AN EMPIRICAL APPROACH TO DETERMINING ROCKWELL HARDNESS MEASUREMENT UNCERTAINTY

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ABSTRACT
Characteristics of the empirically developed Rockwell hardness test make it difficult to determine measurement uncertainty using methods based on mathematical models describing the relationship between the measurand and the influence quantities. An empirical approach to determining Rockwell hardness uncertainty has been developed, which provides a method based on the familiar procedures and practices of Rockwell hardness testing laboratories. The approach views the hardness machine and indenter as a single measuring device, and considers uncertainties associated with the machine repeatability and the usage of the machine over time with varying environmental conditions and with different operators. The approach also considers the measurement bias of the Rockwell hardness machine as compared to reference standards.

1. INTRODUCTION
The *Guide to the Expression of Uncertainty in Measurement* (GUM) [1] provides a method for estimating the uncertainty in a measurement value by quantifying the significant influence quantities of the test, determining the biases and uncertainties of each influence quantity, correcting for the biases, determining and evaluating sensitivity coefficients to convert from the units of the input quantity to the units of the measurand, and finally combining the uncertainties to provide an overall uncertainty in the resultant measurement value. The Rockwell hardness test [2,3,4] involves the time-dependent application of forces to specifically shaped indenters while simultaneously measuring the resultant indentation depths. This is done using machines that can range from entirely mechanical to various combinations of electronic and mechanical components. As a result, it is often difficult to identify all of the significant influence quantities that contribute to the measurement uncertainty. For example, error sources may exist due to the internal workings of the mechanical components of the testing machine or the indenter, which are not easily measurable. Complicating this, the determination of the sensitivity coefficients is a lengthy and difficult undertaking since these conversion factors are dependent on the Rockwell scale and the material being tested, and are usually not linear functions of the hardness value. An additional complication is that the accepted practice for calibrating a Rockwell hardness machine is to not correct for biases in the operational components of the machine as long as they are within stated tolerance limits [2,3]. Although the technique of assessing the separate machine parameters is used for determining Rockwell hardness uncertainty at the highest calibration levels, such as by National Metrology Institutes, it may present an overwhelming challenge to many industrial hardness laboratories.

An empirical approach for determining Rockwell hardness uncertainty has been developed which provides a method that is based on familiar procedures and practices of Rockwell hardness testing laboratories. The approach views the hardness machine and indenter as a single measuring device, and considers uncertainties associated with the overall measurement performance of the Rockwell hardness machine. This paper discusses the general procedure that has been developed, identifies the most significant sources of uncertainty, and finally applies this approach to determining the uncertainty of: (1) the Rockwell hardness machine's measurement "error" determined as part of an indirect verification [2,3]; (2) Rockwell hardness values measured during normal testing; and (3) the certified calibration value of Rockwell hardness reference blocks.

2. GENERAL APPROACH

This approach primarily considers uncertainties associated with repeatability and reproducibility of the measurement, as well as, the measurement bias as compared to reference standards. The procedure follows the common practice [1] of calculating a *combined* standard uncertainty u_c by combining the contributing components of uncertainty u_1, u_2,, …, u_n, such that

$$u_c = \sqrt{u_1^2 + u_2^2 + \ldots + u_n^2}\ . \tag{1}$$

Measurement uncertainty is usually expressed as a combined expanded uncertainty U_c which is calculated by multiplying the combined standard uncertainty u_c by a numerical coverage factor k, such that

$$U_c = k[u_c]\,. \tag{2}$$

A coverage factor is chosen that depends on how well the standard uncertainty was estimated (number of measurements), and the level of uncertainty that is desired. For this analysis, a coverage factor of $k = 2$ is chosen to reflect a confidence level of approximately 95%.

The measurement bias B of the hardness machine is the difference between the expected hardness measurement results and the "true" hardness of a material. It is best estimated from the difference between the average value of a large number of test results and a reference value. When test systems are not corrected for measurement bias, the bias then contributes to the overall uncertainty in a measurement. Ideally, measurement biases should be corrected; however, in practice, this is commonly not done for Rockwell hardness testing. There are a number of possible methods for incorporating uncorrected biases into an uncertainty calculation, each of which has both advantages and disadvantages [5]. A simple and conservative method is to combine the bias with the expanded uncertainty as

$$U = [ku_c + |B|]\,, \tag{3}$$

where $|B|$ is the absolute value of the bias.

