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UNCERTAINTY OF AMPLITUDE DFT SPECTRUM ANALYSIS IN POWER-LINE SYSTEMS

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Abstract – Characterization of measurement by uncertainty becomes a commonly accepted practice. Since in case of several sources of uncertainties which may even be correlated finding the resulting uncertainty according to the Guide to the Expression of Uncertainty in Measurements [1] is not an easy task, computer simulations are a suitable tool respecting the possible correlations of quantities influencing uncertainty. Analysis of DFT spectrum uncertainty was published elsewhere for coherent sampling. We compare correction factors and remaining uncertainties of two methods of the DFT spectrum analysis applicable for non-coherent sampling. Simulations are performed on power-line frequency with predefined harmonic components and noise and ideal quantizer with selectable resolution.

Keywords: Spectrum analysis, interpolated DFT, DFT spectrum uncertainty

1. INTRODUCTION

Spectrum analysis is still an actual theme in measurement. Uncertainty should be applied also to the spectrum measurement. Basic rules of finding uncertainty of measurement gives [1]. In our contribution we compare correction factors and remaining uncertainties of two methods of the DFT spectrum analysis applicable in case of non-coherent sampling of signal [2-4].

The influence of finite length of registers in processors used for FFT spectrum analysis was studied in many papers, e.g. [5-8]. Influence of additive noise on DFT spectrum was analyzed in [9]; effect of signal windowing is shown in [10-14]. Various interpolation methods were proposed to suppress leakage and improve the frequency resolution reducing so errors in DFT analysis [2-4], [15-16].

Uncertainty analysis according to [1] applied to DFT spectrum analysis was published for coherent sampling [17-18]. In [9] problems connected with leakage are not tackled either. We seek the amplitude spectrum components correction factors and uncertainties for non-coherent sampling using computer simulation. A comparison of the uncertainties of DFT amplitude spectrum after application of the two methods of leakage suppression described in [2-4] is presented for multicomponent power-line frequency periodical signal.

Further in the text when speaking about uncertainty, the standard type A uncertainty is meant.

2. SIMULATIONS STRUCTURE

Since in simulations we know both the actual amplitude spectrum of signal and the spectrum estimation using the investigated method (under given simulation conditions), we can find the optimum correction factors. If applied these factors can reduce significantly the spectrum estimation uncertainties.

The procedure of finding the correction factors and their uncertainty is as follows:

- Generation of the defined input signal observing the simulation conditions (THD, SNR, ADC, f_s, f_{sig}).
- Calculation of additive correction factor “acf” (bias changed sign) acf = U_{iMEAS}-U_{iTRUE}, where U_{iMEAS} is the estimated value of the i-th harmonic and U_{iTRUE} is the “true” value of i-th harmonic (known in simulations).
- Calculation of multiplicative correction factor “mcf” as mcf = 1 - δ/100, where δ = (acf/ U_{iMEAS}).100%.
- Uncertainty of the mcf (type A) u_A(mcf) is given as [1] std(mcf). The mcf uncertainty is the same as relative uncertainty of the corresponding spectrum component.

To get results closer to real conditions, magnitudes of harmonic components were chosen according to international EMC standards [19, 20] presenting compatibility levels for three different environments and for harmonic components up to the 50th. Since compatibility levels given in [19, 20] are supposed not to be reached simultaneously, THD computed using them is higher than the nominal THD (THDn in graphs) for each environment. THDn for environments of class 1 (protected power-supply systems with low compatibility levels) is 5%, for environments of class 2 (public and general industrial environment with low-voltage power supply systems) is 8%, and for class 3 environments (industrial environment with in-plant point of coupling) is 10%. Harmonic components in our investigations are reduced proportionally to reach the nominal THD value for each class of environment. Harmonic components phase-shifts are random using uniform probability distribution in the interval (-π, +π), the phase of the fundamental component is kept zero. Number of harmonic components so that the sampling theorem is observed were taken into account in our investigations in simulations using data windowing and interpolation in frequency domain (further *WIFD method*) [2, 4]. Hann window was used in WIFD method, because we have shown in [4] that when using 12 bit DAQ card higher-order windows do not help increasing measurement accuracy. Number of harmonics in simulations using interpolation and re-sampling in time domain (further "*IRTD method*") [3, 4] were limited so that 10 samples per period of the highest analyzed harmonic component be taken (oversampling by factor 5). This should ensure the relative error below 5% in all harmonics [4], but more detailed information about relationship between signal oversampling and correction factor for this method can be found with the help of Fig.5. Linear interpolation has been used in IRDT method. As shown in [4], using spline interpolation does not increase accuracy and is much slower.

Values of unfiltered harmonics were left unchanged, so true values of THD of the analyzed signals were lower than the originally chosen, but nominal THD values are given in figure titles. Filtration of harmonic components above the given limits can be done in case of real signal measurements by corresponding change of corner frequency of antialiasing filter.

The analyzed signals are mixed with external additive noise for given SNR. Simulations are repeated a hundred-times. Since statistical signal processing is used, type A uncertainty is found.

