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METODS OF RATING COMPETITORS FOR QUALITY AWARDS: TENTATIVE COMPARATIVE ANALYSIS

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Abstract – A comparison of three most frequently used methods for quality estimation of complex objects is offered in the paper, namely Overall Integral Index, Analytical Hierarchy Process and Consensus Relation. Such estimation is required, for example, in the European Excellence Model by the European Foundation for Quality Management (EFQM) when making decision on the European Quality Award. Examples of using the estimation methods of five enterprises are given taking into account the Model criteria. Certain advantages of the Consensus Relation method are demonstrated.

Keywords: Quality estimation, Overall Integral Index, Analytical Hierarchy Process, Consensus Relation.

1. INTRODUCTION

The requirements for quality of production nowadays are very high. The high level of quality of an object gives a guarantee to the consumer that the goods are safe and effective in use. Quality Award is the good stimulus to improve production quality at various levels: of the region, of the country and of the whole continent. Then a problem arises of revealing the best enterprises among participants realizing maintenance of quality in all aspects of their activity. In this situation, different methods for estimation and analysis of complex objects properties relieve. Frequently, these methods are essentially of kind of decision making. Many of them are offered as a business software [1]. Among the methods one can find out such techniques as Analytical Hierarchy Process [2, 3]; Overall Integral Index [4]; determination of consensus relation among rankings of estimated alternatives, shortly, Consensus Relation [5, 6]; Point Allocation [1], methods based on several different aggregation functions (linear opinion pool, logarithmic opinion pool, the conjugate method) [7], etc. Thus, the choice of suitable method for estimation of participants by many criteria is not a simple problem for organizers of a particular quality award competition.

A purpose of the paper is to make some tentative comparative analysis of methods suitable for a quality estimation conducted with aim of quality award assignment. The analysis results could help to develop some recommendations on a choice of an appropriate estimation method of the competition participants.

2. WHY THE EFQM EXCELLENCE MODEL NEEDS SOME OTHER METHODS FOR QUALITY ESTIMATION

Let's consider the model of the European Quality Award being accepted now [8].It is based on nine criteria which allow to characterize the achievements of an enterprise in the field of quality. The basis of the model is logic RADAR. The elements of RADAR are Results, Approach, Deployment, Assessment and Review. The elements of Approach, Deployment, Assessment and Review are used when assessing "Enabler" criterion, and the Results element is used when assessing "Results" criterion. The criterion "Enablers" consists of:

1) Leadership;
2) Processes;
3) People;
4) Policy & Strategy;
5) Partnerships & Resources.

The criterion "Results" consists of:

1) Customer results;
2) Key performance results;
3) People results;
4) Society results.

The estimation of the enterprises by criteria of the premium is carried out by the groups consisting of persons, accepting decisions, i.e. experts. Final estimation Q_j of each j participant is formed. It's based on operation of averaging indexes's values of enterprises activity obtained by the formula:

$$Q_j = \sum_{i=1}^{n} m_i A_i , \qquad (1)$$

where A_i is a mean estimation of criterion in %; m_i is weight of criterion.

The estimations are mainly obtained in the ordinal scale. At the same time, it is well-known fact that the addition is inadmissible operation for an ordinal scale values [9]. Ignoring this fact can result in hard mistakes in the work of expert group. Hence it is expedient to choose such method of an estimation which gives the most reliable foundation for quality awards.

3. THREE SPECIFIC TECHNIQUES

The problem of choosing a suitable method for estimation in its turn is rather difficult because of a great variety of similar methods. Besides, as experts' preferences are subjective, as a rule, one can find it difficult to understand whether the decision found by a certain method is optimum and/or correct. It is obvious that each of the methods has its merits and demerits and the task is to reduce the effect of these demerits on the process of estimating of complex objects quality to the minimum influence while the advantages should be involved to the maximum.

