

Novel 3D distinct element limit analysis model for the seismic vulnerability evaluation of historical masonry pagodas of major importance in China

Peixuan Wang¹, Gabriele Milani¹, Carmelo Scuro²

¹ *Department of Architecture, Built Environment and Construction Engineering, Politecnico di Milano, 20133 Milano, peixuan.wang@polimi.it, gabriele.milani@polimi.it*

² *Physics Department, University of Calabria, University of Calabria, 87043 Rende, CS, Italy, carmelo.scuro@unical.it*

Abstract – A new distinct element DEM limit analysis approach is presented and applied to the seismic vulnerability evaluation of an ancient masonry pagoda in China. The numerical model relies on the assemblage of infinitely resistant hexahedral elements and rigid plastic four-nodded interfaces, where all the internal dissipation lumps, constraining cracks to spread between adjoining elements. So, it is neither an upper bound nor a lower bound, but rather a DEM limit analysis. A masonry pagoda, Huqiu pagoda located in China, was selected as a case study to benchmark the model under the application of a seismic load. It is an octagonal masonry pagoda with a total of 7 floors and a height of 48.2m.

The results show that the failure mechanisms found by limit analysis are basically vertical splitting with bending at the base. The results obtained by this research method are consistent with the actual behavior of masonry pagodas, which shows its suitability in direct applications by practitioners.

I. INTRODUCTION

Conservation, preservation, rehabilitation of architectural, cultural, and historical built heritage is a current key issue for our societies, especially in seismic-prone areas such as in China and Italy. Masonry structures are particularly challenging for researchers and engineers who tend to conceive appropriate structural analysis methods, computational tools, and design strategies. In particular, masonry pagodas are one of the most significant monuments in Chinese architectural heritage [1]. They usually represent the culture and history of one nation, which contain an inestimable value. Speaking about building in masonry from a general perspective, it has the advantage of reasonably good compressive strength, weather and fire resistance, low cost, and flexibility of construction. After hundreds of

years, however, the effects of the application of repeated seismic excitation, weather aging, wars, and other artificial destructions, represent a major concern and these monuments have suffered many different typologies of damage. Therefore, trying to protect and repair in particular ancient masonry pagodas has become an important mission for the Chinese cultural heritage.

Limit analysis of structures is an important branch of plastic mechanics. This method considers the material rigid-perfectly plastic with an associated flow rule: a point or part of the structure progressively enters the plastic yielding without leading immediately to structural failure, which is triggered only by the formation of a well-defined, concentrated (global or local) plastic mechanism, finally leading to the collapse. Through the limit analysis method, the bearing capacity of the structure under proportionally increased loading conditions can be directly estimated in one step without knowing the elastic properties of the material, and the applied loads under which it loses stability can be predicted with a high level of accuracy. It can avoid the burden of complex incremental calculations typical of an elastic-plastic analysis and is widely used in engineering practice. At present, limit analysis methods are widely utilized mainly by means of (1) Analytical procedures; (2) Numerical approaches (mainly within the finite element method) [2][3].

In the early 1950s, Drucker and R. Hill made groundbreaking work in the theory and formalized the upper and lower bound theorems.

In 1970, Lysmer was one of the first to discretize a structure with triangular elements and combined it with the linear programming method to obtain a lower bound limit solution. Researchers used the inscribed polygon to approximate the yield criterion. Through such approximation, nonlinear yield conditions can be transformed into linear ones, allowing to combine limit analysis with the simplest operational research method,

which is linear programming [4].

In 1975, American scholar Chen Wai-Fah published the monograph "Limit analysis and soil plasticity", which extended the limit method to the field of geotechnical engineering [5].

From 1976 to 1985, David et al. combined the lower bound limit analysis method with the finite element method and proposed a series of linear programming feasibility methods.

Starting from 1988, based on previous research, Sloan was probably the first to use triangular elements in plane strain to discretize soil, combining linear programming with the lower bound theorem of limit analysis.

