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3D procedural modeling of complex vaulted systems: geometric rules vs SfM based modeling

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***Abstract* – Starting from the geometric genesis of simple and complex vaulted systems, the paper proposes a workflow developing an algorithmic model for the construction of 3D digital surface model of some vaulted systems. The writing of an algorithm allows us to obtain pavilion and cross vaults digital surface models. To test the effectiveness of the models obtained in the application of the algorithms to real case studies, the ongoing phase of the research involved the survey of some complex vaulted systems based on Light Detection And Raging (LiDAR) and Structure from Motion (SfM) techniques. The models obtained by entering the data derived from the survey in the algorithm, allow a comparison between the model derived from dense matching techniques and the algorithmic model. In particular, the results obtained from the analysis of a cross vault system present in a side gothic chapel of the church of Santa Lucia in Cagliari are presented.**

I. INTRODUCTION

The 3D data acquisition and the generation of detailed 3D models are today consolidated processes in the documentation and preservation of built heritage. The development of low cost and open-source software based on image matching techniques has made possible the widespread use of so-called “digital photogrammetry” in 3D data recording and digital metric survey [1].

Geometric modeling process in HBIM environment, due to the complexity of the historical construction elements, often requires the use of local modeling. When the detected geometries are not easily traceable to libraries of pre-compiled objects, the so-called Scan-to-BIM process is a widely used solution.

Objects derived from local modeling may lose a requirement that is fundamental in the BIM philosophy, namely the modifiability and reusability of geometric components.

One of the purposes of the paper is the definition of an algorithm for constructing libraries of objects not natively present in BIM software as vaulted systems. Where local modeling is required, algorithmic modeling offers the advantage of parameterizing the modeling process: starting from a single algorithm it is possible to derive the NURBS models of different vaulted systems, each time setting the specific different values recorded in the metric survey.

Setting up an algorithmic modeling of monocentric and polycentric arcs, we pass to the modeling of the algorithm for the generation of the different types of barrel vaults deriving from cylindrical and conical surfaces, as well as vaults from revolution surfaces. Finally, an additional algorithm allows us to model pavilion and cross vaults. From these assumptions derives a workflow that, starting from the metric survey and from the transcription of the rules that govern the ideal geometry of the vaulted surfaces, allows to reach a library of simple and complex vaulted systems.

The workflow was tested in the metric survey of a ribbed cross vault system in a late Gothic church.

II. STATE OF ART

Usually, in workflows for the implementation of BIM methodologies in historical contexts, in case of particularly complex elements, local modeling is required; this means obtaining objects modeled specifically for the individual case study and difficult to re-use in other areas.

The models can therefore consist of mesh surfaces derived directly from point clouds [2] or obtained as NURBS surfaces based on profiles from sectioning meshes and point clouds [3]. Modeling is often done using external tools that offer greater specificity than native tools in BIM environments; obtained the models, these are imported into BIM software to become part of families [4] [5].

Another limitation that can be found in this methodology is the restricted context in which the objects obtained can be used, even when these are obtained entirely in the BIM environment [6].

Whatever the modeling method chosen, it will be necessary to repeat the entire modeling process for each new object to be analyzed.

The work presented proposes an alternative workflow that assigns the modeling to algorithms specifically developed according to the generating rules derived from historical treatises.

Although this can be done with external tools such as GrassHopper for Rhinoceros [7] it was decided to keep

the whole process within the BIM environment using Dynamo, an integrated tool in Autodesk Revit.
This type of approach therefore bases the modeling process on the formal, typological and historical knowledge and the elements, whose parts are modeled and assembled into local objects, but whose genesis is parametric. A single algorithm, in charge of object modeling, can then generate different objects according to set of parameters set as input; this algorithm is reusable, can be implemented with new features, and can be applied to different typological and geographical areas.

