

NEURAL NETWORKS METHOD IN PRESSURE GAUGE MODELING

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Abstract – The mathematical model of an acoustic wave field in the measuring cavity of pressure calibrator is established. Two ways to the problem solution are posed. The system of two neural networks – RBF and perceptron – is applied to the working hole optimization and the wave field approximation. This new approach based on neural networks methodology seems to be adequate, effective and powerful: it is weakly sensitive to some entrance data perturbation, it gives trained neural networks for a set of problems solution, it is possible to use the same ideas in case of nonlinearity modeling.

Keywords: gauge, boundary optimization, neural networks

There is a need in building some mathematical model of precise apparatus for alternating pressure calibrating. Investigation, development, calibration of similar measuring systems with pressure gauge-converters and creation of corresponding models is the problem of great importance because of their wide applications in different fields of science and technology, e.g. pressure measuring in some pipelines, nozzles, channels and etc.

It is very essential to study some attendant problems:

- error estimation, moving measurement process to a high accuracy,
- taking into account some other permanent and transitory factors (heterogeneity and discontinuity, nonlinearity, time instability, etc.),
- application of new analysis and calculation method in calibration system parameter optimization.

Mathematical modeling of wave processes in measuring working cavity of precise apparatus for alternating pressure calibration is in close connection

- with the solution of boundary value problems for partial differential equations of elliptical type (often for domains with special symmetry),
- with incorrect problems,
- with optimization and variation methods of solution for mathematical physics problems,

- with state-of-the-art information technology – artificial intelligence, especially with neural network technology development and application progress.

We consider some standard calibration apparatus which measuring working cavity is rotationally symmetric, it has axis orthogonal symmetry plane as well. The working cavity is filled with a viscous liquid, for example transformer oil. There is a heavy pressure in the working hollow in order to suppress some cavitation processes. There is a harmonic motion source on the cylindrical part of the cavity boundary – piezoelectric vibration generator. It superimposes alternating pressure over the constant one. We believe that the acoustics wave field in the measuring cavity is time harmonic, axially symmetric and even with respect to the plane of symmetry. There are two precise pressure transducers on the axis of symmetry – standard and to be calibrated. The transducers are supposed to be a spot at the level of adequate accuracy. We ought to vary the part of border containing gauges so that a pressure value upon them should be maximal.

We take up linear approximation for acoustic equations set

$$\frac{\partial \vec{V}}{\partial t} = -\frac{1}{\eta} \nabla p + \nu \Delta \vec{V},$$

$$\frac{\partial p}{\partial t} = -\eta c^2 \nabla \cdot \vec{V},$$

where \vec{V} - velocity of medium part oscillations, p - pressure, η - density, ν - kinematic viscosity, c - sound velocity in medium. After \vec{V} elimination we will come to the hyperbolic (wave) equation for alternating pressure $p = p(\vec{x}, t)$

$$\frac{\partial^2 p}{\partial t^2} = c^2 \Delta p + \nu \frac{\partial}{\partial t} \Delta p.$$

We will study the solution of this equation in some compact axially symmetric domain $\Omega \subset \square^3$ - working measuring hole of calibration device. Special vibration conditions on the domain

boundary $\Gamma = \partial\Omega$ (boundary conditions) should be prescribed: impenetrability of working cavity walls, piezoelectric vibrator on the cylindrical part of the cavity boundary, possible heterogeneities (measuring windows and transducers).

Linearity of the problem gives us an opportunity of using Fourier transform (integral or series expansion), it leads to the consideration of solutions in form $p(\vec{x}, t) = p(\vec{x}, \omega) \exp(i\omega t)$. We have for Fourier-component

$$u = u(x, y, z) \equiv p(\vec{x}, \omega),$$

here $\vec{x} = (x, y, z)$, the following boundary value problem for Helmholtz equation with Neumann boundary condition

$$\begin{cases} (\Delta + k^2)u = 0, & k^2 = \omega^2/(c^2 + i\nu\omega), \\ \left. \frac{\partial u}{\partial n} \right|_{\Gamma} = f. \end{cases}$$

Here u is the pressure in Ω , Δ - Laplace operator, ω - cyclic frequency, $\partial\Omega = \Gamma = \Gamma_0 \cup \tilde{\Gamma}$ - domain boundary, $f|_{\Gamma_0} = f_0$, $f|_{\tilde{\Gamma}} = 0$, $\tilde{\Gamma}$ - the part of the boundary to be optimized, $J[u]$ - to be maximized functional that describes the wave field upon the gauge.

