12th IMEKO TC4 International Symposium
Electrical Measurements and Instrumentation
September 25−27, 2002, Zagreb, Croatia

STUDY OF A STANDARD FOR INFORMATIVE CALIBRATION PROCEDURE IN INSTRUMENTATION CALIBRATION

Maurizio Caciotta

Electronic Engineering Dept. of Roma Tre University - Rome - Italy

Abstract − The calibration cost of an instrument is so expensive that is necessary, to reduce the number of calibrated points, in the scale. There is the necessity to exploit as well as possible the metrological valence of few calibrated points to extend all over the instrumental range. The choice of the number and the position of such few points is the problem faced in the work. Utilizing the Shannon sampling procedure in the transposed domain of the input entities of the instrument, with respect to the time, the truncation error of the cardinal serie has faced utilizing the Helms and Thomas upper bound evaluation. To enhance the efficiency of the procedure, has been utilized the minimax procedure searching the optimal indication on better determination. In every case the efficiency of the procedure is lower on the edge of the instrumental scale. To upgrade the edge calibration, making as even as possible the calibration, upon the scale, one has been utilized the generalized sampling theorem by Papoulis that allows to chose the calibration points more frequent at the edge, with respect of the central part of the scale. An algorithm to guide the calibration procedure is tempted.

Keywords: Sampling Procedures, Truncation Error Upper Bound, Minimax Procedure

1. INTRODUCTION

1.1. Representational Model

In trying to select a component from a system, the object of measurement procedure, it's never possible to insulate only it itself: a number of qualities are pooled by the union of "Objects sets" of both the measurement system and instrument. Such entities, in the intersection of "Entities sets", are correlated by a set of physical laws and these relations establish measurement model, in the intersection of third couple of sets, as shown in Fig.1. The entities outside the intersection of "Entities set", that are correlated by physical laws to entities in the intersection, cause a noise in each quality of the model giving the intrinsic precision (casual noise) of the instrument. The instrument behavior can be quite different from the forecast by the model. One can take into account such a displacement, by a calibration procedure.

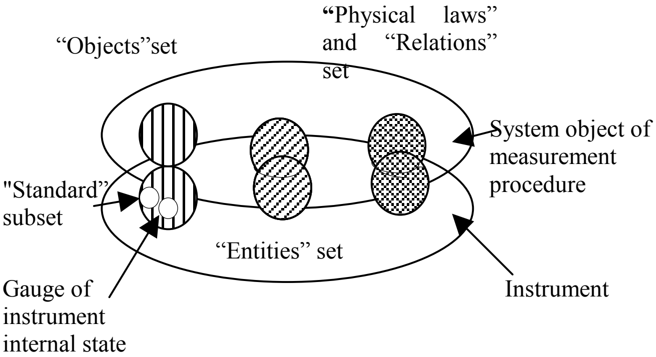

Fig. 1. Representational model of an instrumental measurement procedure

1.2. Calibration General Problem

The model is normally incomplete [9] and this causes a displacement from the correct theoretical projection in numerical field of the chosen entity and the actual measured value: this is the instrumental accuracy. This inaccuracy may be removed by a calibration procedure. The attention is paid to the kind of calibration procedure able to minimize cost-performance ratio. Calibration of all samples can be resolved by a measurement instrument, is a very expensive and time consuming operation [1, 2, 3, 4]. A recent valuation of the cost of a full scale calibration of a resistance, consistent with the sensibility of the available measurement automatic methods is about 2 000 € [13] that can be considered consistent with an electronic non automatic instrument re-calibration [14] which cost is of about 1 000 € for the first point and 100 € for each successive calibration point. A complete calibration of all points of an instrument e.g. with the sensitivity of 10 bits will cost about 100 000 €, that cannot be thought. One deals with the problem of which samples must be calibrated to extend a settled width of the accuracy to the full scale of the instrument. The proposed method considers a transformed domain of the spread between instrument reading and the calibrated quantity supplied as a function of the latter, applying the Fourier Transform. If the sampling time presents an internal "regularity", it is possible to consider the

Shannon theorem as an informative approach to the uncertainty valuation between two calibrated points. Three problems arise: the not uniform distribution of samples, their non infinite samples number and the inversion of procedure when established the calibration quality degree.