III. METHODOLOGY

The study of the historical context and of the historical treatises is an essential step to identify the rules of the constituent geometries of the specific element, without neglecting any evolutions it had undergone over time. After this first essential identification and analysis of the object, the proposed workflow starts from the ideal geometric matrices and from their transcription into the algorithm. This presupposes a functional-typological study of the historical elements and the construction techniques adopted. At the same time, survey operations are carried out using the established techniques of photo-modeling or laser-scanning. Result is a point cloud from which it is possible to obtain the mesh surfaces that approximate in detail the elements to be modeled. From the application of the knowledge gathered previously and from the segmentation of the point cloud, it is possible to obtain measurable profiles that provide the geometric information; the union of the geometric-constructive rules, derived from the treatises, with the dimensional information of the relief constitute the bases for the development of the algorithms. The aim of the work is to obtain, when possible, a single algorithm for each type of element to be modeled so as to guarantee the maximum reusability of the algorithm itself. It then starts with the development of algorithms for model the basic geometric components, that will constitute the most complex elements or from which their modeling takes shape.
The models obtained from each algorithm are therefore the elements entering the subsequent algorithm, which will develop the forms and combine them to obtain new models.
Posing as an alternative to the classic scan-to-BIM workflow, which involves local modeling, it is necessary not only to validate the model by directly comparing the geometries detected, represented by the point clouds, but also to identify and quantify the deviations both between the algorithmic model and the real object, but also between the algorithmic model and the local model; this can be done using the CloudCompare software. This step allows us to make explicit the advantages and criticality of the method in relation to the commonly applied methodology, by relating the degree of precision to the time required for survey and modeling operations and the complexity of the individual operations.

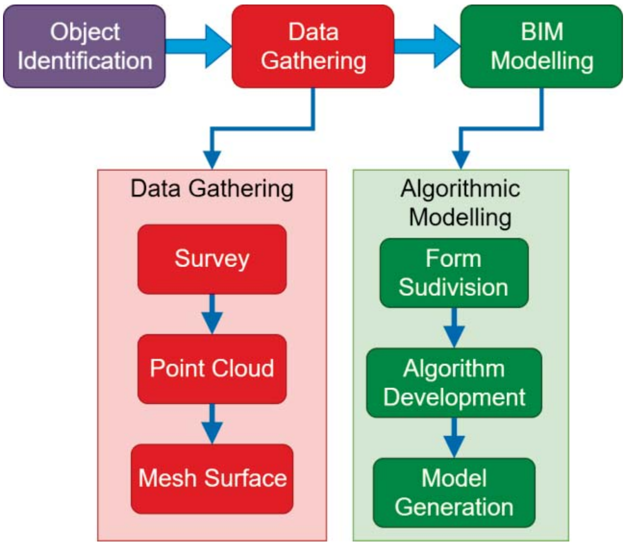

Fig. 1. Algorithmic modeling workflow

Fundamental factors to be taken into account are in fact the survey time, which can be reduced by applying geometric rules and symmetries, and the time and complexity of modeling that are practically canceled where the algorithm is applied to elements already covered. In the case of elements that present new features it is possible to use the existing algorithm as a basis for the implementation of new features.

IV. COMPLEX VAULTED SYSTEMS

The complex vaulted systems can be classified according to their geometric genesis, and in particular to the ridge line joining the keystones of the perimeter arcs and the top of the vault [8].
In cross vaults, a horizontal ridge line characterizes the vaults originating from the intersection of cylinders, thus generating single-curved surfaces, while a curved ridge line suggests a genesis of surfaces of revolution.
In the case of cross vaults resulting from straight cylindrical barrel vaults, the modeling starts from the definition of the latter, and on the arches at the base of their geometry.
The first algorithm therefore has the task of modeling arcs according to the necessary typology, that is to say that of two centers. For this model we have assumed that the centers of the semi-arches lie on the plan of the vault; this feature will then be verified by comparing the models and the point clouds.
Considering the centers lying on the plane implies the possibility of generating the pointed arches having as the only initial parameters the light and the arrow of the arcs to trace; from these parameters it is in fact possible to

obtain the precise position of the centers and build the arcs. The ogival arches thus obtained constitute the profiles that give shape to the barrel vaults, being their generatrix. Considering a cylindrical straight barrel vaults, it is not necessary to contemplate in the specific case the variants of conical or rampant vaults. Taking this into account, we can model the barrel vault as a simple extrusion having as a starting profile the pointed arch obtained as output from the previous algorithm, and as an additional input parameter the distance for which to extrude the profile. It should however be considered that if we wish to implement the vertical or horizontal inclination characteristics in the development of the vault, it is sufficient to define, besides the distance, also the direction of the extrusion to be carried out, all of this by a single algorithm.
Furthermore, since the cross vault has a nearly square plan, it is possible to omit the distance parameter, replacing it with the arch opening, a parameter already requested for the previous phases. The developed algorithm is now able to generate a barrel vault of any length starting from an initial profile.
As we know cross vaults are composite vaults that arise from the intersection of two barrel vaults, which will be the input elements for the algorithm for the generation of the new vault type. From the intersection of barrel vaults it’s possible to obtain not only the cross vaults but also the pavilion vaults, depending on which resulting parts we take, it is necessary to ensure that our algorithm is able to discriminate which, between the surfaces of the intersection, are nails and spindles.