In 1995, in cooperation with Kleeman, Sloan adopted the same method from a kinematic point of view, considering velocity discontinuities between adjoining triangular elements, so obtaining a very accurate upper bound limit solution for a variety of geotechnical problems [6][7]. In the early 90s, mentioning rapidly the advancements of the algorithmic part in limit analysis, Zouain et al. work deserve to be recalled, because the interior point method was adopted extensively to tackle lower bound limit analysis problems. Then, Herskovits improved and proposed a two-stage feasible arc interior point algorithm [8]

In general, the research methods of limit analysis have been greatly developed in the last four decades and have been widely used in the calculation of the bearing capacity of practical engineering problems [9]-[13]. However, there are still problems in the existing algorithms, such as computational complexity, low solution efficiency, and application research far behind theoretical research. Therefore, developing a simple, efficient, and stable numerical approach suitable for the limit analysis of complex engineering structures is still the focus of intensive research in the field of computational mechanics.

The paper provides a new limit analysis method specialized for the seismic safety assessment of masonry pagodas; the analysis model uses infinitely resistant hexahedral elements and rigid plastic interfaces. A masonry pagoda located in China, Huqiu pagoda, was selected as a benchmark to verify the feasibility of the method.

II. DESCRIPTION OF THE CASE STUDY: HU QIU PAGODA

Huqiu pagoda (Fig. 1), also known as Yunyan temple pagoda, is located on Huqiu Mountain on the outskirts of the ancient city of Suzhou, Jiangsu province, China. On March 4, 1961, Huqiu pagoda was announced by the State Council of the People's Republic of China as the first batch of national key cultural relics protection units.

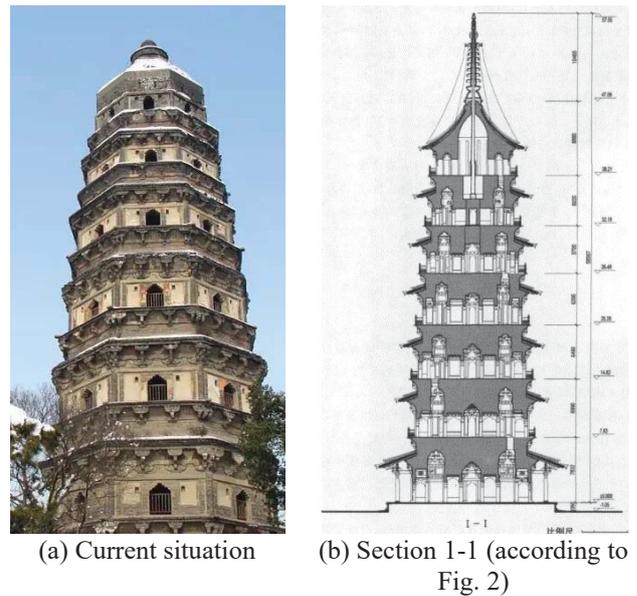


Fig. 1. Huqiu pagoda

It is an octagonal masonry pagoda with a total of 7 floors and a height of 48.2m [14]. The bottom floor is 13.81m long from north to south and 13.64m long from east to west (Fig. 2, Table 1). The pagoda body and base are all made of bricks, and the outer eaves are a mixed structure of brick and wood (they are not considered in the present computations because their structural role under seismic loads is secondary). The pagoda body is composed of three parts: the outer wall, the cloister, and the central room. The corners of each layer of the outer wall are cylindrical. Each side facade is divided into three rooms by eaves and columns, one of which is the pagoda gate, and the two sides are windows. There is an aisle from the gate on the outer wall to the cloister; the inside of the aisle is the central room, which is octagonal, and opens on all sides from east, west, north, and south. From the pagoda gate, people can walk into the central room through the aisle. The planes of the central rooms are all square except the second and seventh are octagonal. According to original estimates, the entire weight of Huqiu pagoda is about 61,000kN.