2. NON INFINITE NUMBER OF SAMPLES: TRUNCATION ERROR

In the scientific literature there are many works about the truncation error valuation. They are magistrally summed up by M. Unser [5] that in references quotes at least fourteen recent papers of the more important. They are synthesized by a kernel function that allows to calculate truncation error considering the chosen interpolation function. In the kernel approach is foreseen an integration all over the signal frequency range. This truncation error valuation is very efficient for time depending signals for whom it is possible to think in frequency terms. This approach can be used for a measures set linked by quantities different from the time, considering voltage as a transposed domain of frequency. To apply an informative interpretation to the calibration collected data, it needs consider methods that are less possible bound to the idea of the frequency spectrum. H.D. Helms and J.B. Thomas [6] propose the computing of an upper bound of the truncation error [H.T.] that has the advantage of weak link with the (transposed) frequency, that is important, considering the structure of calibration data. The calibration function $G(V_c)$ in the calibration spread domain, considering an uniform sampling ΔV_c, can be written as:

$$G(V_c)=\sum_{i=0}^{N} G(i\cdot\Delta V_c)\,\mathrm{sinc}\left[\frac{2\pi}{\Delta V_c}(V_c - i\cdot\Delta V_c)\right] \qquad (1)$$

with

$$\mathrm{sinc}(x)=\frac{\sin(x)}{x} \qquad (1')$$

The upper bound of truncation error foreseen by the H.T. theory in the interval between two calibrated points is:

$$\Delta G(j)=\frac{2M}{(1-r)\pi^2}\left(\frac{1}{j}+\frac{1}{N-j}\right) \qquad (2)$$

with j=1,…,N, being j the generic, r the ratio of Nyquist frequency occupied by signal and

$$M\overset{\Delta}{=}\max\left|G(V_c)\right| \qquad (3)$$

This result is obtained considering an analytical function, in the finite Laplace's plane s. One can apply the Cauchy's integral formula

$$\frac{f(V_c)}{\sin\left(\frac{2\pi V_c}{\Delta V_c}\right)}=\frac{1}{2\pi j}\oint_{c}\frac{f(s)}{(s-V_c)\sin\left(\frac{2\pi V_c}{\Delta V_c}\right)}ds \qquad (4)$$

being C the contour around a generic pole s. In the case of a sampled function f(t), samples must be considered poles with a total residual value computable as:

$$Q(V_c)=-\sum_{n=N_1}^{N_2}\frac{(-1)^n f(n\Delta V_c)}{\frac{2\pi}{\Delta V_c}\left(\frac{n\Delta V_c}{2}-V_c\right)} \qquad (5)$$

being N_1 the number of samples before the considered voltage V_c and N_2 the number after. The Cauchy's formula will be modified as:

$$\frac{f(V_c)}{\sin\left(\frac{2\pi V_c}{\Delta V_c}\right)}-Q(V_c)=\frac{1}{2\pi j}\oint_{c_a+c_b}\frac{f(s)}{(s-V_c)\sin\left(\frac{2\pi s}{\Delta V_c}\right)}ds \qquad (6)$$

where integration contours, become the imaginary axes, C_a and C_b, from -∞ to ∞ positioned before and after the time window of observation of signal. The left term of the last relation, becomes, for Helms and Thomas:

$$\frac{f(V_c)-Q(V_c)\sin\left(\frac{2\pi V_c}{\Delta V_c}\right)}{\sin\left(\frac{2\pi V_c}{\Delta V_c}\right)}=\frac{\Delta G(V_c)}{\sin\left(\frac{2\pi V_c}{\Delta V_c}\right)} \qquad (7)$$

where $\Delta G(V_c)$ is the truncation error, being, the numerator, the difference between the theoretical function and the limited sampled one. $\Delta G(j)$ is upper bounded by the

$$\Delta G(j)=\frac{4M}{\pi}\left(\int_0^{\infty}e^{-\pi j(1-r)\alpha}d\alpha+\int_0^{\infty}e^{-\pi(N-j)(1-r)\alpha}d\alpha\right) \qquad (8)$$

