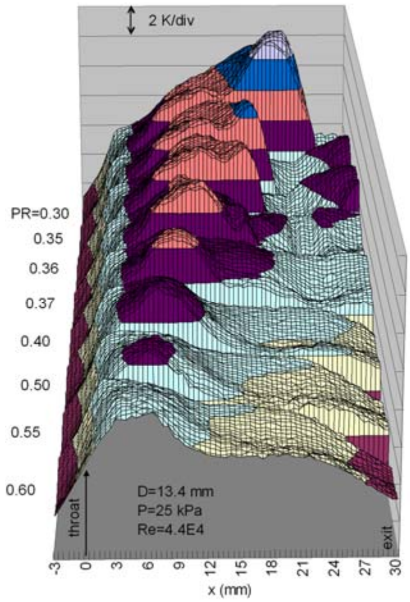


Fig. 10 Dependence of the flow fields of a toroidal throat Venturi nozzle on pressure ratio (PR)

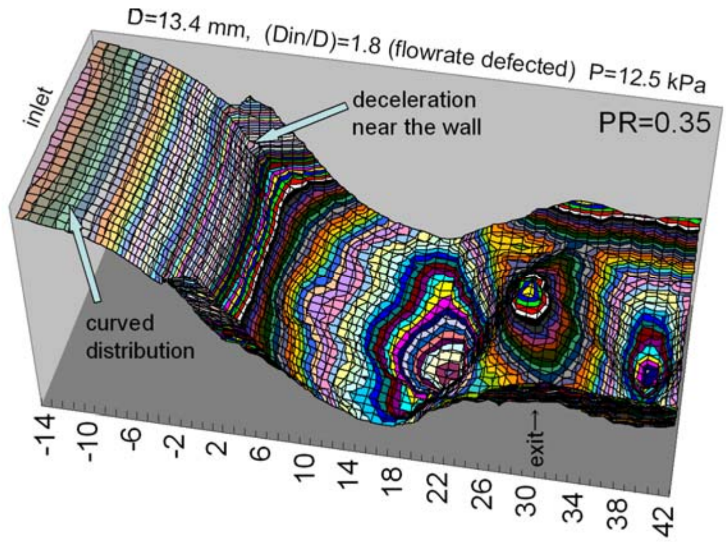


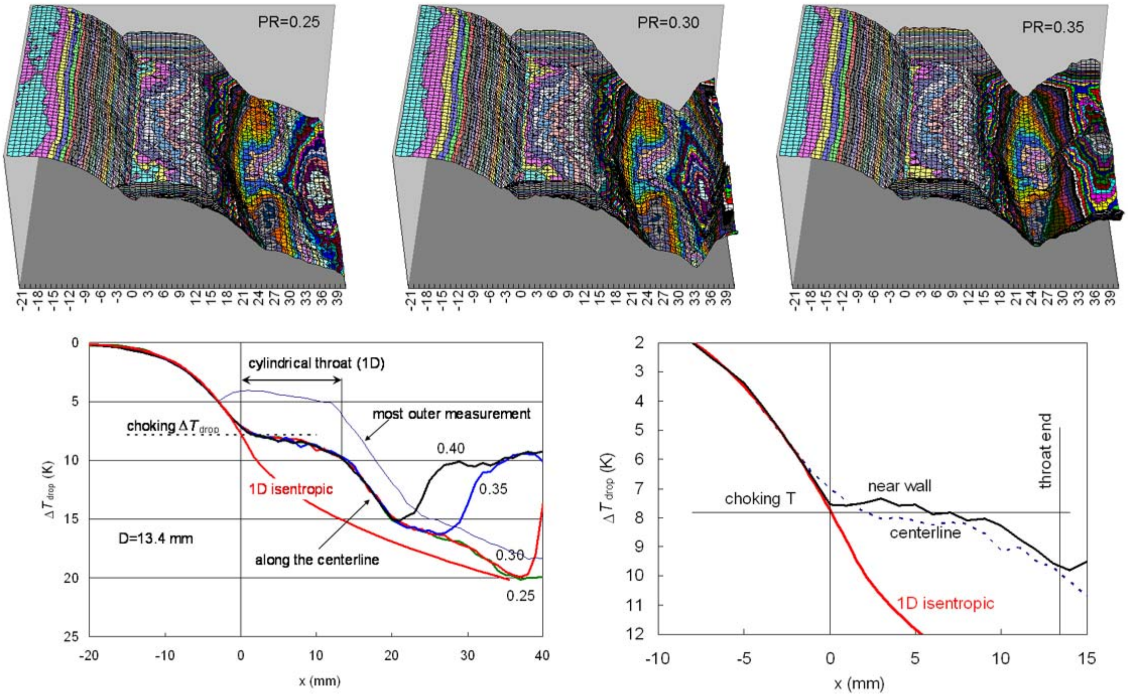
Fig. 11 Flow field with flowrate defect owing to too small D_{in} (D_{in}/D=1.8).