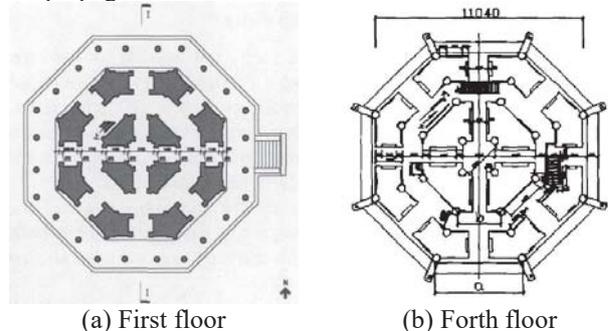


Fig. 2. Plans of Huqiu pagoda. (Source: Internet and [14])

Table 1. The geometric dimensions of Huqiu pagoda [14]

Floor	Height	Length of outside plane	Length of inside plane
1	7.8	5.47	2.49
2	6.37	5.21	2.59
3	6.06	4.90	2.49
4	5.95	4.57	2.48
5	5.48	4.23	2.16
6	5.2	3.79	1.90
7	10.6	3.36	1.99

The pagoda has suffered several modifications, for instance the wooden eaves and the spire have been destroyed, and the existing part is mainly constituted by masonry. According to the research carried out so far, since its construction, the base of the pagoda progressively subsided unevenly, which causes its body to tilt to the north, and the top of the pagoda's eccentricity has reached more than 2.34 m. After the partial collapse of the bottom of the pagoda from 1953, until 1986, Suzhou city carried out two maintenance and emergency reinforcement projects for the Huqiu pagoda. These two projects were compiled into the "Suzhou Yunyan Temple Pagoda (Huqiu pagoda) Maintenance and Reinforcement Project Report". Due to the serious inclination of the Huqiu Pagoda and the urgent need for protection, the existing research mainly focuses on reinforcement, repair, and testing, while the design research of the pagoda is still lacking.

III. DESCRIPTION OF THE NUMERICAL MODEL

The analytical description of the proposed numerical approach adopted is postponed to a future research report for the sake of conciseness, whereas here it is worth mentioning that the procedure is very similar to that proposed by one of the authors in [15] for the analysis of double curvature structures. In such a model, one of the authors of this paper proposed a kind of lower bound Limit analysis method by means of discretization with infinitely resistant hexahedrons and interfaces. The model is slightly different from Ref. [15], because the internal static quantities acting on the interfaces are derived by integration directly performed at the structural level here and many hexahedrons along the thickness can be considered. The formulation is self-dual, so velocities and rotation rates of the elements can be used as primal variables instead of internal actions on interfaces; in this regard, the proposed model is neither an upper bound nor a lower bound method.

IV. HU QIU PAGODA RESULTS

A. Mechanical properties and features

For the analyses, the paper used a double Mohr-Coulomb failure criterion with tension and compression cutoffs. According to Fig. 3, the mechanical properties are the following: $f_t=0.05$ MPa, $c=0.05$ MPa, $f_c=2.5$ MPa, $\phi=30^\circ$. The model of Huqiu pagoda was created in a

common commercial FE software, used only in the pre-processing phase (Fig. 4). In the model studied here, 10349 interfaces, 4073 elements, and 5808 nodes are present.

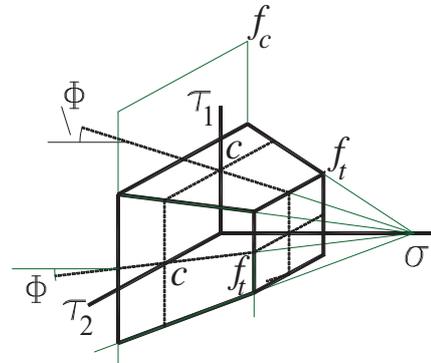


Fig. 3. Double Mohr-Coulomb failure criterion with tension and compression cutoff.