To evaluate r and M is difficult: they must be both over-estimated, depending on the number N+1 of samples considered [7]. In every case the $\Delta G(j)$ is an upper bound of calibration accuracy in the interval. It is quite plausible, considering the shape of cardinal functions, be maximum at the center of the interval and zero at their extremes. So it is valid, for each interval:

$$E(V_c)\cong\Delta G(j)\cdot\sin\left[\pi\left(\frac{V_c}{\Delta V_c}+1-j\right)\right] \qquad (9)$$

with

$$j=CEIL\left(\frac{V_c}{\Delta V_c}\right) \qquad (9')$$

being CEIL the approximation to the greater close integer.

3. MINIMAX PRINCIPLE APPLIED TO UPPER BOUNDS

Since one considers upper bounds, in the valuation of accuracy, when it can be computed in different ways, one is authorized to consider the minimum between all possible upper bound. So, with N+1 samples, in the generic j interval, for which there are N_1 samples lower and N_2-1 samples upper, being $N_1+N_2=N$, it is possible to consider all combinations with $n_1 \leq N_1+1$ and $n_2 \leq N_2$ for whom is possible to calculate an upper bound based on experimental data.

$$\Delta G_{n_1,n_2}(j) = \frac{2M_{n_1,n_2}}{\left(1-r_{n_1,n_2}\right)\pi^2}\left(\frac{1}{j-n_1}+\frac{1}{N-n_2-j}\right) \qquad (10)$$

As shown in Fig 2 there is possible to consider all interval combination varying n_2 and n_1, and after choosing

$$\Delta G(j) = \min \Delta G_{n_1,n_2}(j) \qquad (10')$$

3.1. M and r evaluation

As shown in the (10) for each upper bound is performed experimental M and r extended valuation depending on local behaviour. This problem is important essentially in r evaluation because the trend is in utilising the lower number of sample as possible that can give a r value greater than one causing an aliasing phenomenon. This problem is under study, but can be resolved considering the folding effect of the spectrum of the data.

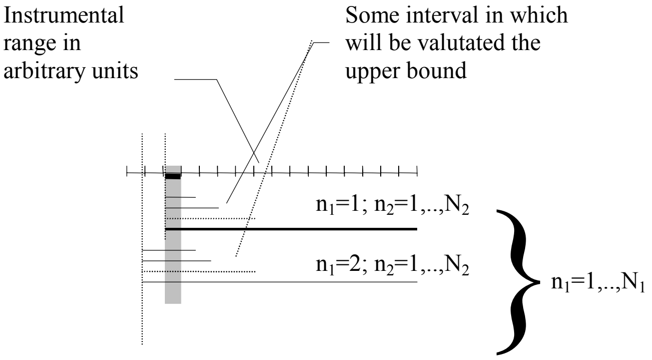

Fig. 2. Interval to which is applied upper bound valuation

In Fig. 3 is shown the behavior of $\Delta G(j)$ calculated for various values of n_1 and n_2.

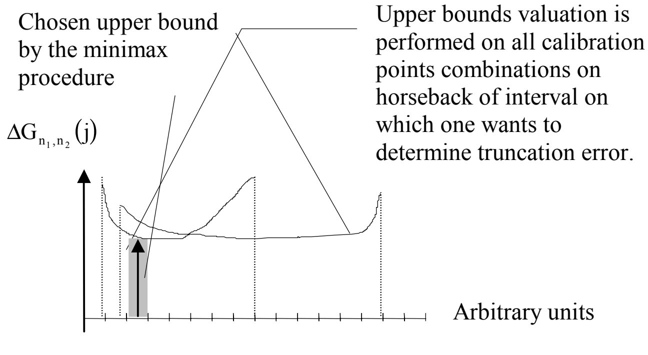

Fig. 3. Minimax procedure application

This procedure, valid for each interval between two calibrated points, can be extended to all the instrumentation range. Generally, the approximate results are shown in Fig. 4.