2.6 Dependence of the Flow Field of Cylindrical Throat Venturi Nozzle on PR

Figs. 12 show some examples of the dependence of the flow field in a cylindrical throat critical nozzle on the pressure ratio (PR). The length of the cylindrical part is 1*D* where *D*=13.4 mm at the upstream pressure of 50 kPa. It is interesting that when PR is changed, the strong shock jumps between the nodes formed by the oblique shock system like jumping on steppingstones. The bottom left figure shows the distributions along the centerline of the nozzle, which indicates that the choking point is slightly behind the throat inlet. The temperature distribution deviates from the 1D isentropic theory shown in red from slightly before the throat to have its choking point delayed. Theory suggests that the Fano flow should be generated in the critical flow in a cylindrical tube and the choking point should be at the exit of the cylinder, which conflicts with the measurement. The contradiction should be investigated by carrying out further measurements. As shown in the bottom right figure, the flow near the wall reached the sound velocity at the halfway in the throat. The decelerated flow towards the wall at the throat is completely different from the toroidal throat.

2.7 Dependence of the Flow Field in a quadrant Venturi Nozzle on PR

Figs. 13 show the dependence of the flow field in a quadrant critical nozzle with *D*=13.4 mm. on the pressure ratio (PR). In the figures, the largest extreme of the vertical axis depends on PR and is indicated at the right bottom in each figure. At low PR, the flow is accelerated quite largely right behind the throat but is suddenly decelerated to the subsonic flow. Since the flow velocity is very low in the right half of the measurement area, there must be fast flow existed outside the measurement area, that is, there must be a ring flow formed in the cylindrical tube behind the throat (see Figs. 2). It is interesting that a small moving shock is formed in the high speed area when PR=0.225 and 0.300. At the larger PRs, shock cells are formed in the center. As the PR is increased, the total length of the shock cells gets shorter and finally they disappear.



Figs. 12 Flow fields of a cylindrical throat critical nozzle. Centerline distributions in the bottom left figure. Comparison with near wall distribution in the bottom right figure.

Figs. 14 compare the flow field of the quadrant and Venturi nozzles. In the quadrant nozzle, the flow is more accelerated than the Venturi nozzles before the throat whereas the short and long Venturis have no such acceleration before the throat and agree very well with the theory. The premature acceleration in the quadrant nozzle is considered to be resulted in by the expansion waves projected from the corner at the throat. The short Venturi has only the toroidal portion without the conical diffuser thus the length of the diffuser is only 0.105 mm. Calibrations of these nozzles by the primary standard also explain these phenomenon, that is, the discharge coefficients of the quadrant nozzle is larger at low PR, however, those of the both Venturi are identical. It is mentioned that the quadrant nozzle has a dependence of the discharge coefficient on PR. The corner in the short Venturi also projects the expansion wave, however, since it is already in the supersonic flow, the influence of the expansion wave propagates only towards the downstream.

Conclusion

Principle of RTA (Recovery Temperature Anemometry) to measure transonic flow velocity field in a small space with very high space resolution is introduced. The measurement results by RTA on critical nozzles of various shapes reveal very interesting phenomena specific to their shapes and explain the phenomena very well.