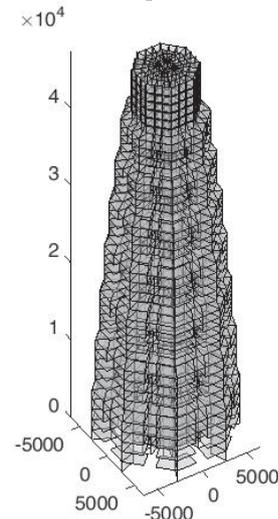


Fig. 4. Huqiu pagoda analysis model

B. Limit analysis results

Limit analyses were carried out under the application of horizontal loads mimicking the effect of an earthquake in a static fashion, in order to have an insight into the seismic vulnerability of the structure. The horizontal load directions considered are along four principal straight lines oriented with respect to one of the principal axes of the octagonal cross-section: 0° , 15° , 30° , and 45° . The along the height load distributions are the following: G1, principal distribution of forces proportional to the mass and linearly proportional to the height of the structure (the so-called "inverted triangle" distribution); G2, secondary distribution of forces proportional to the mass and constant along the height of the structure. The Azimut/Elevation Angles chosen to represent the results in terms of failure mechanisms are $45^\circ/45^\circ$, $0^\circ/0^\circ$, and $15^\circ/30^\circ$. The results as far as the collapse loads are

concerned are reported in Table 2 and Table 3 and Fig. 5.

Collapse mechanisms found for G1 and G2 distributions are similar but, as expected, G1 cases exhibit a lower collapse total shear at the base. Although each direction has its own features in the damage due to the peculiar distribution of the internal walls, the collapse mechanisms exhibit recurrent characteristic paths of pagodas damaged by earthquake actions. A vertical crack is visible, and in almost all mechanisms it is visible the activation of an ultimate bending at the base with the cross-section separated into two parts, one in tension and the other in compression..

The limit analysis demonstrated also that the geometrical characteristics of the pagoda's structural typology strongly influence the ultimate behaviour in case of horizontal loads, in this specific case probably because the central room creates weak regions in the cross-section.

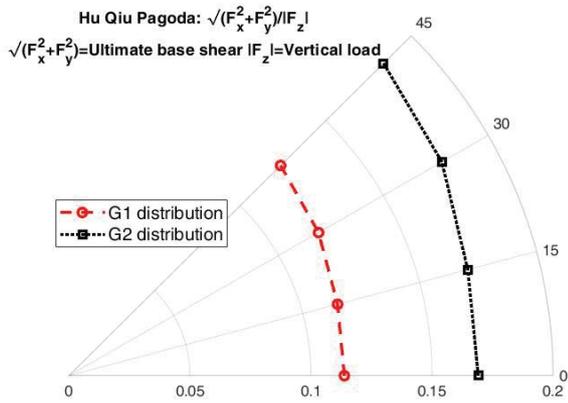


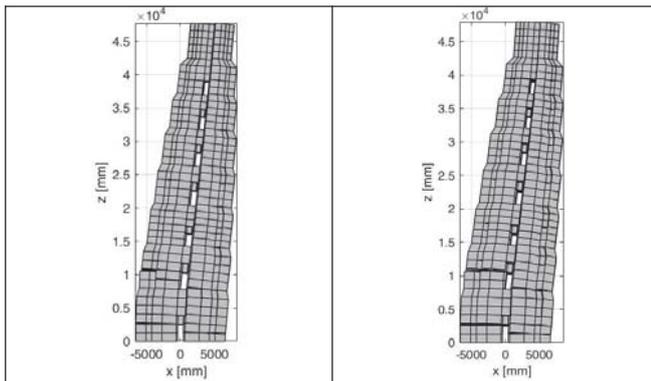
Fig. 5. Numerical results obtained with the limit analysis model

Table 2. Numerical results of G1 distribution.