A method to achieve this can be to use an arc parallel to the generatrix of the passing through the latter's centerline as a support element. By identifying the type of intersection between the support arch and the individual surfaces, it is possible to identify the nails, when this intersection is a single point, or the spindles, if the intersection is an arc.

"Taught" to our algorithm to recognize nails and spindles it is sufficient to select the useful surface and through a polar series go to model the cross vault; with the same procedure the pavilion vault can be obtained.

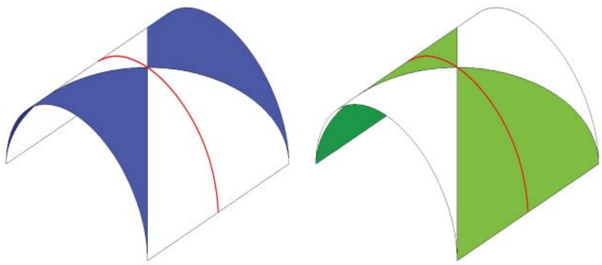

Fig. 2. Recognition of vault components

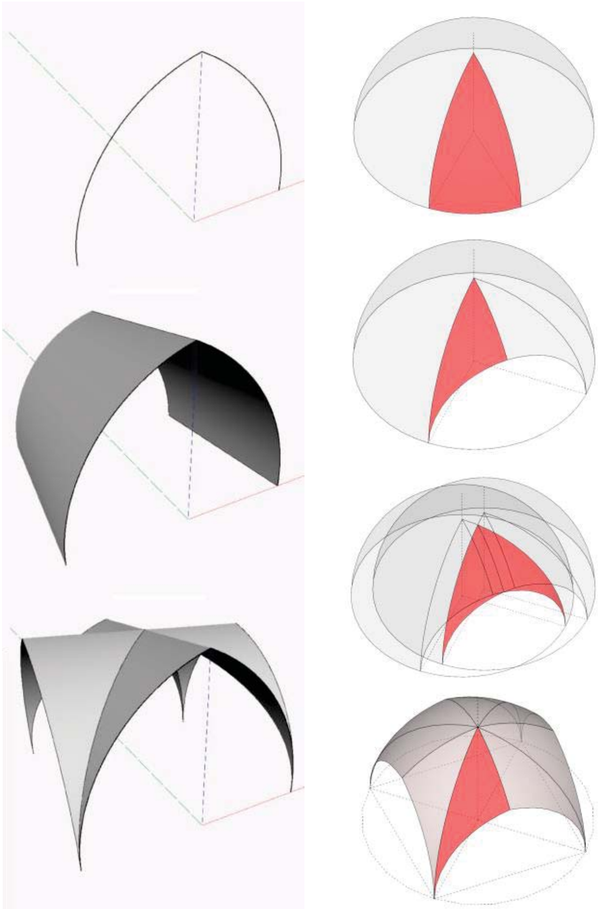

Fig. 3. Algorithm development steps for groin vault, based on translation surfaces on the left and on revolution surfaces on the right

The cross vaults originated from surfaces of revolution have as their basis the diagonal arc from whose revolution a hemisphere is generated; from the point of view of algorithmic modeling we start from the definition of this arc, which requires a center, a starting point and a final point for its design.
The center coincides with the intersection of the diagonals of the perimeter of the space to be covered, the initial point is one of the vertices of the perimeter and the final point lies on the vertical axis of the center. Once this is done, the surface is generated by the arc revolution.
Extending the sides of the perimeter it is possible to dissect the hemisphere obtaining a ribbed vault which, segmented, generates the nails of the desired vault. In the case where the cross vault has perimeter arches with two centers it is possible to imagine the nails as portions of ribs determined by the intersection of several systems like the one described. The amount of the overlap is determined by the position of the centers of the semi-arches that make up the perimeter of the vault.

V. CASE STUDY

The described method was applied during the study of the Santa Lucia church in Cagliari, referred in particular the Gothic ribbed vaulted system that covers one of the side chapels.

One of the main peculiarities of the system under examination is the configuration that provides for the covering of the room used as a chapel through the use of a complete cross vault juxtaposed to a half vault with which it shares one of the perimeter arches. From a first planimetric analysis it is possible to exclude that the half-vault can be part of a cross vault equal to the full one that joins it. This led to the choice of treating the two portions of the system as distinct elements, excluding at the moment the compilation of a linear algorithm, but preferring a development for parallel branches that deal with the modeling of full vaults and portions of them.