Resulting corrections are influenced by fundamental signal frequency (possible leakage in original signal) and level of external noise (characterized by the SNR value). Further by signal distortion (described by the THD value), by sampling frequency (corresponding here to the number of samples per signal period) and by resolution of the ADC (ideal quantizer is supposed).

Uncertainty of results of IRTD method [3-4] depends also on the accuracy of finding the fundamental signal period. Method of "integrated zero-crossing" (*IZC*) [3, 21] was used for this task. Its standard uncertainty in our case is less than 0.001% for SNR=70 dB and about 0.01% for SNR=50 dB [20].

3. SIMULATION RESULTS

Selected results of our simulations are presented in Fig.1 to Fig.7. Results for both WIFD and IRTD methods are presented for identical or similar simulation conditions, so that the comparison is more straightforward. Results are given for more than three sampled periods of signal because of Hann window used in WIFD method. Spectrum analysis by means of WIFD method for sampling three or less periods is unusable, uncertaities are too high even for SNR=70 dB.

64, 128 and 256 samples per period were chosen in most simulations, so corresponding sampling frequencies are 3200, 6400 and 12800 Hz.

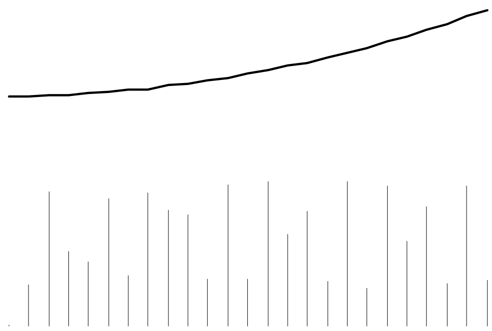

Fig.1 The mcf values and uncertainties of IRTD method (simulation conditions see text in figure)

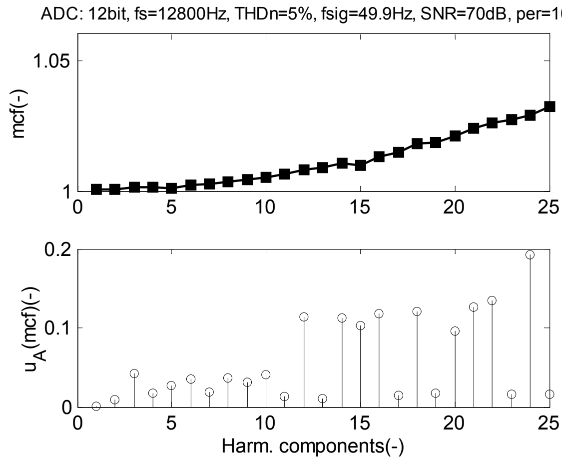

Fig.2 The mcf values and uncertainties of IRTD method for lower THD value

Simulation results for various THD values (various classes of environments) show that relative uncertainties are indirectly proportional to the signal THD (see Fig.1 and 2). For given standard deviation σ_x of white additive noise in time domain the corresponding additive noise of DFT amplitude spectrum has standard deviation $\sigma_{DFT}=\sigma_x$/SQRT(N/2), where N is the DFT length. That is a consequence of DFT linearity. and it was shown in [9] for coherent sampling. If signal THD is lower for the same SNR value, amplitude spectral components are lower, but their additive noise in frequency domain has practically unchanged value (the fundamental harmonic has decisive influence on the signal RMS value.

4. CONCLUSION

Properties of two methods described in [2 – 4] usable for DFT spectrum analysis were investigated by means of simulations from the point of view of sensitivity of their uncertainty to various measurement conditions.

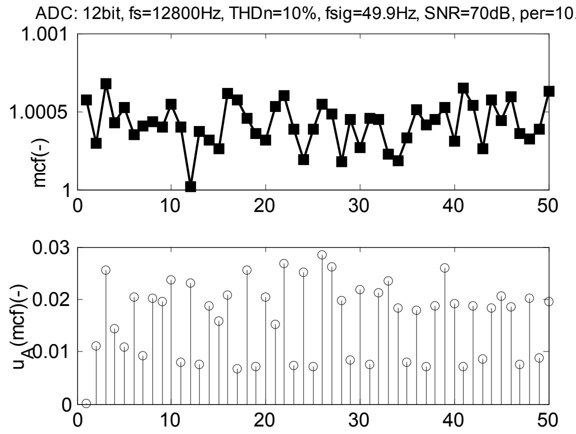

Fig.3 The mcf values and uncertaintis of WIFD method (simulation conditions see text in figure)

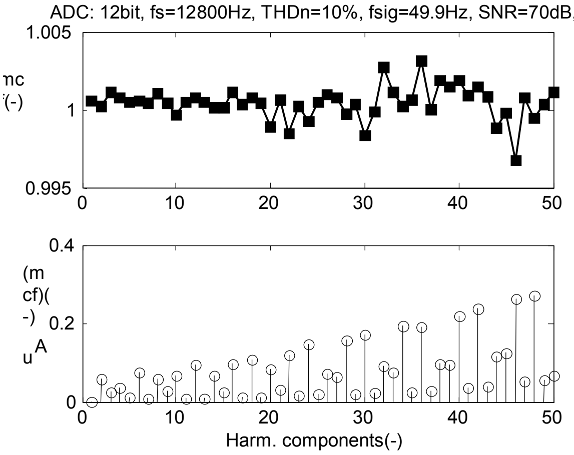

Fig.4 WIFD method, for signal periods sampled, other conditions as in Fig.3.