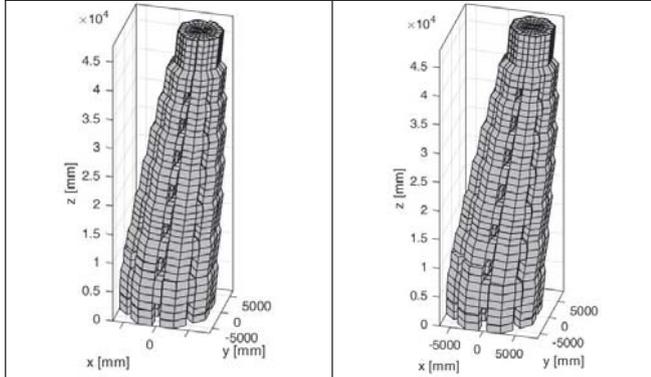
D	G1-di ag/g	A/EA[°]	D	G1-di ag/g	A/EA[°]
0°	0.1138	45°/45°	15°	0.1149	45°/45°
D	G1-di ag/g	A/EA[°]	D	G1-di ag/g	A/EA[°]
0°	0.1138	0°/0°	15°	0.1149	0°/0°

D	G1-di ag/g	A/EA[°]	D	G1-di ag/g	A/EA[°]
0°	0.1138	15°/30°	15°	0.1149	15°/30°

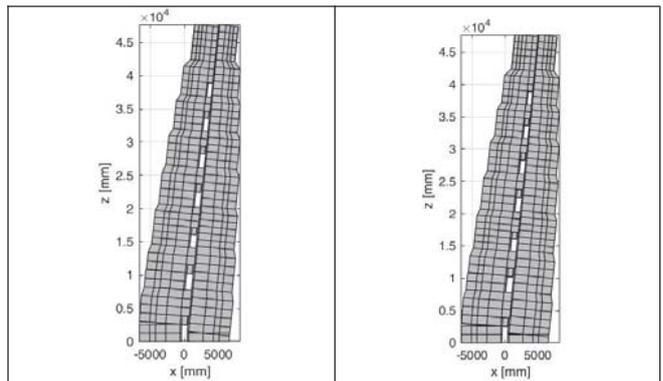
D	G1-di ag/g	A/EA[°]	D	G1-di ag/g	A/EA[°]
30°	0.1191	45°/45°	45°	0.1237	45°/45°
D	G1-di ag/g	A/EA[°]	D	G1-di ag/g	A/EA[°]
30°	0.1191	0°/0°	45°	0.1237	0°/0°



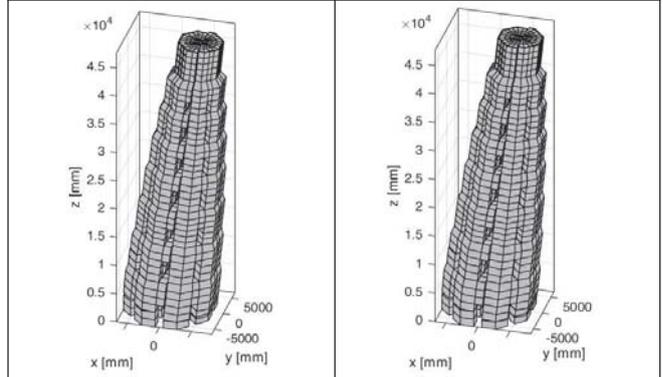
D	G1-di ag/g	A/EA[°]	D	G1-di ag/g	A/EA[°]
30°	0.1191	15°/30°	45°	0.1237	15°/30°



