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Numerical model for damage identification in brittle materials monitored by Acoustic Emission acquisition systems

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***Abstract* – A numerical method for damage identification in brittle materials is presented. The method is based on the time of arrival of the elastic wave, generated by the damage formation, to a group of sensors positioned on the boundary of a material sample. The equations of motion for the elastic waves are the starting point, including a body force term which accounts for the sudden formation of a crack. Then a discretization scheme for the 2D equations of elasticity is developed, which allows to obtained a suitable numerical model. Some numerical experiments are performed, to assess the validity of the method, reproducing the readings of some sensor positioned on the boundary of the sample. The location of the damage and the time of formation are identified by an appropriate nonlinear least square problem.**

I. INTRODUCTION

Recently a lot of research activity has been devoted to the detection of damage in a brittle material by means of Acoustic Emission (AE) [1, 2, 3, 4, 5], which naturally occurs in a material when there is a crack, or dislocation source. The response of concrete to a load, both in compression and in tension, is influenced by the presence of cracks. Indeed, it is an established fact that the stress-strain response of concrete is closely related with the formation of microcracks [6, 7, 8]. The presence of macroscopic cracks is also an important factor, since they reduce the characteristic resistance of the material. It is generally assumed that each fracture is the result of many microcracks that join to compose a greater fracture. The most important crack will be characterized by an AE signal with maximum amplitude. This signal can be recorded by specific acoustic emission sensors that operate on resonance frequencies in the range from 50 to 400 kHz [9, 10, 11].

Different methods for studying and analyzing these signals can be found in the literature. One of the most recent and interesting methods, links the b value to the AE signals and is based on the Gutenberg Richter law (GBR), typically used in seismology for the study of earthquakes [12, 13]. The GBR law defines the relationship between the magnitude and the total number of earthquake events detected in a region during a pre-established time interval. The b-value parameter, defined within the framework of the GBR law, is used to select the AE signals that identify critical damage events [14, 15]. In particular, the critical AE signals are selected under the condition that the b value is close to 1, because in this case the AE signals have the highest amplitude. This technique can be used to detect the important cracks that develop in a concrete sample during different typologies of tests, for example compressive test or three point test. An aspect which might affect the propagation of AE in a porous material, such as concrete, is the aggression of the material by some chemical pollutant [16, 17]. It is also interesting to note that the propagation of elastic waves in a material could give rise to chaotic phenomena, similar to the ones described in other research domains [18, 19, 20, 21, 22].

Another interest issue is the localization of the damage which generates an AE. Several approaches are available in literature [23, 24]. These different methods are characterized by uncertainty about the position of cracks and must be improved for brittle materials subject to compression.

This paper presents a localization method for crack detection in a concrete sample based on the time of arrival of the elastic wave, generated by the crack formation, to a group of sensors positioned on the boundary of the sample. The following section deals with the equation of motion for the elastic wave, with a body force term which accounts for the sudden formation of a crack. Next, we described a discretized version of the equations of elasticity which is suitable for numerical simulations. A localization method is described on the last section, and some numerical experiments are performed to assess the validity of the method.

II. THE EQUATIONS OF ELASTICITY

We consider a sample of brittle material, subject to a compression stress along the vertical direction which will produce a localized crack, whose position and formation time are unknown. Let $\mathbf{u}(\mathbf{x}, t) \in \mathbb{R}^3$ denote the displacement field, which depends on space coordinates $\mathbf{x} \in \Omega \subset \mathbb{R}^3$, and time $t > 0$, where Ω is the region occupied by the sample. Following [25], the displacement field satisfies the dynamic elastic equilibrium equations

$$\rho \frac{\partial^2 \mathbf{u}}{\partial t^2} = \nabla \cdot \sigma + \mathbf{f}, \tag{1}$$

where ρ is the density, depending on space, σ is the stress tensor and $\mathbf{f}$ is a body force term, depending on space and time. We consider Hooke's law for isotropic media, which provides a linear constitutive relation between the stress tensor σ and the strain tensor $\varepsilon := \frac{1}{2}(\nabla \mathbf{u} + \nabla \mathbf{u}^T)$, given by

$$\sigma = \lambda \mathrm{Tr}(\varepsilon)\mathbf{I} + 2\mu\varepsilon, \tag{2}$$

where λ and μ are Lamé's first and second parameter.