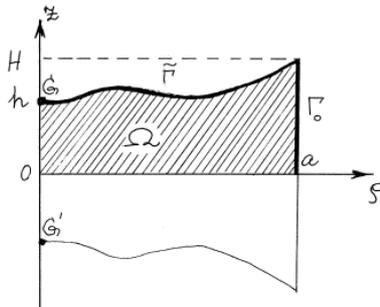


Fig.1. Region Ω

Symmetry of the problem leads to a solution of the following character

$$u(x, y, z) = u(\rho, z) = u(\rho, -z)$$

in the plane region $\Omega \subset \square^2$ which is set in new ρ, z variables, where $\rho = \sqrt{x^2 + y^2}$, as $\Omega: 0 \leq z \leq \varsigma(\rho), 0 \leq \rho \leq a$; boundary parts are given now as

$$\Gamma_0: \rho = a, 0 \leq z \leq H;$$

$$\tilde{\Gamma}: z = \varsigma(\rho), 0 \leq \rho \leq a,$$

$$\varsigma(a) = H, \varsigma(0) = h > 0, \varsigma'(0) = 0.$$

We get corresponding Neumann boundary condition in the form of

$$\frac{\partial u}{\partial \rho}(a, z) = f_0(z), 0 \leq z \leq H;$$

$$\frac{\partial u}{\partial \rho}(0, z) = 0, 0 \leq z \leq h;$$

$$\frac{\partial u}{\partial z}(\rho, \varsigma(\rho)) - \varsigma'(\rho) \frac{\partial u}{\partial \rho}(\rho, \varsigma(\rho)) = 0,$$

$$\frac{\partial u}{\partial z}(\rho, 0) = 0, 0 \leq \rho \leq a.$$

Among different ways for optimization conditions upon gauge G modeling we choose the following

$$J[u] = u(G) = u(0, h)$$

(e.g. functional

$$J[u] = \left(\int_{\sigma} |u|^p d\Gamma \right)^{1/p} + \left(\int_{\sigma} |\nabla u|^q d\Gamma \right)^{1/q}$$

gives another one, here $\sigma \subset \tilde{\Gamma}$ - part of the boundary occupied by the pressure gauge G).

So we get the variation problem $Max \rightarrow J[u]$ on the grounds of corresponding differential condition (boundary value problem). An original method of boundary Γ optimization, alternating pressure u computation was given in [1]. The boundary integral equation $\mu = K\mu + \gamma$ for interior limit $\mu = u|_{\tilde{\Gamma}}$ of the solution u on the unknown to be optimized part of the boundary $\tilde{\Gamma}$ was obtained, and as a result of extremum necessary condition for functional $J[u]$ some ordinary differential equation for part $\tilde{\Gamma}$ with coefficients that depend on μ was derived as well. That is why an iterative process for the pair $(\mu, \tilde{\Gamma})$ computation was developed.

We offer for consideration in the paper a new approach to the problem solution based on heterogeneous neural network projecting and training technology [3] that is to our mind an efficient and adequate one. We are going to approximate the solution $u(\rho, z)$ via RBF (radial basis function) neural networks, in other words we represent it in the form $\sum_{i=1}^N c_i \varphi(a_i \| \vec{x} - \vec{x}_i \|)$, where $\varphi \in C^\infty$ - so-called basis function: for example,

$\varphi(r) = \exp(-r^2)$ or $\varphi(r) = (1 + r^2)^{-1}$ and some others, $\|\cdot\|$ denotes as usually a norm (distance), (c_i, a_i, \bar{x}_i) - some parameters to be determined. In our case we have got

$$u(\rho, z) = \sum_{i=1}^N c_i e^{-a_i[(\rho-\rho_i)^2 + (z-z_i)^2]},$$

here $\{(\rho_i, z_i)\}_{i=1}^N$ - RBF neural network center set.