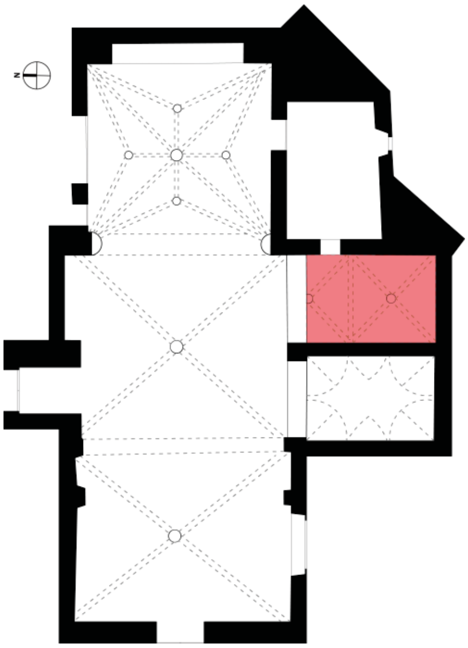

Fig. 4. Planimetric identification of the studied vaulted system

The proposed algorithmic modeling is based on the use of classical geometric-constructive schemes derived from historical treatises [9]. The starting planimetric diagram will therefore be the one that foresees once set on a quadrangular basis and point of maximum elevation identified by the intersection of its diagonals. The keys of the perimeter arches are supposed initially in a median position along the sides of the vault itself. This makes it possible to reduce the basic points for the construction to only four spring points, and thus allowing the position of the remaining points to be identified by geometric constructions.

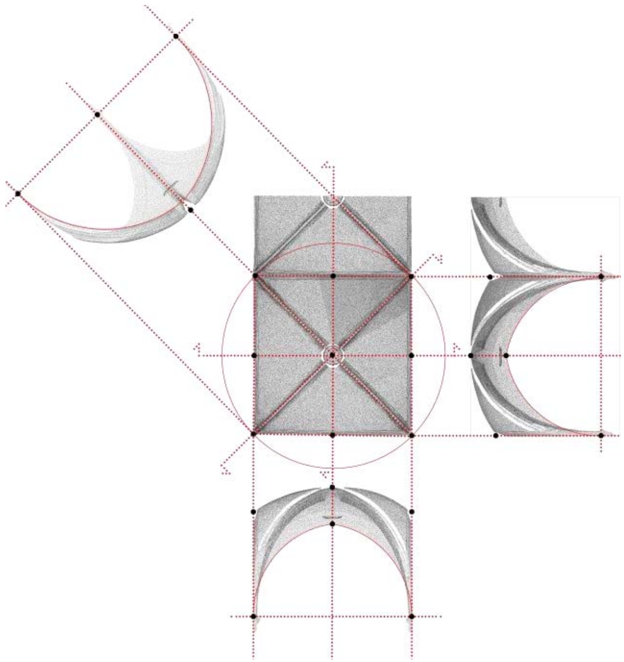

Fig. 5. Plan and sections of the point cloud of the vault. in red continuous line the constituent arcs, in red dotted line the projections of main points

Precisely this type of setting involves a different approach for the partial vault, as this last need to directly identify the position of the keystones of the two non-complete perimeter arcs. Unifying the two settings does not entail such advantages as to justify the greater complexity of the algorithm that would arise, suggesting instead a parallel development of the two components.

From the study of the point cloud it is clear that the vaults under study are composed of surfaces with multiple curvatures [10]. It is possible to see how the ridge lines, joining the keystones of the perimeter arcs with the top of the vault, have a curvilinear and non-linear shape. Examples of this typology of vaults can be found in the treatises even if with a different element: in the treatises the geometrical constructions mostly refer to a single-centered diagonal arc, often with a round arch, while in the case of vaults in Santa Lucia, these have diagonal arcs with two centers.

This characteristic has already been found in a previous case study in which algorithmic modeling was applied to complex vaulted systems: the church of Santa Maria del Monte. In that case the vault analyzed was a star-vault with five gems, with perimeter and diagonal arcs at two centers, characteristics shared with the vaults analyzed in S. Lucia.

It was therefore decided to modify the algorithm already developed for previous case and make it suitable for the modeling of the vaults now analyzed; this choice allows

the development of an algorithm suitable for the more generic case, ideal for this first phase of the research. This also demonstrates the flexibility offered by algorithmic modeling which, starting from previously studied cases, allows the evolution of the algorithm for other, different elements.
The modeling involves dividing the vault into eight surfaces that constitute it, defining each of them using the three profile arcs that delimit the surfaces. Arcs that can be defined by 3 points, start, end and midpoint; as seen above these points are identified by geometric construction starting from the four points on which the vault is set. By a regular subdivision of the initial plan shape, it's possible to define a grid of 25 point corresponding to the position where the elevation is taken.