In component form, let $\mathbf{u} = (u_1, u_2, u_3)$, $\mathbf{x} = (x_1, x_2, x_3)$, and use the notation $u_{a,t} = \partial u_a/\partial t$, $u_{a,b} = \partial u_a/\partial x_b$, with $a, b = 1, 2, 3$. Using Einstein's convention of implicit summation for repeated indices, we have

$$\varepsilon_{ab} = \frac{1}{2}(u_{a,b} + u_{b,a}), \quad \sigma_{ab} = \lambda \varepsilon_{cc}\delta_{ab} + 2\mu\varepsilon_{ab},$$

where δ_{ab} is the Kronecker delta, and equation (1) becomes

$$\rho u_{a,tt} = (\lambda + \mu)u_{c,ca} + \mu u_{a,cc} + f_a, \quad a = 1, 2, 3. \tag{3}$$

The body force components f_a will be chosen to model a sudden separation of two sides of a fault localized in a point, which may think of as an infinitesimal planar crack with a prescribed normal direction. Following the classical paper by Burridge and Knopoff [26], let us assume that the displacement and its derivatives present a discontinuity across a surface Σ embedded in Ω. Let $\mathbf{n}$ denote the normal direction to Σ, and $[u_a](\mathbf{x}, t)$, $[u_{a,b}](\mathbf{x}, t)$ the discontinuity of u_a, $u_{a,b}$ across Σ in the direction $\mathbf{n}$. Then, after extending the relevant example in [26], the body force equivalent is given by

$$f_a(\mathbf{x}, t) = -H(t)R_{ab}(\mathbf{n})D_{bc}R_{dc}(\mathbf{n})\delta_{,d}(\mathbf{x} - \mathbf{x}_0), \tag{4}$$

where $\mathbf{n}$ is the normal direction to the crack, $\mathbf{x}_0$ is the location of the crack, R_{ab} are the components of the rotation matrix which takes $\mathbf{n}$ to the unit vector of the z-axis, D_{ab} are the components of the diagonal matrix $D = \mathrm{diag}(\lambda, \lambda, \lambda + 2\mu)$, and $\delta_{,a}$ is the derivative of a Dirac delta function with respect to x_a. The parameter $\mathbf{n}$ comprises three angular variables in three dimensions (Euler angles), or an angular variable in two dimensions, and the parameter $\mathbf{x}_0$ comprises three coordinates in three dimension, or two coordinates in two dimensions, for a total of 6 parameters in three dimensions, or 3 parameters in two dimensions.

The final equations are supplemented with homogeneous Neumann boundary conditions on the boundary of the space domain, and with zero initial data.

As is well known, the equations of linear elasticity have two characteristic velocities,

$$c_p = \sqrt{\frac{\lambda + 2\mu}{\rho}} \quad c_s = \sqrt{\frac{\mu}{\rho}}$$

corresponding to dilatational and isochore waves, respectively. We will use the primary velocity c_p to evaluate the ratio between the distance of a sensor from the source of the crack, and the corresponding time of travel.

III. THE NUMERICAL MODEL

In this section, we derive a discretized version of equations (3), with a body force given by the expression (4), in the case of space dimension 2. Then, the sensors will be represented by a set of points on the boundary, and the reading of a specific sensor by the the first time at which the displacement vector at that point becomes different from zero.

