Fluid Dynamics Model for Liquid Flow Calibration Facility

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Abstract: Some designs for liquid flow calibration facility are studied by employing liquid dynamics theory. The flow equation in test pipe ,The overflow formula in water tower and the module of liduid film attached in the tank are put forward and calculated. This paper provides theoretical references for designing liquid flow calibration facility.

Keywords: Metrology; Liquid flow calibration facility; Fluid dynamics theory; Calculated model

1. Introduction

Liquid Flow Calibration Facility is designed for calibrate flowmeters to make it more accurate. In order to get better performance and decrease the uncertainty, the majority engineers tend to focus on the structure of the facility and selecting or manufacture of the key equipments, such as reference flowmeter, volumetric tank, electronic scale, etc. But put less concern about designing and calculation of the facility with the assistance of fluid dynamics aspect within their design and construction procedure. However, the uncertainty of facility including flow stability is no only related to the key equipments which mentioned earlier, but also to do with whether the principle of the fluid dynamic has been largely involved in designing and regular base running system.For instance, how to design and arrange pipe lines for the high water tank, in order to meet the requirement of maximum flow rate and velocity? What's the relationship between the structure of overflow channel for high water tank and fluctuation of flow?How much effect will elbow resistances (or other local resistances)make in flow performance of the facility?How to solve the problem of water film error in volumetric tank? In fact, those questions are all involved the principle of the fluid dynamic. This article is going to show couple of fluid dynamics calculation model, which based on the principle. The formula and calculate processes is attached in some of the model. In this article, it will also discuss the problems, which need to take careful consideration and principles should follow up, while doing the liquid flow calibration facility design, construction and further installation and usage.

2. Fluid Dynamics Model and it's Calculation

2.1 Flow Equation

Liquid flow calibration facility with high water tank method, is allso called the facility with constant water head method. Suppose the test pipeline of the facility is made up of n pipes in series connections, m local drags and z reference flowmeters of pipes in parallel, using the Bernoulli equation from the high water tank surface to the nozzle outlet of the diverter and going through some simplification, we can get

$$\Delta H =\left(\frac{d}{d^{'}}\right)^2 \frac{V^2}{2g} + d^2 \left[\lambda \sum_{i=1}^{n}\left(\frac{l_i}{d_i^3}\right) + \sum_{j=1}^{m}\left(\xi_j \frac{1}{d_j^2}\right)\right]\frac{V^2}{2g} + \sqrt{\frac{Q}{\sum_{i=1}^{z}\left(K_i \middle/ \sqrt{l_i}\right)}} \qquad (1)$$

where

$$K_i = \sqrt{\frac{g\pi^2 d_i^5}{8\lambda}}$$

In this equation, the flow velocity of the nozzle outlet of the diverter and the series pipelines, as well as the velocity of before (or after) the drag devices are converted to the flow velocity V of the test pipeline. According to Eq. (1), we can get the flow equation of the liquid flow calibration facility

$$Q = \frac{1}{\frac{d}{d'}\sqrt{1+d'^2\left[\lambda\sum_{i=1}^{n}\left(\frac{l_i}{d_i^3}\right)+\sum_{j=1}^{m}\left(\xi_j\frac{1}{d_j^2}\right)\right]}} \times A\sqrt{2g}\left[\Delta H - \sqrt{\frac{Q}{\sum_{i=1}^{z}\left(\frac{K_i}{\sqrt{l_i}}\right)}}\right]^{\frac{1}{2}} \qquad (2)$$

where
Q: flowrate,
   H: validated water height (height between the water level surface of the high water tank to the diverter's fishtail nozzle,
A, d: the pipe cross-section area and pipe diameter, respectively,
d': the equivalent diameter of the pipe diverter's nozzle,
λ: frictional resistance coefficient of the facility's pipeline,
l_i, d_i: the length and diameter of pipe I in the series connection,
ξ_j, d_i: local resistance coefficient No. j and the pipe's diameter before (or after) the local resistance object,
K_i, d_i: the modules of flow and diameters of the pipes in parallel.

If the paralleled pipes are not long, nor is the local resistance big, $\sqrt{Q\Big/\sum_{i=1}^{z}\left(\frac{K_i}{\sqrt{l_i}}\right)}$ in this formula can be neglected.

Suppose that there is a design parameter of the facility like this: $\Delta H = 20m$,parameters of the downcomer and main pipeline are   200,40m =0.02 ,test pipeline: 100,10m =0.02 ,and there are 4 elbows with 90°angles   =0.137   in the facility, and other local resistances are $\sum \xi$ =1~10,20,30,40 respectively. The calculated maximum flow rate is shown in table 1.