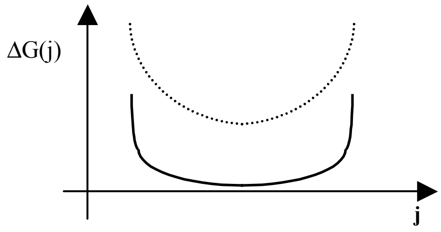

Fig. 4. Upper bound behaviour without (dotted line) and with (bold line) minimax application

In Fig. 5 is shown the number of interval coherent with the minimax chose of the upper bound.

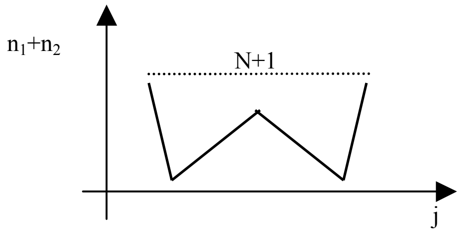

Fig. 5. Windowing corresponding to Fig. 2

This process gets the benefit all over the calibrations range already performed, but blow up number of points growing procedure level, doubling calibration points. From (2) it is easy to see how it is always possible to choose a number of calibration points at "right" and at "left" of actual interval, that will give, as shown in Fig. 4, different behavior depending on experimental samples.

4. SAMPLING DISTRIBUTION

Generally speaking, it is only possible to foresee that corner values of truncation error, will be greater than central ones. Present values will depend on experimental value of calibration spreads. Surely to thicken calibration points at the furthermost limits of the instrumental range, must render more uniform the truncation error general behavior. This is the matter to use a generalization of Sampling Theorem due to Papoulis [8]. He proposes a reconstruction by samples assembled in bunches around a time base:

$$G(V_c) = \sum_{n=-\infty}^{+\infty}\sum_{k=1}^{m} G_k\left(n\Delta V_c + \alpha_k\right) y_k\left(V_c - n\Delta V_c\right) \qquad (11)$$

where

$$y_k\left(V_c\right) = \int_{-\frac{m}{2}}^{-\frac{m}{2}+1} Y_k\left(\frac{2l\pi}{\Delta V_c}, V_c\right) e^{j\frac{2l\pi}{\Delta V_c}}\, dl \quad (k=1,...,m) \qquad (11')$$

If $\omega_T = \dfrac{2l\pi}{\Delta V_c}$, the $Y_k(\omega_T, V_c)$ are solutions of the system:

$$\sum_{i=1}^{m} H_i\left[\omega_T + (k-1)c\right] Y_i\left(\omega_T, V_c\right) = e^{j(k-1)cV_c} \quad (k=1,...,m) \quad (12)$$

with $H_i(.)$ filtering functions that in the case of bunches becomes

$$H_i(\omega_T) = e^{j\omega_T \alpha_i} \tag{13}$$

Rebuilt functions are different from cardinals ones and are dependent on α_i. To evaluate a generic upper bound that can be assigned to all the interval j, is not necessary to know effective function $y_k(.)$: this is necessary only if one wants evaluate the behavior of upper bound in each zone of the considered interval. Each of bunch elements can be considered as a Shannon sequence for whom it is possible to value the H.T. upper bound. For each reduced sequence the upper bound is greater than that calculable with all samples, but there still applying the minimax criterion one chooses the minimum of upper bound foreseen by each of the m regular samples that builds a brunch. Considering a bunched situation for each of their components, can be, in the left terms of the Cauchy's integral formula, settled:

$$\frac{f_k(V_c)}{\sin\left(\frac{2\pi V_c}{\Delta V_c}\right)} - Q_k(\xi) = \frac{\Delta G_k(V_c)}{\sin\left(\frac{2\pi V_c}{\Delta V_c}\right)} \tag{14}$$