The two-dimensional version of system (3) in a rectangular sample $\Omega = [0, l_x] \times [0, l_y]$, with homogeneous Neumann boundary conditions and zero initial value is:

$$\begin{cases} \rho u_{tt} = [(\lambda + 2\mu)u_x + \lambda v_y]_x + [\mu v_x + \mu u_y]_y + f, \\ \rho v_{tt} = [\mu v_x + \mu u_y]_x + [\lambda u_x + (\lambda + 2\mu)v_y]_y + g, \\ \quad \text{for } (x, y) \in \Omega,\ t > 0, \\ u_x = v_x = 0, \quad \text{for } x = 0, l_x,\ y \in [0, l_y],\ t > 0, \\ u_y = v_y = 0, \quad \text{for } x \in [0, l_x],\ y = 0, l_y,\ t > 0, \\ u = v = u_t = v_t = 0, \quad \text{for } (x, y) \in \Omega,\ t = 0. \end{cases} \tag{5}$$

Here, for simplicity, $(u, v)^T$ denotes the displacement vector, depending on space variables $\mathbf{x} = (x, y)^T$ and time t, and $(f, g)^T$ denotes the body force. Let θ be the angle formed by the infinitesimal crack line Σ with the x-axis, so that Σ is $(\cos\theta, \sin\theta)^T$ is the tangent vector, and $\mathbf{n} = (-\sin\theta, \cos\theta)^T$ the normal vector, and let $\mathbf{x}_0 = (x_0, y_0)^T$ be the location of the crack. Then, the rotation matrix R and the diagonal matrix D are given by

$$R = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}, \quad D = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda + 2\mu \end{bmatrix},$$

and the expression (4) which gives the corresponding body force becomes

$$\begin{bmatrix} f \\ g \end{bmatrix} = -H(t)(\lambda I + 2\mu \mathbf{n}\mathbf{n}^T)\begin{bmatrix} \partial_x \\ \partial_y \end{bmatrix}\delta(\mathbf{x} - \mathbf{x}_0). \tag{6}$$

For the numerical simulations, we use a representation of the Dirac delta,

$$\delta(\mathbf{x} - \mathbf{x}_0) \approx \eta_\varepsilon(\mathbf{x}) \equiv \frac{1}{2\pi\varepsilon} e^{-\frac{|\mathbf{x} - \mathbf{x}_0|^2}{2\varepsilon}},$$

so that (6) will be replaced by

$$\begin{bmatrix} f \\ g \end{bmatrix} \approx H(t)(\lambda I + 2\mu \mathbf{n}\mathbf{n}^T)(\mathbf{x} - \mathbf{x}_0)\frac{1}{\varepsilon}\eta_\varepsilon(\mathbf{x}). \tag{7}$$

The maximum of both components of this function is $\frac{c_p}{2\pi e^{1/2}\varepsilon^{3/2}}$, so we divide the above expression by this constant, to obtain an approximation of the theoretical body force with reasonable magnitude.

To integrate numerically (5)-(6), we use a box-integration method with two staggered grids. We subdivide the interval $[0, l_x]$ in n_x subintervals of equal length $h_x = l_x/n_x$, the interval $[0, l_y]$ in n_y subintervals of equal length $h_y = l_y/n_y$, and introduce the primary grid points

$$\begin{aligned}&(x_i, y_j) = (ih_x, jh_y),\\ &i = 0, 1, \ldots, n_x,\ j = 0, 1, \ldots, n_y,\end{aligned}$$

and the dual grid points

$$\begin{aligned}&(x_{i+\frac{1}{2}}, y_{j+\frac{1}{2}}) = (x_i + \tfrac{1}{2}h_x, y_j + \tfrac{1}{2}h_y),\\ &i = 0, 1, \ldots, n_x - 1,\ j = 0, 1, \ldots, n_y - 1.\end{aligned}$$