Further in the paper the attempt to find the method which would allow awarding the premium in the field of quality with the greater confidence is discussed. In this relation, only three methods for the analysis are chosen from all the variety of methods: Overall Integral Index (OII); Analytical Hierarchy Process (AHP); Consensus Relation (CR).

The choice of these particular methods is due to their wide application. Besides it is interesting to compare very different but at the same time similar methods [1]. Similar methods are AHP and OII, method CR differs from both of them to a great extend by its approach.

Let us carry out the preliminary comparative analysis of three methods using a set of their characteristics taken from [1]. These characteristics are: scaling; preference elicitation; weighting of particular attributes; synthesis of resulting estimation; and description structure of object in question. Results of this analysis are shown in Table 1.

TABLE 1. The comparative characteristic of methods

Characteristic of method	AHP	Overall Integral Index	Consensus relation
Scaling	ratio; priorities	order, priorities	order
Preference elicitation	pairwise comparison	pairwise comparison	distance between rankings
Weighting of particular attributes	normalized ratio via eigenvalues	normalized relative attributes	no
Synthesis of resulting estimation	additive, eigenvectors	additive, multiplicative	linear order relation as consensus
Description structure of object in question	hierarchic	hierarchic	is not important

Let us consider the basic features of each of compared methods in detail.

3.1. Analytical Hierarchy Process (AHP)

It is described in details in [2] and is rather widely used, see, for example [1, 3]. The method is a regular procedure for hierarchical representation of elements (a tree of criteria), determining features of any complex object. Paired comparison is the way to define importance factors. The result of comparison is estimated in scores. On the basis of such comparisons factors of importance of criteria, estimations of alternatives are calculated and general estimation as the weighed sum of criteria estimations is determined.

To establish relative importance of elements in hierarchy the scale of relations is used. Value 1 is given to the relation of the objects which have identical importance when compared in pairs. Value 9 is given to the relation of the objects when one object is superior to the other. Values 3, 5, 7 are used in interim situations when compared in pairs. In case when the compromise is necessary values 2, 4, 6, 8 can be used. The given scale allows a Decision Maker (DM) to bring some numbers to conformity with the degrees of preference of one object under comparison to another.

The method provides estimation of rejection degree from consistency. When such deviations exceed the established limits, DM should recheck them in a matrix of pair comparisons.

3.2. Overall Integral Index

It represents construction in space of criteria of the scalar function to refer to each of the objects estimation of its quality [4, 7]. Such function, if it is coordinated with the initial (individual) criteria is the generalized integrated criterion of objects estimation. Convolution of individual criteria not only enables to allocate the best objects, but also to specify a place of each object in their aggregation. The generalized criterion looks like this:

$$W = \sum_{i=1}^{n} m_i x_i \ , \qquad (2)$$

where m_i is weight of each index; x_i is individual index.

One can see, the formula (2) coincides with the formula (1) by which the estimation of the competitor of the premium on quality is performed. The method also uses a criteria tree in which complex indexes are divided into individual ones. The example of a criteria tree for the model of the European Quality Award is shown in Fig 1. In this figure m_{11} and m_{12} are complex indexes "Enablers" and "Results" respectively, the others values of m with three-value indexes are weights of individual indexes, and $x_1,\ldots, x_9$ correspond to nine criteria described in Section 2.

In this method a specialized predictional expert estimations processing obtained by systematized poll of highly qualified experts is carried out. A numerical estimation on each of criteria is given to each of the alternatives by experts. Each of the criteria gets the quantitative weights characterizing their relative importance. Weights are multiplied by criterion's estimation and the resulting numbers are summarized: in this way the value of an alternative is determined. Then the alternative with the greatest index of value is to be chosen.

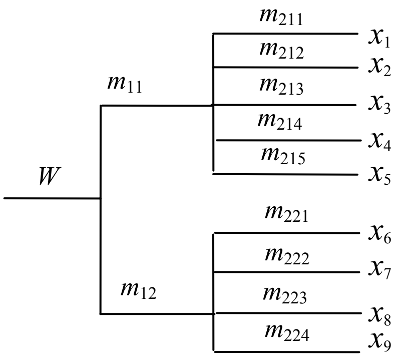

Fig. 1. Tree of criteria in OII.