Ideally, an individual measurement uncertainty should be determined for each hardness scale and hardness level of interest since the contributing components of uncertainty may vary depending on the scale and hardness level. In practice, this is not practical. In many cases, a single uncertainty value may be applied to a range of hardness levels based on the laboratory's experience and knowledge of the operation of the hardness machine. Also, because several approaches may be used to evaluate and express measurement uncertainty, a brief description of what the reported uncertainty values represent should be included with the reported uncertainty value.

3. SOURCES OF ERROR

This section describes the most significant sources of error in a Rockwell hardness measurement. In later sections, it will be shown how these sources of error contribute to the total measurement uncertainty. The sources of error to be discussed are: (1) the hardness machine's lack of repeatability; (2) the non-uniformity in hardness of the material under test; (3) the hardness machine's lack of reproducibility; (4) the resolution of the hardness machine's measurement display; and (5) the uncertainty in the certified value of the reference block standards. An estimation of the measurement bias will also be discussed.

3.1 Single hardness measurement - uncertainty due to lack of repeatability (u_{Repeat})

Imagine a material exists that is ideally uniform in hardness over its entire surface, and that hardness measurements are repeatedly made on this material over a short period of time without varying the testing conditions (including the operator). The *repeatability* of the hardness machine making these measurements is defined as its ability to continually produce the same hardness value each time a measurement is made. In actuality, all test instruments, including hardness machines, exhibit some degree of a lack of repeatability.

Even if a material could be found that was perfectly uniform in hardness, each subsequent measurement value would differ from all other measurement values (assuming sufficient measurement resolution). Therefore, the lack of repeatability prevents the hardness machine from being able to always measure the true hardness of the material, and hence contributes to the uncertainty in the measurement. For a future single hardness measurement, the standard uncertainty contribution u_{Repeat} due to the lack of repeatability, may be estimated by making a number of hardness measurements on a uniform test sample, such as a reference block, and calculating the standard deviation of the measurement values as

$$u_{Repeat} = \sigma_{Repeat}\,, \tag{4}$$

where σ_{Repeat} is the standard deviation of the n hardness values. In general, the estimate of uncertainty due to the lack of repeatability is improved as the number of hardness measurements is increased. When evaluating repeatability as discussed above, the influence of non-uniformity in the hardness of the test sample should be minimized as much as possible. The laboratory is cautioned that if the determination of repeatability is based on tests made across the surface of the material, then it will likely include a significant uncertainty contribution due to the material's non-uniformity. A machine's repeatability is better evaluated by making hardness measurements close together (within spacing limitations [2,3]), or by testing material for which the hardness non-uniformity has been modeled [6].

3.2 Average of multiple measurements - uncertainty due to lack of repeatability and material non-uniformity ($u_{Rep\&NU}$)

In practice, hardness measurements are often made at several locations and the values averaged in order to estimate the average hardness of the material as a whole. For example, this is done when making quality control measurements during the manufacture of many types of products; when determining the machine "error" as part of an indirect verification [2,3]; and when calibrating a reference block. How well the calculated value estimates the true average hardness of the material is influenced by both the measurement error due to the lack of repeatability and a sampling error due to the non-uniformity in the hardness of the material. When the average of multiple hardness measurement values is calculated, the combined uncertainty contributions due to the lack of repeatability in the hardness machine and the non-uniformity in the test material, may be estimated from the "standard deviation of the mean" of the hardness measurement values as

$$u_{Rep\&NU} = \frac{\sigma_{Rep\&NU}}{\sqrt{n}}\,, \tag{5}$$

where $\sigma_{Rep\&NU}$ is the standard deviation of the n hardness values.