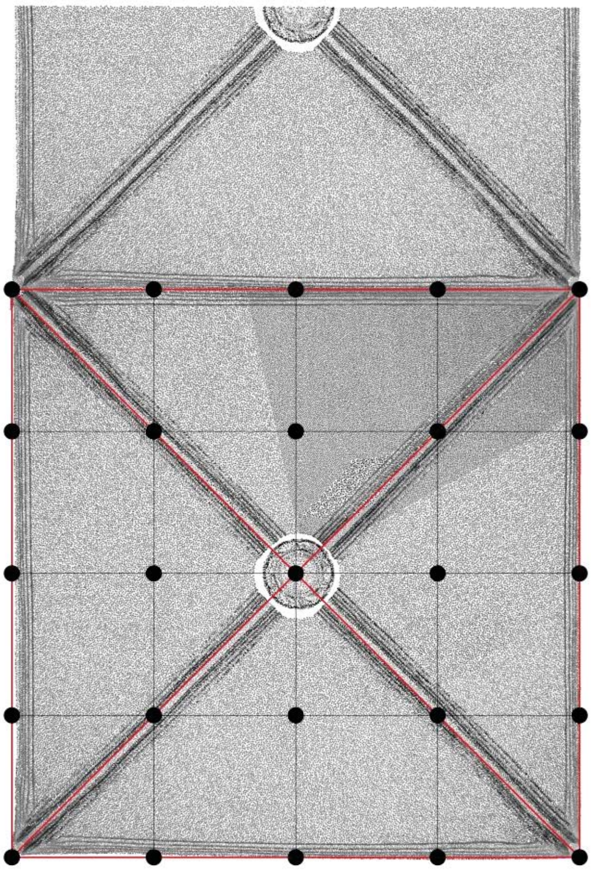

Fig. 6. Grid of the 25 reference points needed for algorithmic modeling of a groin vault

The same procedure can be repeated for the partial vault, whose base points can be identified with the two spring points in common with the complete vault and the two opposite corners, at the limits of the room. The resulting model of the complete vaulted system is reported.

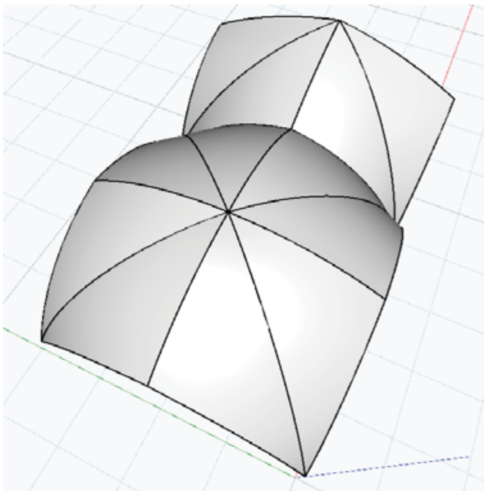

Fig. 7. Algorithmic model of the vaulted system of the side chapel in Santa Lucia

VI. CONCLUSIONS

The proposed study constitutes the first step of a ongoing research that examines complex vaulted systems based on surfaces of revolution and their implementation in a scan-to-BIM workflow through the use of algorithmic modeling. In this first part of the research we tried to demonstrate how algorithmic modeling allows us to represent complex systems [11], in particular within BIM environments; this also through a generic configuration based on the geometrical construction schemes of the elements. In the case of the church of Santa Lucia we have seen how it is possible to make an analogy between the ribs that make up the nails and ribbed vault; as an initial approach, aimed at a general typological study, it was decided to set up the modeling algorithm so as to be able to deal with a wide range of case studies. In the following phases it will certainly be necessary to apply the methodology to further similar cases, and through these will deepen the investigation concerning the double-curved surfaces on which the genesis of the studied vaults is based; particular interest will be devoted to the relationships between the sphere, hypothesized as the genesis of the ribs, and the dimensions of the covered environment, in addition to the location of its center.

In the specific case a more detailed validation of the model will be necessary, for example by computing the distances between point clouds and algorithmic models using software such as CloudCompare. Already at this stage of the research, however, it is interesting to note how algorithmic modeling allows easier study of complex geometries thanks to the simplicity of modification, the possibility of a direct comparison with the point cloud and the setting up of construction schemes linked, not to single case, but to the type of element studied.

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