While making a criteria tree, coordination of groups of indexes indicated by the experts and coordination of indexes with each other is carried out. If the conditions of consistency are broken the experts are to reconsider their judgments.

3.3. Consensus Relation

Let some n enterprises will be characterized by m criterion. Let a set $A = \{a_1, a_2, ..., a_n\}$, $|A| = n$, be a set of enterprises. By every criterion, the enterprises can be ranked in order of preference. We have the relation set $\mathrm{A} = \{\alpha_1,...,\alpha_m\}$, where $|\mathrm{A}| = m$. Every *ranking* (preference relation $\succsim$) $\alpha = \{a_1 \succ a_2 \succ ...\sim a_s \sim a_t \succ ...\sim a_n\}$ includes $\succ$, a strict preference relation π, and $\sim$, an indifference relation ν, so that $\alpha = \pi \cup \nu$. The relation π is complete, transitive, irreflexive, and antisymmetric, and the relation ν is reflexive and symmetric. Such the relation α is known to be called *preorder*. How to obtain preorder relations describing many properties of objects is discussed in greater detail in [9]. The relation set A can be titled a *preference profile* for the given m properties.

A single preference relation can be determined that would give an integrative characterization of the enterprises. Let a space Π be a set of all $n!$ strict (linear) order relations $\succ$ on A. Each linear order corresponds to one of permutations of first n natural numbers $\mathbf{N_n}$. We will consider a permutation $\beta \in \Pi$ of the enterprises $a_1, ..., a_n$ to represent the preference profile A and will call it *consensus ranking*. It is desirable that, in some sense, β would be nearest to the every of rankings $\alpha_1, ..., \alpha_m$.

It is clear that the problem described above is very similar to the problem of voting or group decision where A is a set of alternatives or candidates which are ranked by group of m individuals [5]. For a profile, finding the consensus ranking is possible due to measure of distance between pairs of rankings firstly introduced by Kemeny [10] and discussed in many papers, see, for instance, in [5, 11]. The ranking α can be represented by an $(n\times n)$ *relation matrix* $[a_{ij}]$ whose rows and columns are labeled by the enterprises a and

$$a_{ij} = \begin{cases} 1 & \text{if } a_i \succ a_j \\ 0 & \text{if } a_i \sim a_j \\ -1 & \text{if } a_i \prec a_j \end{cases}. \tag{3}$$

Properties of the relation matrix are connected to those of the corresponding relation. The Kemeny distance function $d(\alpha_k, \alpha_l)$ between two rankings α_k and α_l is defined by formula

$$d(\alpha_k,\alpha_l) = \frac{1}{2}\sum_{i,j=1}^{n} | a_{ij}^k - a_{ij}^l | =$$

$$= \sum_{i<j} | a_{ij}^k - a_{ij}^l | . \tag{4}$$

It can be then defined a 'distance' between a ranking α and a profile A as follows:

$$D(\alpha, \mathrm{A}) = \sum_{k=1}^{m} d(\alpha, \alpha_k)$$

$$= \sum_{i<j}\sum_{k=1}^{m} | a_{ij}^k - a_{ij} | = \sum_{i<j}\sum_{k=1}^{m} d_{ij}^k . \tag{5}$$

Like in the case of the relation matrix, we can define an $(n\times n)$ *profile matrix* $P = |p_{ij}|$ which can represent in compact form all the rankings. In the profile matrix

$$p_{ij} = \sum_{k=1}^{m} d_{ij}^k, \quad i, j = 1,...,n, \tag{6}$$

$$\text{where } d_{ij}^k = \begin{cases} 0 & \text{if } a_i^k \succ a_j^k \\ 1 & \text{if } a_i^k \sim a_j^k \\ 2 & \text{if } a_i^k \prec a_j^k \end{cases}. \tag{7}$$

It is easy to obtain (7) from (5) assuming $a_{ij} = 1$.