Introducing the notation

$$\begin{aligned}&u_{i,j}(t) = u(x_i, y_j, t),\ i = 0, \ldots, n_x,\ j = 0, \ldots, n_y,\\ &v_{i,j}(t) = v(x_{i-\frac{1}{2}}, y_{j-\frac{1}{2}}, t),\ i = 1, \ldots, n_x,\ j = 1, \ldots, n_y,\end{aligned}$$

after a standard finite difference approximation with three point for the second order time derivative, with time step τ, we obtain the following scheme:

$$\begin{aligned}u_{i,j}^{k+1} &= 2u_{i,j}^k - u_{i,j}^{k-1}\\ &+ \tfrac{c_p^2\tau^2}{h_x^2}(u_{i-1,j}^k - 2u_{i,j}^k + u_{i+1,j}^k)\\ &+ \tfrac{c_s^2\tau^2}{h_y^2}(u_{i,j-1}^k - 2u_{i,j}^k + u_{i,j+1}^k) + \tau^2 f(x_i, y_j, t_k)\\ &+ \tfrac{(\lambda+\mu)\tau^2}{h_x h_y \rho}(v_{i+1,j+1}^k - v_{i,j+1}^k - v_{i+1,j}^k + v_{i,j}^k)\end{aligned} \tag{8}$$

$$\begin{aligned}v_{i,j}^{k+1} &= 2v_{i,j}^k - v_{i,j}^{k-1}\\ &+ \tfrac{c_s^2\tau^2}{h_x^2}(v_{i-1,j}^k - 2v_{i,j}^k + v_{i+1,j}^k)\\ &+ \tfrac{c_p^2\tau^2}{h_y^2}(v_{i,j-1}^k - 2v_{i,j}^k + v_{i,j+1}^k) + \tau^2 g_{i-\frac{1}{2},j-\frac{1}{2}}^k\\ &+ \tfrac{\lambda+\mu}{h_x h_y \rho}(u_{i-1,j-1}^k - u_{i-1,j}^k - u_{i,j-1}^k + u_{i,j}^k),\end{aligned} \tag{9}$$

where the superscript denotes the index of the time t_k. The above scheme provides the update for the unknowns on the grid points at time t_{k+1} in terms of the same unknowns at times t_{k-1}, t_k. To enforce homogeneous Neumann conditions for u and v, we introduce the ghost nodes $x_{-1} = -h_x$, $x_{n_x+1} = l_x + h_x$, $y_{-1} = -h_y$, $y_{n_y+1} = l_y + h_y$, and assume the following mirror conditions with respect to the boundary :

$$\begin{aligned}&u_{-1,j} = u_{1,j}, \quad u_{n_x+1,j} = u_{n_x-1,j}, \quad j = 0, \ldots, n_y,\\ &u_{i,-1} = u_{i,1}, \quad u_{i,n_y+1} = u_{i,n_y-1}, \quad i = 0, \ldots, n_x,\\ &v_{0,j} = u_{1,j}, \quad v_{n_x+1,j} = v_{n_x,j}, \quad j = 0, \ldots, n_y + 1,\\ &v_{i,0} = u_{1,j}, \quad v_{i,n_y+1} = v_{i,n_y}, \quad i = 0, \ldots, n_x + 1.\end{aligned}$$

IV. DAMAGE DETECTION

The location and orientation of a sudden crack, described by the vectors $\mathbf{x}_0$ and $\mathbf{n}$, as well as the formation time t_0, will be now assumed unknown. We present a method to estimate these crack unknowns by the arrival time of the elastic wave, produced by the crack, to some sensor at the boundary of the sample. We assume to have N sensors, at the positions $\mathbf{x}_i = (x_i^1, x_i^2, x_i^3)^T$, with $i = 1, 2, \ldots, N$. The i-th sensor detects the arrival of an elastic wave at time t_i, $i = 1, \ldots, N$. We assume for simplicity that only a crack develops, at the position $\mathbf{x}_0 = (x_0^1, x_0^2, x_0^3)^T$, at time t_0. As a working hypothesis, we also assume that the orientation of the crack is not relevant for the effectiveness of the localization procedure that we are going to present. We aim to identify $\mathbf{x}_0$ and t_0 from the reading of the arrival times t_i, $i = 1, \ldots, N$.