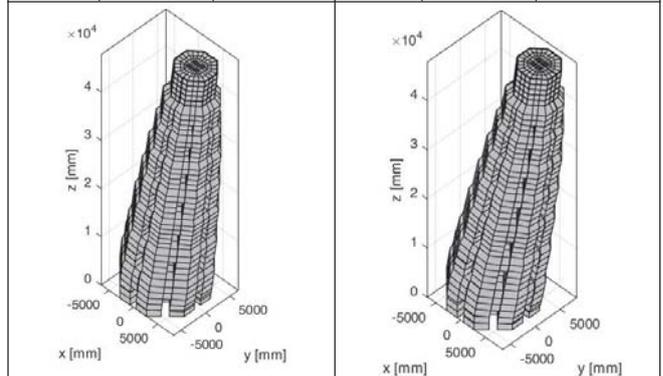
D: Direction, G2-di: G2 distribution, A/EA: Azimut/Elevation Angles.



D	G2-di ag/g	A/EA[°]	D	G1-di ag/g	A/EA[°]
0°	0.1691	15°/30°	15°	0.1706	15°/30°



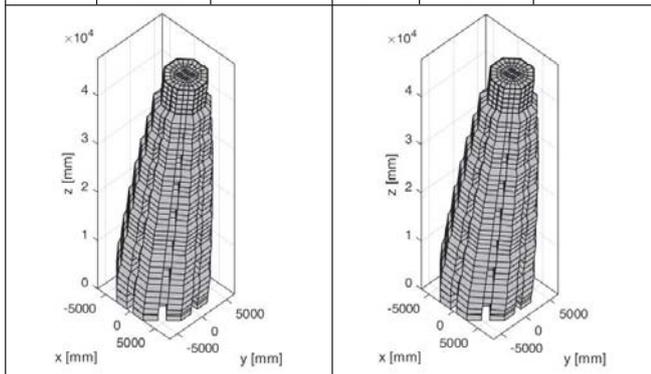
D	G2-di ag/g	A/EA[°]	D	G1-di ag/g	A/EA[°]
30°	0.1779	45°/45°	45°	0.1838	45°/45°



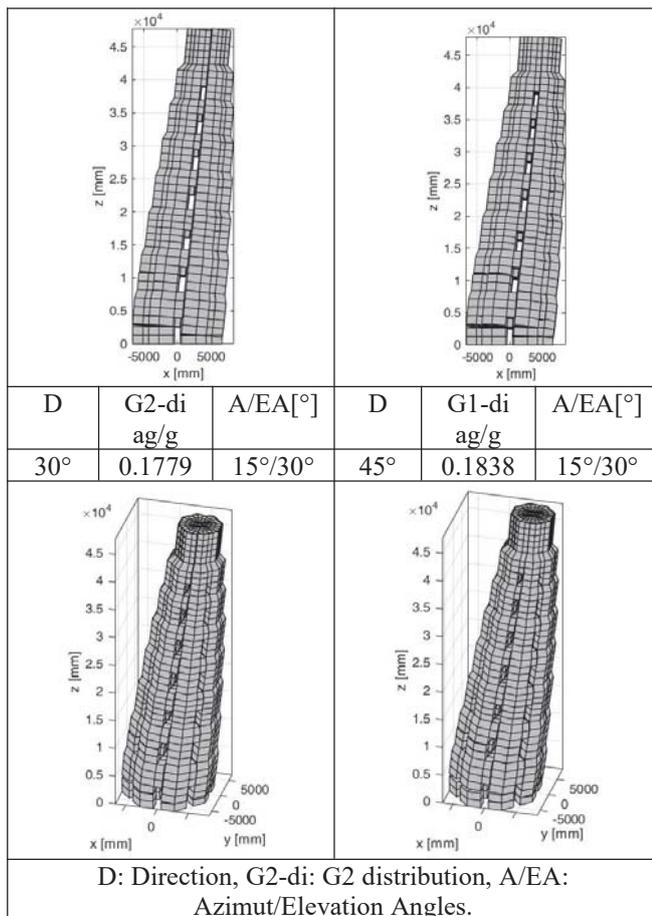
D	G2-di ag/g	A/EA[°]	D	G1-di ag/g	A/EA[°]
30°	0.1779	0°/0°	45°	0.1838	0°/0°

Table 3. Numerical results of G2 distribution.