We ought to stress that in the same manner one could investigate more complicated (asymmetric, nonsmooth, uneven, nonlinear, incorrect, etc.) problem. We will find parameter values from corresponding functional $I[u]$ minimization (boundary value problem for u is replaced with variation problem for I): in our case we set

$$I[u] = \delta_1 \cdot \int_{\Omega} |\Delta u + k^2 u|^2 d\Omega + \delta_2 \cdot \int_{\tilde{\Gamma}} \left| \frac{\partial u}{\partial z} - \zeta' \frac{\partial u}{\partial \rho} \right|^2 d\Gamma + \delta_3 \cdot \int_{\Gamma_0} \left| \frac{\partial u}{\partial \rho} - f_0 \right|^2 d\Gamma,$$

$\delta_i > 0$ - penalty coefficients.

We give neural network approximation for unknown boundary part $\tilde{\Gamma}$ by means of perceptron with one hidden layer

$$\zeta(\rho) = \sum_{i=1}^n b_i \gamma(d_i \rho - e_i) + b_0,$$

where nonlinear so-called activation function γ is given as follows $\gamma(s) = s/(1 + |s|)$, another form of activation function γ is possible as well, e.g. $\gamma(s) = th(s)$; coefficients b_0, b_i, d_i, e_i are some perceptron parameters, n - a number of neuron units.

After problem discretization we get the functional analogues in the form of

$$I[u] = \delta_1 \cdot \sum_{j=1}^M |\Delta u + k^2 u|^2(\rho_j, z_j) + \delta_2 \cdot \sum_{j=1}^{\tilde{M}} \left| \frac{\partial u}{\partial z} - \zeta' \frac{\partial u}{\partial \rho} \right|^2(\rho_j, z_j) + \delta_3 \cdot \sum_{j_0=1}^{M_0} \left| \frac{\partial u}{\partial \rho} - f_0 \right|^2(\rho_{j_0}, z_{j_0}),$$

here three test point (point of reference) ensembles are used:

$\{(\rho_j, z_j)\}_{j=1}^M$ - inside region Ω , $\{(\rho_j, z_j)\}_{j=1}^{\tilde{M}}$ - on the boundary part $\tilde{\Gamma}$, $\{(\rho_{j_0}, z_{j_0})\}_{j_0=1}^{M_0}$ - on the boundary part Γ_0 ; $\delta_i > 0, i = 1, 2, 3$, - penalty coefficients; let I_0 denotes the first sum in I ;

$$J[u] = \delta_4 \cdot |u(0, h)|^{-2} + \delta_5 \cdot (\zeta(a) - H)^2 + \delta_6 \cdot \sum_{i=1}^{m_G} (\zeta(\rho_i) - h)^2,$$

here $\delta_i > 0, i = 4, 5, 6$, - penalty coefficients; some additional function $\zeta(\rho)$ requirements are described by the third term of functional via $\{(\rho_i, \zeta(\rho_i))\}_{i=1}^{m_G}$ - a set of points upon the gauge G .

We will organize neural network training process (adjusting its parameters according to the functional minimization) in the following manner:

1. The initial state of the boundary part $\tilde{\Gamma}$ is given as an input of one hidden layer perceptron with some initially prescribed coefficients (weights) so that it is close to the horizontal line.
2. Initial RBF neural network centers are variables distributed uniformly random or regularly (according to some special rule) in an initial domain - rectangle $\Pi \supset \Omega$ (or some rectangle neighbourhood). Coefficients c_i and a_i are considered as independent uniformly random parameters.
3. We preset a set of boundary points of reference with some definite given distance between them, distribute interior point of reference set uniformly random within the domain and calculate the functional I .
4. We run a functional I minimization process on the basis of some iteration method changing after a number of steps the RBF neural network coefficients (weights). The random search method was successful in our investigation.
5. We change perceptron coefficients and corresponding boundary part $\tilde{\Gamma}$ minimizing functional J .
6. The inside domain random points of reference are generated anew taking into consideration the boundary variation. We avoid sticking together of RBF neural network centers in this way. Such a test point set regeneration