In sense of the measure (5), the consensus linear ranking β is the closest relation (so called *median*) to the preference profile, i.e.

$$\beta = \arg\min_{\alpha} \sum_{k=1}^{m} d(\alpha, \alpha_k) . \tag{8}$$

Every permutation of states a corresponds to transposition of the profile matrix rows and columns. Hence, the problem (8) means the *determination of such a transposition of distance matrix rows and* *columns that the sum of elements of the upper triangle submatrix is minimal*.

Unfortunately, the problem may have more than one optimal solution. In the case of several solutions available, one can attract additional considerations to choose among them.

4. EXAMPLES OF ESTIMATIONS BY THE THREE METHODS

It is useful to compare methods being analyzed and to see how they work in a simple model example. Let five enterprises chosen for the quality award be estimated by nine criteria described in Section 2. For this purpose the group of three highly qualified experts was organized. The group ranks five enterprises by each of the three methods.

4.1. Overall Integral Index

Each of the three experts sets weights of indexes with the help of $n \times n$ matrix of indexes pair comparisons. For example, the matrix for group of indexes $m_{211} \ldots m_{215}$ (see Fig. 1) has been made by the first expert:

	x_1	x_2	x_3	x_4	x_5
x_1	10	9	7	6	5
x_2		10	8	7	6
x_3			10	7	6
x_4				10	8
x_5					10

Then, relative estimations of weight of each index are usually calculated by means of the matrix of pair comparisons in several ways. For example, the some of these calculations are shown in Table 2, where weight of x_1 relative to itself is $m_{x1} = 1$; weight of x_2 relative to x_1 is $m_{x2/x1} = 0{,}9$; and so on.

TABLE 2.

	x_2/x_1	x_3/x_2	x_4/x_3	x_5/x_4
x_1	0.9	0.78	0.86	0.83
x_2		0.8	0.87	0.86
x_3			0.7	0.86
x_4				0.8
Average values	0.9	0.79	0.81	0.84

Individual average value of the relative weight $m_{x3/x2}$ is, for example, (0.78 + 0.8)/2 = 0.79. All the values are in the bottom row of Table 2 and are used to calculate normalized weight factors. For example, $m_{x3/x1} = 0.7$; $m_{x3/x2/x1} = 0.79 \cdot 0.9 = 0.71$; and the average relative weight is $\overline{m}_{x3/x1} = (m_{x3/x1} + m_{x3/x2/x1})/2 = 0.705$. Then $\overline{m}_{x4/x1} = 0.58$; $\overline{m}_{x5/x1} = 0.49$; and so on. The obtained values are normalized.

Further procedure is repeated for each of the indexes. Other experts make the same procedure in parallel. Average estimations of weights of indexes are normalized by division of each into the sum of estimations. For every index the average value is calculated by three experts. Normalized final weights of indexes made by three experts are as follows:

$\hat{m}_{x1} = 0.27$; $\hat{m}_{x2} = 0.24$; $\hat{m}_{x3} = 0.19$; $\hat{m}_{x4} = 0.16$; $\hat{m}_{x5} = 0.13$.

The overall integral index of quality W is calculated by formula (2). For instance, a value the integral index of quality W_1 for the first enterprise is

$W_1 = 0.64(10\times0.27+9\times0.24+8\times0.19+7\times0.16+6\times0.13)+0{,}36(10\times0.34+9\times0.29+8\times0.21+7\times0.16) = 8.47$;

where $x_1 = 10$; $x_2 = 9$; etc.

This value and the values indexes of the rest competitors $W_2, \ldots, W_5$ are shown in Table 3.

TABLE 3.

Enterprise	Value of overall index
1	8,47
2	6,81
3	6,44
4	5,36
5	4,24

The following distribution of places between enterprises corresponds to these results: 1, 2, 3, 4, 5.