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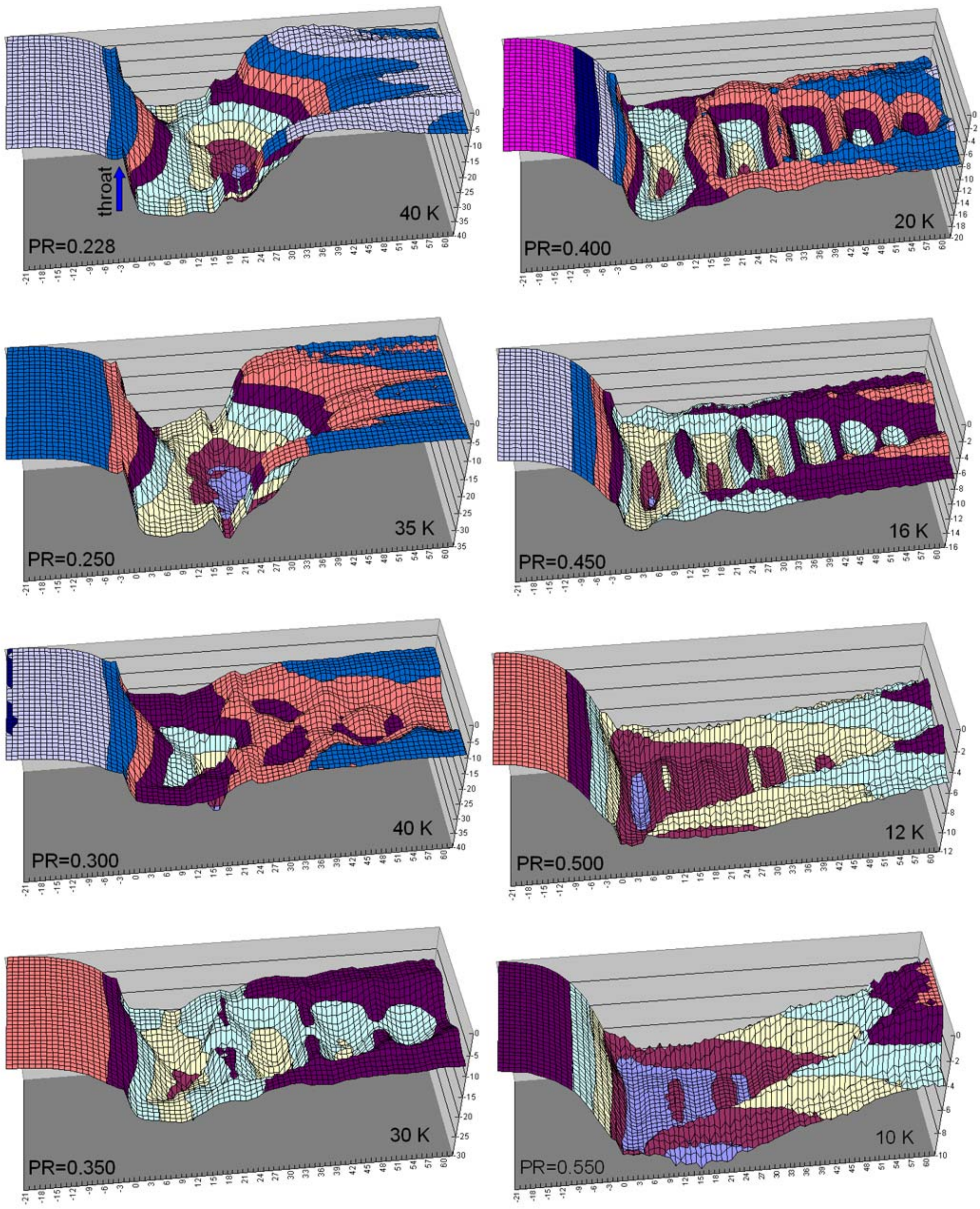
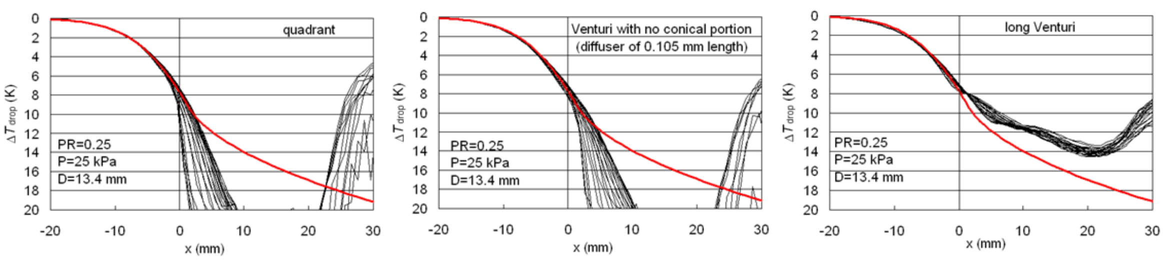


Figs. 13 Flow fields of a quadrant critical nozzle. D=13.4 mm at the pressure of 50 kPa.



Figs. 14 Comparison of the flow fields in the quadrant and Venturi nozzles. The red line by 1D isentropic theory. The short Venturi nozzle has only the toroidal portion without the conical diffuser. All D=13.4 mm at PR=0.30 at 25 kPa.