<p>makes computation process more stable, decreases the number of test points and gives as an opportunity to estimate errors via statistical methods.</p> <p>7. We repeat steps 5 and 6 of iteration process a given number of times or continue calculations up to the level of adequate accuracy: the functional I is less than a prescribed positive number ε and the functional J variation is small enough.</p>	<table border="0"> <tr><td>500</td><td>100</td><td>2315.53</td><td>1692.79</td><td>0.21</td><td>4.69</td></tr> <tr><td>20</td><td>125</td><td>41.49</td><td>81.35</td><td>1.03</td><td>0.96</td></tr> <tr><td>50</td><td>125</td><td>153.38</td><td>205.73</td><td>33.91</td><td>0.03</td></tr> <tr><td>75</td><td>125</td><td>177.42</td><td>195.40</td><td>1.77</td><td>0.56</td></tr> <tr><td>100</td><td>125</td><td>326.58</td><td>265.53</td><td>5.67</td><td>0.17</td></tr> <tr><td>150</td><td>125</td><td>432.16</td><td>401.00</td><td>0.43</td><td>2.33</td></tr> <tr><td>200</td><td>125</td><td>562.49</td><td>463.16</td><td>0.55</td><td>1.83</td></tr> <tr><td>250</td><td>125</td><td>842.84</td><td>722.39</td><td>0.35</td><td>2.87</td></tr> <tr><td>500</td><td>125</td><td>1907.77</td><td>1393.30</td><td>0.21</td><td>4.79</td></tr> </table>	500	100	2315.53	1692.79	0.21	4.69	20	125	41.49	81.35	1.03	0.96	50	125	153.38	205.73	33.91	0.03	75	125	177.42	195.40	1.77	0.56	100	125	326.58	265.53	5.67	0.17	150	125	432.16	401.00	0.43	2.33	200	125	562.49	463.16	0.55	1.83	250	125	842.84	722.39	0.35	2.87	500	125	1907.77	1393.30	0.21	4.79
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We give here some computation results for the following parameter values:

$$M = 1536, \tilde{M} = M_0 = 100, m_G = 8;$$

$$\delta_1 = 0.5, \delta_i = 1, i = 2, \dots, 6$$

Tab. 1. Iteration process

N RBF	Iter. Num.	I_0	I	J	$u(G)$						
20	25	191.81	198.36	0.73	1.32	20	175	37.72	70.74	0.99	1.01
50	25	724.75	565.92	5.38	0.18	50	175	130.35	189.83	38.10	0.03
75	25	487.72	389.13	1.32	0.76	75	175	161.33	176.81	1.74	0.58
100	25	1270.76	993.78	1.47	0.65	100	175	262.92	201.08	7.43	0.13
150	25	1238.52	835.28	0.32	2.99	150	175	368.27	351.15	0.47	2.14
200	25	2262.29	1465.49	0.40	2.48	200	175	499.23	433.12	0.60	1.66
250	25	2625.67	1804.74	0.20	5.17	250	175	630.19	568.22	0.41	2.43
500	25	6527.36	4506.75	0.28	3.62	500	175	1396.28	1038.91	0.21	4.66
20	50	98.36	132.20	1.01	0.98	20	200	35.14	67.37	0.93	1.08
50	50	346.51	339.85	15.63	0.07	50	200	135.27	186.88	30.98	0.03
75	50	289.64	268.91	1.66	0.59	75	200	155.21	170.95	1.85	0.53
100	50	724.88	590.07	2.50	0.38	100	200	236.32	182.35	8.02	0.12
150	50	770.90	581.77	0.33	3.00	150	200	361.87	345.23	0.50	2.01
200	50	1045.69	793.12	0.47	2.13	200	200	468.65	401.26	0.66	1.52
250	50	1619.72	1239.52	0.27	3.68	250	200	625.47	568.22	0.45	2.26
500	50	3524.33	2638.27	0.21	4.55	500	200	1293.55	981.87	0.24	4.23
20	75	69.66	110.55	1.02	0.97	20	225	34.05	66.18	0.88	1.13
50	75	283.83	289.17	22.68	0.04	50	225	119.45	174.51	8.02	0.12
75	75	235.28	238.82	1.84	0.54	75	225	144.09	161.82	1.88	0.52
100	75	567.86	434.84	3.76	0.27	100	225	168.03	144.48	8.98	0.12
150	75	560.15	470.17	0.39	2.54	150	225	302.12	306.17	0.51	1.98
200	75	767.86	609.02	0.49	2.01	200	225	471.71	400.82	0.69	1.42
250	75	1240.01	988.43	0.31	3.20	250	225	627.88	539.70	0.44	2.23
500	75	2866.30	2073.30	0.21	4.80	500	225	1248.48	910.62	0.22	4.61
20	100	52.02	94.08	1.03	0.97	20	250	30.38	61.86	0.86	1.17
50	100	195.28	236.48	26.65	0.04	50	250	105.55	164.97	8.98	0.11
75	100	209.88	218.61	1.83	0.55	75	250	130.15	151.35	1.86	0.54
100	100	427.84	324.27	5.00	0.20	100	250	193.09	160.26	8.27	0.12
150	100	521.72	444.02	0.40	2.46	150	250	291.76	302.14	0.52	1.92
200	100	604.48	492.88	0.53	1.87	200	250	413.17	360.44	0.77	1.29
250	100	1025.88	847.99	0.31	3.30	250	250	570.31	512.84	0.45	2.22