Table. 1 The calculated maximum flow rate in the conditions of various local resistances

Other Local Resistances $\sum \xi$	5	10	15	20	40
Maximum Flow RateQ(m^3/h)	165.8	138.5	118.7	107.3	81.7

From the above calculation, we can also get that the local coefficient of resistance in the pipes changes 0.1,when flow rate has a variation of 0.28%. Something worth consideration is that the

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closed diverter's coefficient of resistance might change due to structural and processing technique limits.

2.2 Overflow Weir's Length in the High Water Tank

With valves and other local resistance object's impact on the flow stability are all neglected, the facility's flow stability depends on the fluctuation of flow in the high water tank. Normally, in designing the liquid flow calibration facility using the high water tank method, a comparatively long overflow channel and a notch of over flow with the fill width & thin plate weir structure should be chosen, References 4 and 5 use numerical solution to discuss the relation between overflow and the surface level of the weir, but not the length of the overflow channel. we use the fill width & thin plate weir formula as the calculation formula[3] of weir's surface level as shown below

$$ h = \left[\frac{3}{2} \frac{q}{C_e b \sqrt{2g}} \right]^{\frac{2}{3}} - 0.0012 \qquad (3) $$

where
h : Height of water level above the weir
q : Overflow
b : The width of the weir
C_e : Flow coefficient (C_e can be set at 0.602)

Fig.1 shows the calculation results of Eq.(3). It can be seen that with certain overflow amount, the longer the overflow channel, the lower the water surface climbs, the smaller surface fluctuation that flow variation causes. The calculation allso shows that when overflow is set at $50m^3/h$, length of the weir is set at $25m$, $10m$ and $5m$ respectively, the surface above the weir climbs $4.6mm$, $8.5mm$ and $13.5mm$ accordingly. 1% change of overflow will cause 0.67% of surface fluctuation. Because the rising amount of the surface level is relevant to the amount of overflow, the more overflow, the higher liquid surface rises. Therefore, The flow of upper water supply should be adjusted according to different test flow to ensure that water tank overflow should stay low and as stable as possible. Moreover, the weir of the overflow tunnel should be designed long enough so as to reduce flow fluctuation considerably, like adopting labyrinth structure.
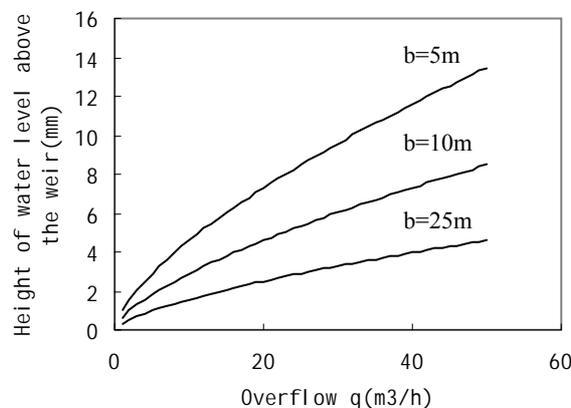

Fig. 1 The relationship between the height above the weir and overflow

Suppose the effective surface height of water tank is 20*m* and the surface fluctuation is 1.35*mm* (surface fluctuation is 0.67%), it will cause an instant flow rate fluctuation of 0.3% in the test pipeline. So we can see that the design of water tank overflow weir and control of water supply (in order to control overflow) plays a vital role in enhancing the facility's stability.

2.3 Design of Elbow Connection

When liquid passes through elbow connections("elbow" for short below), the inside and outside of the elbows show diffuse and contract effects alternatively which makes liquid separate from the wall and results in vortex region and secondary flow (symmetrical rotating flow with the pipe's axis as the center). The resistance of the elbow is mainly caused by the vortex region formed in the inside wall. The smaller the elbow pipe's radius-of-curvature, the bigger this diffuse and contract effect, the more violent the secondary flow it causes, the bigger the drag losses. As a result, the flow calibration facility "should use as few elbows as possible or no three dimensional elbows at all", which is one of the design principle of liquid flow calibration facility. However, it should also be noticed that in current flow calibration facilities, standard elbows which have the same pipe diameter *d* and radius-of-curvature *r* are mostly used. Standard elbow not only has big resistance but causes secondary flow which is not easy to eliminate.
The local resistance coefficient of the elbow can be calculated as shown below[1]

$$\xi = \left[0.131 + 0.163 \left(\frac{d}{r} \right)^{3.5} \right] \frac{\theta}{90} \tag{4}$$

The calculation demonstrates that when *d*/*r*　1.0 and *d*/*r*　0.4, the elbow's resistance is 0.294 and 0.137 respectively. It is recommended that we'd better use elbows of large radius-of-curvature to reduce resistance and increase flow stability.