The value of the upper bound of the calibration error is given by the relation:

$$E_{tot}(n \cdot m, \alpha_k) = \sum_{n=-\infty}^{+\infty} \sum_{k=1}^{m} \left| \min \Delta G_{k_{operating\_on\_j=nm+k}}(n, \alpha_k) \right| y_k(V_c - n\Delta V_c + \alpha_k) \tag{15}$$

The behavior of the accuracy of the calibration is of the kind shown in Fig. 6. The accuracy at the center of the scale is lower, but is more uniform with samples brunched at the corners. Results suggested by Fig. 4 and Fig. 5, that are the mean behavior obtained from many application of the method, are that is convenient to grow the calibration points around the edges of the instrument range to obtain a more uniform calibration quality extension of the real calibrated points. In every case, some brunches will be present in the central part of the instrument scale that make always such a part more accurate than the other ones.

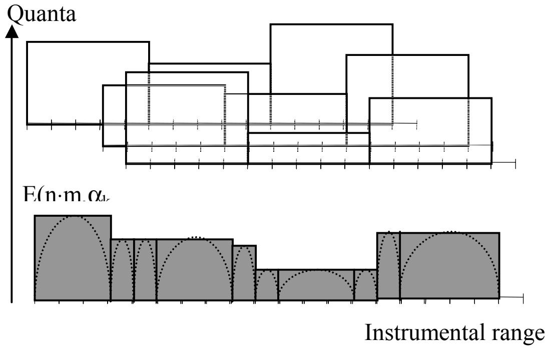

Fig. 6. In grey the "brunch" calibration; others draws are the regular sampling that built the brunch

5. THE CALIBRATION ALGORITHM

Is not possible to know in advance what will be the number of points necessary to determine a wanted calibration quality. Normally, one chose initially the number of samples for each of brunches that will be utilized, considering the certified class of the instrument. A fundamental criterion can be of economical origin. In fact a calibration cannot be more expensive than a 10% of the original cost of the instrument. Considering actual prices of calibration it is possible to affirm that the number of total calibration points will be:

$$.1 \cdot Cost = 1000 + 100 \cdot (N - 1) \tag{16}$$

with the cost expressed in €. This means that only instruments that have costs greater than 13 000 €, one can be considered to calibrate with the minimum of four points. The procedure proposed can be applicable with a minimum number of six point, that make possible to consider only instruments with a cost grater than 15 000 €. The procedure applicable is reported diagram flux shown in Fig. 7.