4.2. Analytic Hierarchy Process

A matrix of pair comparisons of indexes is made by experts, if calculations are performed by this method. For example, for the group of indexes $x_1, \ldots, x_5$ this matrix is as follows:

	x_1	x_2	x_3	x_4	x_5
x_1	1	3	3	4	2
x_2	(1/3)	1	3	5	7
x_3	(1/3)	(1/3)	1	7	5
x_4	(1/4)	(1/5)	(1/7)	1	1
x_5	(1/2)	(1/7)	(1/5)	1	1

Component estimations of eigenvector in the rows are calculated. For example, in the first row of the matrix the estimation looks like

$$\sqrt[5]{1\times3\times3\times4\times2} = 2.35.$$

To get the estimation of vectors the result for every row is normalized. We get the vector of priorities which is shown in Table 4.

Further the eigenvalue λ_{max} is calculated for the given matrix (in our case $\lambda_{max} = 6.38$) and the index of agreement (IA) is evaluated by the formula:

$$IA = (\lambda_{max} - n) / (n - 1), \quad (9)$$

where n is dimension of the matrix. In our case IA = 0.345.

TABLE 4.

	Vector of priorities
x_1	0.36
x_2	0.31
x_3	0.20
x_4	0.07
x_5	0.06

Then the ratio of consistency (RC) is obtained by division IA by the number corresponding to random consistency (RC1) of a matrix of the same order. For a matrix of the fifth order RC1 = 1.12 [2], then

RC = (IA) / 1.12 = 0.31.

The size of RC is acceptable as it makes about 20 % of RC1.

The process is repeated for each group of criteria and objects. Then each column of vectors of priorities is multiplied by the priority of corresponding criteria and the result is summed in each of the rows. The total result is shown in Table 5:

TABLE 5.

Enterprise	Vector of priorities
1	0.81
2	0.71
3	0.56
4	0.41
5	0.33

The following distribution of places between enterprises corresponds to these results: 1, 2, 3, 4, 5. It is just the same as in the case mentioned above.

4.3. Consensus relation

Using this method, experts have determined the following rankings for the enterprises:

1 expert: 1~3 2 4 5
2 expert: 1 2 3 4~5
3 expert: 1~2 3 4~5

In this case the preference profile matrix is as follows:

$$[p_{ij}] = \begin{bmatrix} 0 & 0 & 5 & 0 & 4 \\ 6 & 0 & 6 & 4 & 6 \\ 1 & 0 & 0 & 0 & 1 \\ 6 & 2 & 6 & 0 & 6 \\ 2 & 0 & 5 & 0 & 0 \end{bmatrix}$$

The search of the optimum linear order according to criterion function (8) with the help of special algorithm [11] on this matrix has given the following distribution of places between the enterprises: 1, 2, 3, 4, 5.

The calculation of a percentage ratio of the importance of the enterprises obtained by the three methods is illustrated in Table 6.

TABLE 6. Percentage ratio of enterprises importance

Enterprise	Overall integral index, %	AHP, %	Consensus relation, %
1	27,05	28,77	20
2	21,74	25,18	20
3	20,57	19,95	20
4	17,10	17,49	20
5	13,54	11,62	20

The following formula is used for the calculation:

$$\tau = \frac{M_i}{\sum_{i=1}^{n} M_i} \cdot 100 \qquad (10)$$

where τ is value of index of importance of the enterprise for a method in %; M_i is value of priority of i-th enterprise for some certain method.

5. CONCLUSION

The method of consensus relation has one advantage in comparison with other two methods. It does not contain laborious procedure of weight's assigning and their checking on compatibility. Though the task of finding the relation of the linear order by the criterion function (8) represents a complicated problem [11], there are algorithms which allow quickly to find the solution for rather small values n.

The example demonstrates that all the three methods lead to practically identical results. However, reliability of the method of consensus relation is axiomatically proved. This allows to consider it as a potential method to be used in models of quality awards. More serious recommendations should be supported by computing experiments with plenty of examples and also the organization of real trial competitions where the method of consensus relation would be used.

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