3.3 Uncertainty due to lack of reproducibility (u_{Reprod})

The reproducibility of a hardness machine can be thought of as how well the measurements agree under changing testing conditions. Influences such as different machine operators and changes in the test environment often affect the performance of a hardness machine. The level of reproducibility is best determined by monitoring the performance of the hardness machine over an extended period of time, during which the hardness machine is subjected to the extremes of variations in the testing variables. It is very important that the test machine be in statistical control, as demonstrated by a control chart, during the assessment of reproducibility. An assessment of a hardness machine's lack of reproducibility may be based on periodic monitoring measurements of the hardness machine, such as daily verification measurements. The uncertainty contribution may be estimated by

$$u_{Reprod} = \sigma_{Reprod} \, , \tag{6}$$

where σ_{Reprod} is the standard deviation of the averages of each set of monitoring measurement values made over a period of time. This estimate of uncertainty also includes a contribution due to the machine's lack of repeatability and the non-uniformity of the monitoring test block; however, these contributions are based on the averages of measurements and should not significantly over-estimate the reproducibility uncertainty. As with the estimation of the uncertainty due to a lack of repeatability, reproducibility may be alternatively estimated by testing material for which the hardness non-uniformity has been modeled [6].

3.4 Uncertainty due to the resolution of the hardness measurement display (u_{Resol})

The finite resolution of the measurement display of all hardness machines prevents the hardness machine from providing an accurate hardness value. However, the influence of the display resolution on the measurement uncertainty is usually only significant when the hardness display resolution is no better than 0.5 Rockwell units, such as for some dial displays. The uncertainty contribution u_{Resol} due to the influence of the display resolution may be described by a rectangular distribution and estimated as

$$u_{Resol} = \frac{r/2}{\sqrt{3}} = \frac{r}{\sqrt{12}} \, , \tag{7}$$

where r is the resolution limit within a hardness value can be estimated from the measurement display in Rockwell hardness units.

3.5 Standard uncertainty in the certified average value of the reference block (u_{RefBlk})

Reference test blocks provide the link to the Rockwell standard to which traceability is claimed. All reference test blocks should have a reported uncertainty in the certified hardness value. This uncertainty contributes to the measurement uncertainty of hardness machines calibrated or verified with the reference test blocks. Note that the uncertainty reported on reference test block certificates is usually stated as an *expanded uncertainty.* Since this analysis uses the standard uncertainty, the uncertainty in the certified value of the reference test block may be calculated as

$$u_{RefBlk} = \frac{U_{RefBlk}}{k_{RefBlk}} \, , \tag{8}$$

where U_{RefBlk} is the reported expanded uncertainty of the certified value of the reference test block, and k_{RefBlk} is the coverage factor used to calculate the uncertainty in the certified value of the reference standard.

3.6 Measurement bias (B)

The measurement bias B of a Rockwell hardness machine may be estimated from the "error" of the hardness machine, as determined from the results of an indirect verification as specified in Rockwell hardness test methods standards [2,3], as

$$B = \overline{H} - \overline{H}_{RefBlk} \, , \tag{9}$$

where $\overline{H}$ is the mean hardness value as measured by the hardness machine during the indirect verification, and $\overline{H}_{RefBlk}$ is the certified average hardness value of the reference test block standard used for the indirect verification.

4. UNCERTAINTY CALCULATION: INDIRECT VERIFICATION

As part of an indirect verification specified in test method standards, the "error" of the hardness machine is determined from the average value of measurements made on a reference test block. This value provides an indication of how well the hardness machine can measure the "true" hardness of a material. Since there is always uncertainty in a hardness measurement, it follows that there must be uncertainty in the determination of the machine "error." This section provides a procedure that the verification agency can use to estimate the uncertainty U_{Mach} of the measurement "error" of the hardness machine, determined as the difference between the average of the measurement values and the certified value of the reference blocks used for the verification. The contributions to the standard uncertainty of the measurement "error", u_{Mach}, are: $u_{Rep\&NU}(Ref.\ Block)$ from Eq. 5, which may be determined from the hardness measurements made on the reference test block used to determine the "error" of the hardness machine; u_{Resol} from Eq. 7; and u_{RefBlk} from Eq. 8. The combined standard uncertainty u_{Mach} and the expanded uncertainty U_{Mach} are calculated by combining the appropriate uncertainty components described above as

$$u_{Mach} = \sqrt{u^2_{Rep\&NU}(Ref.\ Block) + u^2_{Resol} + u^2_{RefBlk}}\ ,\ \text{and} \tag{10}$$

$$U_{Mach} = ku_{Mach}\ , \qquad \text{where } k = 2. \tag{11}$$

This expanded uncertainty U_{Mach} may be reported by a verification agency to its customer as an indication of the uncertainty in the machine "error" reported as part of the indirect verification of the Rockwell hardness machine. Because the approach described in this paper incorporates the uncorrected measurement bias in the calculation of measurement uncertainty, the value of u_{Mach} may be used by the customer to estimate his own measurement uncertainty as an estimate of the uncertainty in determining the bias, as shown below.