Since the speed of the elastic wave in the direction of the front propagation is c_p, we have

$$c_p(t_i - t_0) = \|\mathbf{x}_i - \mathbf{x}_0\|, \quad i = 1, 2, \ldots, N. \tag{10}$$

To solve for $(\mathbf{x}_0, t_0)$, we need to minimize the residuals

$$r_i = c_p(t_i - t_0) - \|\mathbf{x}_i - \mathbf{x}_0\|, \quad i = 1, 2, \ldots, N,$$

that is, to minimize their quadratic norm,

$$S := \sum_{i=1}^N r_i^2 = \sum_{i=1}^N (c_p(t_i - t_0) - \|\mathbf{x}_i - \mathbf{x}_0\|)^2. \tag{11}$$

Introducing the vectors $\mathbf{X}_i = (x_i^0, x_i^1, x_i^2, x_i^3)^T$, with $x_i^0 = c_p t_i$, $i = 0, 1, \ldots, N$, the function S becomes

$$S = \sum_{i=1}^N (X_i^0 - f_i(\mathbf{X}_0))^2,\ f_i(\mathbf{X}_0) = X_0^0 + \|\mathbf{x}_i - \mathbf{x}_0\|, \tag{12}$$

This is a nonlinear least squares problem which can be solved by the Gauss-Newton method.

Assuming to know an approximation $\mathbf{X}_0^{(k)}$ of the solution for $k \geq 0$, we set

$$\mathbf{X}_0^{(k+1)} = \mathbf{X}_0^{(k)} + \Delta\mathbf{X}_0^{(k)}, \tag{13}$$

and solve for $\Delta\mathbf{X}_0^{(k)}$ the following approximate gradient equations:

$$J\big(\mathbf{X}_0^{(k)}\big)^T J\big(\mathbf{X}_0^{(k)}\big)\Delta\mathbf{X}_0^{(k)} = J\big(\mathbf{X}_0^{(k)}\big)^T \mathbf{r}\big(\mathbf{X}_0^{(k)}\big), \tag{14}$$

where

$$\mathbf{r}(\mathbf{X}_0) = \begin{bmatrix} X_1^0 - f_1(\mathbf{X}_0) \\ \vdots \\ X_n^0 - f_n(\mathbf{X}_0) \end{bmatrix},\ J(\mathbf{X}_0) = \begin{bmatrix} \nabla_{\mathbf{X}_0} f_1(\mathbf{X}_0) \\ \vdots \\ \nabla_{\mathbf{X}_0} f_n(\mathbf{X}_0) \end{bmatrix},$$

and

$$\nabla_{\mathbf{X}_0} f_i(\mathbf{X}_0) = \begin{bmatrix} 1 & -\frac{\mathbf{x}_i^T - \mathbf{x}_0^T}{\|\mathbf{x}_i - \mathbf{x}_0\|} \end{bmatrix}.$$

We have solved the discretized system (8)-(9) in a square with size 15 cm, with 250×250 grid points, that is, space step $h_x = h_y = 0.06$ cm, and time step $\tau = 0.08$ μs. We have used Lamé's parameters $\lambda = 9.7078$ GPa, $\mu = 12.2883$ GPa, and density $\rho = 2200 \text{ kg/m}^3$. A crack formation has been simulated at the position $(x_0, y_0) = (5.3550, 8.9550)$, between a couple of adjacent grid points, one on the primary grid and the other one on the dual grid, at a random time $t_0 = 3.75$ μs, with an angle $\theta = \pi/4$ between the crack line and the x-axis. To validate the presented localization method, we have simulated four sensors, located in the positions $(x_1, y_1) = (4.92, 0)$, $(x_2, y_2) = (0, 9.9)$, $(x_3, y_3) = (9.9, 14.94)$, $(x_4, y_4) = (14.94, 4.92)$. For the simulation performed, the method gives an estimated location of the crack at (5.4438,8.8606), and an estimated formation time 11.7156 μs. These values are remarkably close to the real crack location (5.355,8.955) and formation time $t_0 = 12$, with a relative error of about 1%.

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