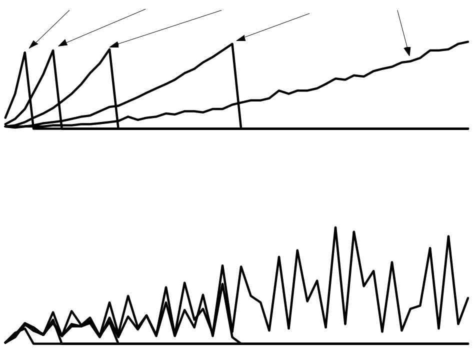

Fig.5 IRTD method, results for 5 different sampling frequencies.

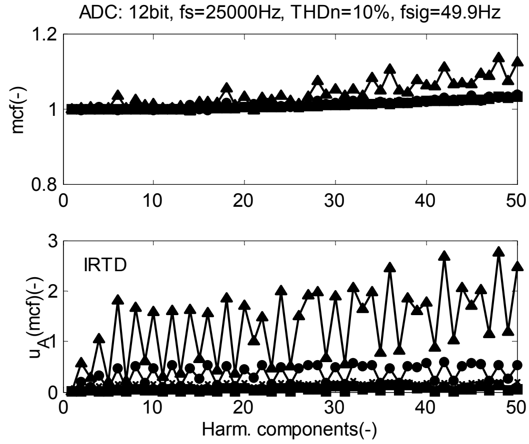

Fig.6 IRTD method, simulation results for four SNR values (30, 40, 50 and 70 dB)

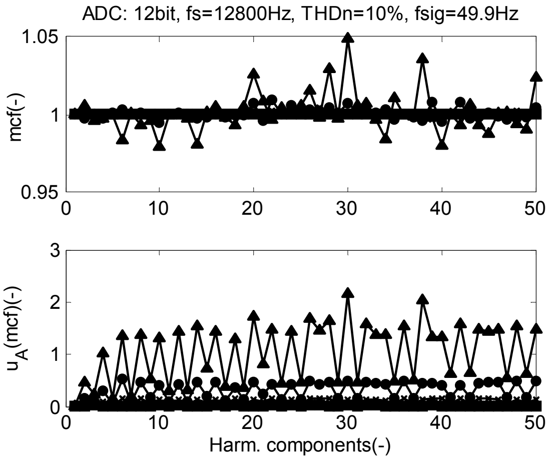

Fig.6 WIFD method, simulation results for four SNR values (30, 40, 50 and 70 dB)

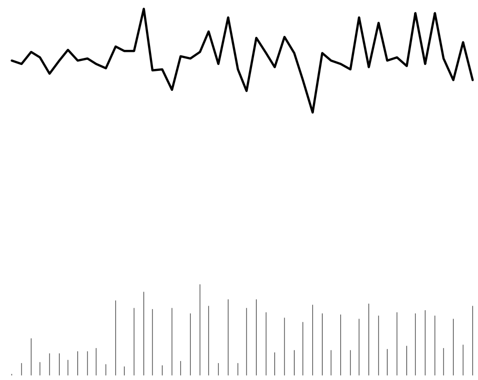

Fig.7 WIFDmethod, effect of lower THD (compare with Fig. 3)

Simulations were performed in MATLAB environment. They allow finding of optimum correction factors for individual harmonic components in DFT spectrum analysis for the case of non-coherent sampling using interpolation in either time domain or frequency domain and to evaluate spectrum components' uncertainty for given SNR. The correction factors are found for defined spectrum analysis conditions (spectrum components corresponding to compatibility levels given in [19, 20], SNR, number of samples per signal period, ADC resolution and signal fundamental frequency).

Numerical values of correction factors inform about bias of amplitudes of individual harmonics, their uncertainty reflects the influence of additive noise. If measurement of spectrum is performed in comparatively low noise (SNR>50dB), application of correction factors in IRTD method can reduce substantially the measurement uncertainty. Simulations were performed on signals defined in [19, 20].

The described methodology of finding correction factors and their uncertainty using numerical simulation is generally applicable for evaluation of measurement uncertainty.

Method using interpolation in time domain [3, 4] requires oversampling – sampling frequency at least 5-times higher than follows from sampling theorem. The higher the oversampling, the lower the correction factors (and spectrum bias). For 50 Hz signals containing 50 harmonics sampling frequency should be therefore at least 25 kHz if all 50 harmonics should be estimated with uncertainty below 5%. Such sampling frequency is nowadays easily achievable even by low-cost PC DAQ plug-in boards (e.g. the NI 6023E board has maximum sampling frequency 200 kSa/s). Therefore the only unpleasant consequence is increase in processing time and demands on memory capacity. If it is sufficient to estimate uncertainties of several lowest harmonics, corresponding setting of the anti-aliasing filter could provide for signal frequency band limitation according to the sampling frequency used.

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