500	250	1138.31	829.12	0.22	4.57	500	400	803.88	595.68	0.17	5.74
20	275	25.82	59.81	0.85	1.17	20	425	19.56	54.07	0.73	1.37
50	275	87.57	152.52	9.97	0.10	50	425	72.53	128.39	12.16	0.08
75	275	123.40	143.81	1.92	0.53	75	425	100.03	114.87	1.67	0.60
100	275	150.27	133.94	7.96	0.13	100	425	124.59	112.90	4.81	0.21
150	275	290.64	285.81	0.56	1.80	150	425	183.83	228.14	0.62	1.61
200	275	386.74	336.63	0.82	1.21	200	425	250.49	242.08	1.16	0.84
250	275	565.27	492.70	0.46	2.21	250	425	384.53	370.10	0.55	1.85
500	275	970.76	733.77	0.23	4.46	500	425	724.31	552.78	0.18	5.64
20	300	27.12	57.40	0.84	1.20	20	450	21.38	54.96	0.70	1.41
50	300	90.53	149.37	10.56	0.09	50	450	70.46	124.63	11.88	0.08
75	300	111.17	133.93	1.99	0.51	75	450	91.26	110.08	1.61	0.62
100	300	133.36	125.34	8.16	0.12	100	450	102.97	101.90	4.71	0.21
150	300	244.81	266.51	0.56	1.78	150	450	191.04	223.02	0.64	1.57
200	300	353.03	302.92	0.84	1.17	200	450	271.41	245.98	1.27	0.79
250	300	469.21	426.89	0.47	2.16	250	450	396.11	372.83	0.56	1.77
500	300	891.47	665.67	0.22	4.56	500	450	754.56	544.61	0.10	10.19
20	325	26.45	57.78	0.81	1.23						
50	325	93.57	150.00	10.74	0.09						
75	325	113.29	128.80	1.82	0.55						
100	325	133.72	123.11	7.16	0.14						
150	325	234.02	257.81	0.57	1.74						
200	325	321.71	287.88	0.92	1.09						
250	325	485.15	431.20	0.52	1.91						
500	325	890.02	661.01	0.21	4.84						
20	350	23.84	56.00	0.78	1.28						
50	350	81.96	140.49	11.18	0.09						
75	350	109.88	125.20	1.80	0.56						
100	350	122.40	118.14	6.21	0.17						
150	350	202.63	241.74	0.58	1.72						
200	350	312.25	282.65	0.96	1.02						
250	350	462.86	414.67	0.54	1.85						
500	350	914.00	661.61	0.19	5.28						
20	375	21.92	56.45	0.76	1.31						
50	375	91.22	142.36	11.60	0.09						
75	375	97.75	121.85	1.70	0.59						
100	375	151.66	128.45	5.46	0.19						
150	375	215.68	247.07	0.59	1.69						
200	375	304.88	276.50	1.06	0.94						
250	375	441.19	402.65	0.55	1.80						
500	375	902.78	654.35	0.18	5.56						
20	400	21.55	53.71	0.74	1.36						
50	400	72.04	131.78	11.64	0.09						
75	400	94.17	114.99	1.72	0.60						
100	400	116.30	109.77	5.09	0.19						
150	400	203.40	233.63	0.58	1.71						
200	400	291.10	265.74	1.13	0.89						
250	400	430.64	401.04	0.54	1.88						