D	G2-di ag/g	A/EA[°]	D	G1-di ag/g	A/EA[°]
0°	0.1691	45°/45°	15°	0.1706	45°/45°



D	G2-di ag/g	A/EA[°]	D	G1-di ag/g	A/EA[°]
0°	0.1691	0°/0°	15°	0.1706	0°/0°



V. CONCLUSION

This paper proposed an innovative limit analysis numerical technique combined with the distinct element method for the seismic vulnerability evaluation of a masonry pagoda in China, which required the cooperation of a commercial FE software for the pre-processing phase and a robust linear programming solver. Applying this technique to the Huqiu pagoda, it has been found that fast and reliable predictions of the collapse mechanisms triggering under the application of a variety of horizontal loads can be obtained. The applicability by common practitioners in standard engineering practice is straightforward.

ACKNOWLEDGEMENTS

The author Peixuan Wang who is studying in Politecnico di Milano was funded by the China Scholarship Council CSC, which is gratefully acknowledged by Politecnico di Milano, Department of Architecture Built Environment and Construction Engineering.

REFERENCES

- [1] H.L. Xu, Chinese Ancient Pagoda Shape, China Forestry Publishing, 2006.
- [2] D.C. Drucker. A more fundamental approach to plastic stress-strain relations. *International Journal of Solids & Structures*, 1995. 32(10): 1433-1457.
- [3] R. Hill. *The mathematical theory of plasticity*. London: Oxford University Press, 1950.
- [4] Lysmer J. Limit analysis of plane problems in soil mechanics. *Journal of the Soil Mechanics & Foundations Division*, 1970. 96:1311-1344.
- [5] Wai-Fah Chen. *Limit analysis and soil plasticity*. Elsevier Scientific Publ. Co. 1975.
- [6] Q.W. Huang. *Numerical limit analysis lower bound method based on cone programming theory and its application*. Tongji University, 2007.
- [7] A.V. Lyamin, S.W. Sloan. Upper bound limit analysis using linear finite elements and non-linear programming. *International Journal for Numerical and Analytical Methods in Geomechanics*, 2002, 12(2): 61-77.
- [8] N. Zouanin, J. Herskovits, L.A. Borges, et al. An iterative algorithm for limit analysis with nonlinear yield functions. *International Journal of Solids & Structures*, 1993. 30(10): 1397-1417.
- [9] G. Milani. Fast vulnerability evaluation of masonry towers by means of an interactive and adaptive 3D kinematic limit analysis with pre-assigned failure mechanisms. *International Journal of Architectural Heritage*, 2019, 13(7), pp. 941–962.
- [10] M. Valente, G. Milani. Effects of geometrical features on the seismic response of historical masonry towers. *Journal of Earthquake Engineering*, 2018, 22-S1, pp. 2-34.
- [11] V. Sarhosis, G. Milani, A. Formisano, F. Fabbrocino. Evaluation of different approaches for the estimation of the seismic vulnerability of masonry towers. *Bulletin of Earthquake Engineering*, 2018, 16(3), pp. 1511-1545.
- [12] M. Valente, G. Milani. Seismic assessment of historical masonry towers by means of simplified approaches and standard FEM. *Construction and Building Materials*, 2016, 108, pp. 74–104.
- [13] F. Minghini, G. Milani, A. Tralli. Seismic risk assessment of a 50m high masonry chimney using advanced analysis techniques. *Engineering Structures*, 2014, 69, pp. 255–270.
- [14] Y. Zhang. *A Study on the Shape and Style of the Pagoda of Huqiu in Suzhou*, Ms phd thesis, Dongnan university, 2008.
- [15] G. Milani. Simple lower bound limit analysis model for masonry double curvature structures. *Computers and Structures*, 2022, 269, 106831.