2.4 'Water film' Error of the Volumetric Tank

'Water film' of volumetric tank refers to the liquid residual on the volumetric tank's wall that is not drained in time due to fluid viscosity, that's how it's called 'water film'. Let's first estimate the error caused by the 'water film'. Suppose a volumetric tank has a capacity of 10*m*³ and the volumetric tank's exterior height aside from the neck is 5.66*m* (the volumetric diameter is 1.5*m*), then the error would be 0.027% if the water film thickness is 0.1*mm* (percentage of water film against total water capacity). This error must be eliminated by extension of time.

Now let's build a mathematical model of water film on fluid dynamics theory. As shown in fig.2, a differential equation of equilibrium of forces of the water film is acquired according to Newton's law of viscosity

$$\frac{d^2 v}{dy^2} + \frac{\rho g}{\eta} = 0 \tag{5}$$

The time 'water film' forms, its boundary shearing stress is 0, so the equation's boundary condition is

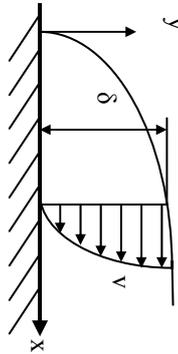$$y = 0, v = 0$$

$$y = \delta, \tau = \eta \frac{dv}{dy} = 0$$

Fig. 2 The fluid dynamics model of water film

In solving Eq. (5), we can get the velocity distribution of the water film and its average velocity

$$v = \frac{\rho g \delta^3}{\eta} \left[\left(\frac{y}{\delta} \right) - \frac{1}{2} \left(\frac{y}{\delta} \right)^2 \right]$$

$$v_m = \frac{1}{\delta} \int_0^\delta v \, dy = \frac{\rho g \delta^2}{3\eta} \tag{6}$$

where,
η : dynamic viscosity
δ : the thickness of water film.

Suppose the volumetric tank has a diameter *D*, therefore the water film flow is

$$q_m = v_m \pi D \delta = \frac{\pi \rho g D \delta^3}{3\eta} \tag{7}$$

Therefore the average time of draining water filmcan be derived

$$t = \frac{D \delta H}{q_m} = \frac{3\eta}{\pi \rho g \delta^2} H \tag{8}$$

where
H and *D* : the height and diameter of the tank respectively.

From Eq. (8), we can see the draining time of liquid is relevant to the tank's height rather than its diameter. Therefore, in designing the facility, the volumetric tank should be designed as "chunky" as possible in order to reduce the tank's height *H* and shorten the draining time of water film liquid as a result.

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Table 2 shows thickness, error and draining time of the water film. The calculated results show that in order to reduce the volumetric tank's water film error to 0.01%, the draining time must be prolonged at least 220.5 seconds.

Table. 2 Draining time and 'water film' error

Thickness of Water Film mm	0.01	0.05	0.1	0.2	0.4	0.8
Error %	0.003	0.013	0.027	0.053	0.107	0.213
Draining Time s	5512.3	220.5	55.1	13.8	3.4	0.86

3. Conclusion

This paper uses liquid dynamics theory to conduct discussion on several design problems of liquid flow calibration facility and reaches the following conclusions:

a. This paper provides flow calculation equation of liquid calibration facility. The calculated results show that if the facility's local resistance coefficient changes $\Delta\xi$ 0.1, it will result in flow error of 0.28%.

b. While neglecting the impact of elbows, valves and other local resistances on flow fluctuation, the tank's surface fluctuation will cause flow fluctuation. When liquid surface has a fluctuation of 0.67%, it will cause instant flow fluctuation of 0.3% in the test pipeline.

c. The facility should use as few elbows as possible and elbow of big radius-of-curvature is favored. Calculation shows that using elbow of big radius-of-curvature *r/d* =0.4 rather than standard elbow can reduce the resistance coefficient by half.

d. A fluid dynamic model of volumetric tank's 'water film' is built. The calculation results show that, with a standard volumetric tank whose capacity is $10m^3$, in order to reduce the volumetric tank's water film error to 0.01%, the draining time must be prolonged at least 220.5 seconds.

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