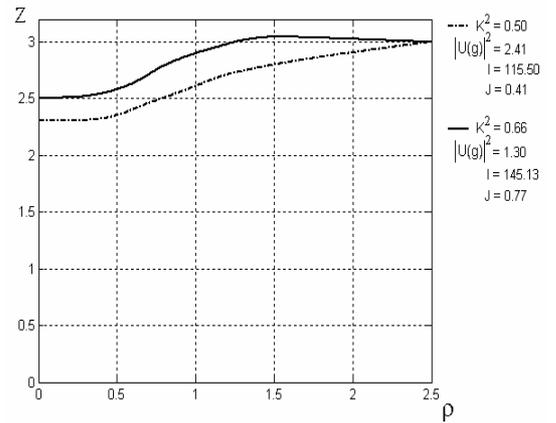


Fig. 2. Optimized region

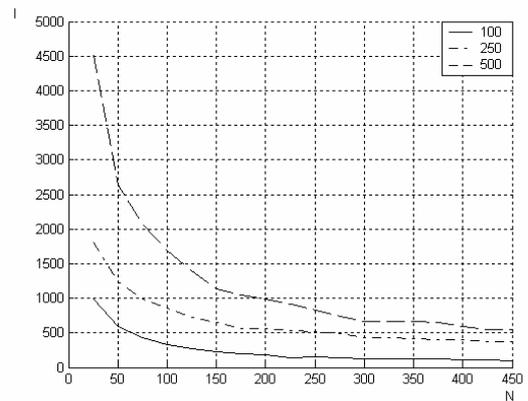


Fig. 3. I convergence property

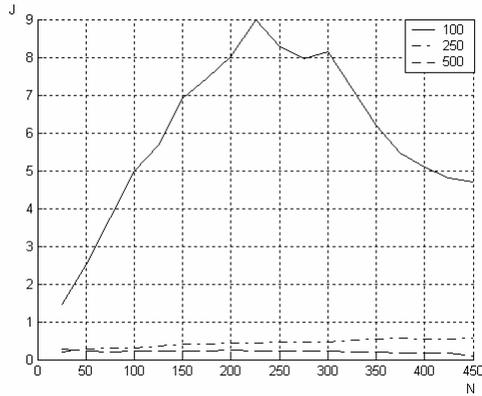


Fig. 4. J convergence property

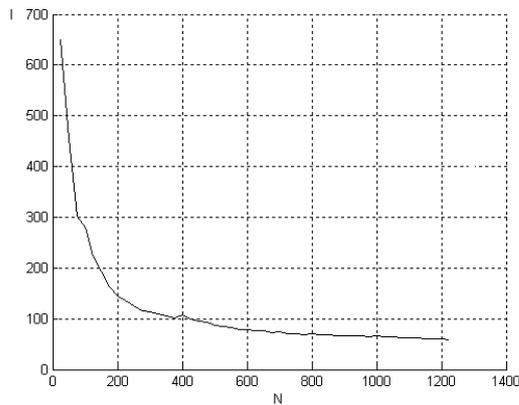


Fig. 5. I iteration process (N RBF = 75)

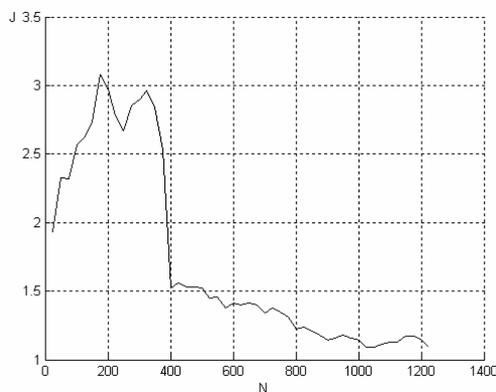


Fig. 6. J iteration process (N RBF = 75)

Our new approach to the described problem solution is based on some ideas and methods offered in [2]. The neural networks point of view applied by us for the problem solution has got some essential advantages over the approach to the question suggested in [1] in the following aspects:

- interference immunity of the neural networks method – the result (solution) poorly varies at small changes of the entrance data (boundary conditions, oil stuff properties, time instability);
- there is no need at the solution of a set of problems to retrain neural networks each time anew, it is enough to use already trained network for some sufficiently close entrance data and (if it is necessary) to finish teaching a little up to the level of required accuracy;
- nonlinear problem solution by using neural networks is practically the same as a linear one – it allows us to consider a line of new problem statements (some of them are non-classical ones) that are inaccessible to study for the approach developed in [1].

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