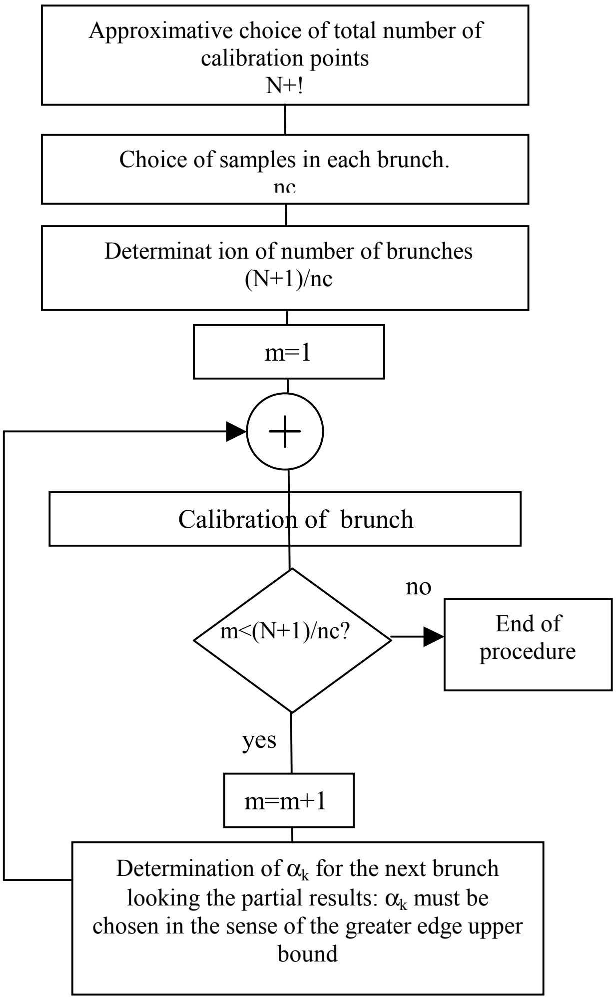

Fig. 7. Flow chart of the calibration procedure

6. PERSPECTIVES

The procedure described until now is under test, applied to 16 bit A/D converter ADT112. The application of Generalized Papoulis Sampling Extension allows limit the calibration points from $2^{(m-1)}+2$, foreseen by the simple application of informative procedure based on Shannon theorem, to $nc \cdot m$ calibration points, where m is the calibration level. Now the possibility to manage the non uniform sampling [10, 11, 12], perhaps will allow more efficient procedure, saving time and money.

REFERENCES

[1] C. Cobelli, E.R. Carson, L. Finkelstein and M.S. Leaning, “Validity of Measurements obtained by model identification and parameter estimation*”, Proc of 4th IMEKO Symp. on Measurements and Estimation* Bressanone 1984;

[2] L. Finkelstein, K.T.V. Grattan (eds), “State and advance of measurement and instrumentation science*”, Proc IMEKO TC1 and TC7 colloquium,* City University of London Sept 1993;

[3] G. Zingales, C. Narduzzi, “The role of artificial intelligence in measurement”, *Proc of 8th Int IMEKO TC7 Symp. on Artificial Intelligent Based Measurements and Control*, Kyoto, Japan, Sept 1991, Livingston, Scotland 1992;

[4] L. Finkelstein, “Intelligent and knowledge based instrumentation-An examination of basic concepts”, *Measurements,* vol 14 N1 September 1994;

[5] M. Unser, "Sampling-50 Years after Shannon", *Proc. of IEEE,* vol. 88, n. 4, April 2000, pp 569/587

[6] H.D. Helms, J.B. Thomas, "Truncation Error of Sampling Theorem Expansion", *Proc. of I.R.E.,* vol. 50, 1962, pp 179/184

[7] M. Caciotta, "The Application of Shannon Sampling Theorem to the Optimization of Calibration Procedure of Instruments", *Proc. of 10th Int. Symp. On "Development in Digital Measuring Instrumentation",* Naples, Sept. 1998, pp 503/507

[8] A. Papoulis, "Signal Analysis", *Mac Graw Hill,* 1977, pp193/196

[9] M. Caciotta, “Calibrazione ottimale della strumentazione: un approccio informativo*” Atti del Congresso SIT - Metrologia e Qualità,* pp. 3/13, Torino, February 1999

[10] M. Unser, J. Zerubia, "A generalized sampling without band limiting constraint", *IEEE Trans. Circuit Syst.*vol 45, pp 959/69, Aug. 1998

[11] G. Say Song, I.G.H. Ong, "Reconstruction of bandlimited signals from irregular samples" *Signal Process,* vol 46 n°3, pp 315/29, 1995

[12] Y. M. Lin, G. G.: Walter, "Irregular sampling in wavelet subspaces", *J. Fourier Anal. Appl.,* vol. 2, pp 181/9, 1995

[13] D.W. Braudaway, "The Cost of Calibration", *Proc. of VIMS 2002Int. Symp. On "Virtual and Intelligent Measurement Systems",* Anchorage, May 2002

[14] www.nrc.ca; website of National Research Council of Canada

---

Author: Prof. Maurizio Caciotta, Electronic Engineering Dept., University "Roma Tre", via della Vasca Navale, 84, 00146 Roma - Italy - Phone: +39-06-55177029; Fax: +39- 06-5579078; E-mail: caciotta @misure.ele.uniroma3.it