5. UNCERTAINTY CALCULATION: ROCKWELL HARDNESS MEASUREMENT VALUES

The uncertainty U_{Meas} in a hardness measurement value may be thought of as an indication of how well the measured value agrees with the "true" value of the hardness of the material. Measurement laboratories and manufacturing facilities typically measure the Rockwell hardness of a test sample or product by making multiple hardness measurements across the surface. The average hardness value of the measurements is then reported as an indication of the hardness of the tested material. The contributions to the standard uncertainty, u_{Meas}, of the average measurement value are: $u_{Rep\&NU}(Material)$ from Eq. 5, which may be determined from the hardness measurements made on the test material; u_{Reprod} from Eq. 6; u_{Resol} from Eq. 7; and u_{Mach} from Eq. 10. The combined standard uncertainty u_{Meas} and the expanded uncertainty U_{Meas} are calculated by combining the appropriate uncertainty components described above for each hardness level of each Rockwell scale as

$$u_{Meas} = \sqrt{u^2_{Rep\&NU}(Material) + u^2_{Reprod} + u^2_{Resol} + u^2_{Mach}}\ ,\ \text{and} \tag{12}$$

$$U_{Meas} = ku_{Meas} + |B|\ . \tag{13}$$

where $|B|$ is the absolute value of the bias and k = 2. The value of U_{Meas} is an estimate of the testing facility's uncertainty in the value of the hardness of the test material.

5.1 *Single measurement*— In the special case that the measurement uncertainty of a single hardness measurement is to be reported, rather than for an average of multiple hardness values, the uncertainty contribution $u_{Rep\&NU}(Material)$ should be replaced with u_{Repeat}, calculated using Eq. 4. The uncertainty in the value of a single hardness measurement is independent of any test material non-uniformity because it is a measurement of the hardness at a single test location. Thus, the measurement uncertainty calculated using u_{Repeat} is applicable to any single measurement made at any location on the material. The value of u_{Repeat} may be calculated from the measurements made during the indirect verification; however, the caution given in 3.1 should be considered.

6. UNCERTAINTY CALCULATION: CERTIFIED VALUE OF REFERENCE BLOCKS

Standardizing laboratories engaged in the calibration of reference test blocks should determine the uncertainty in the reported certified value. This uncertainty U_{Cert} provides an indication of how well the certified value would agree with the “true” average hardness of the test block. Test blocks are certified as having an average hardness value based on calibration measurements made across the surface of the test block. This analysis is essentially identical to the analysis given above in section 5 for measuring the average hardness of a product. In this case, the product is a reference test block. The contributions to the standard uncertainty u_{Cert} of the certified average value of the test block are: $u_{Rep\&NU}(Block)$ from Eq. 5, which may be determined from the calibration measurements made on the test block; u_{Reprod} from Eq. 6; u_{Resol} from Eq. 7; and u_{Mach} from Eq. 10. The combined standard uncertainty, u_{Cert}, and the expanded uncertainty U_{Cert} are calculated by combining the appropriate uncertainty components described above for each hardness level of each Rockwell scale as

$$u_{Cert} = \sqrt{u^2_{Rep\&NU}(Block) + u^2_{Reprod} + u^2_{Resol} + u^2_{Mach}}\text{ , and} \tag{14}$$

$$U_{Cert} = ku_{Cert} + |B|. \tag{15}$$

where $|B|$ is the absolute value of the bias and k = 2. The value of U_{Cert} is an estimate of the uncertainty in the reported certified average hardness value of a reference test block.

7. SUMMARY

An empirical approach for determining Rockwell hardness uncertainty has been developed that provides a practical method for use by industrial hardness laboratories. This method determines uncertainty using measurement data acquired from the indirect verification of the hardness machine as well as from control-chart type monitoring of the machine. The significant sources of uncertainty have been identified, and the approach was applied to measurement values from an indirect verification, normal testing, and the calibration of reference blocks.

8. REFERENCES

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