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The metrological research of Machu Picchu site. Application of a cosine quantogram method for 3D laser data.

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The aim of research and metrological analysis of the Machu Picchu site is to verify the hypothesis on the functioning of the imperial system of length measurement, which were used by Incas during measurement and construction process. The results of 3D laser scanning of the Machu Picchu site provides crucial, precise output for metrological analysis. Main objective is to verify the hypothesis assuming the existence and usage of one, common imperial unit. As far as the research method is concerned, I propose using statistical model of cosine quantogram, which has successfully been employed during the analysis of architectural sites of Mediterranean culture as well as European medieval urbanism. The questions and problems of research concerning the existence and usage of base unit or units of measure are formed on the foregoing assumptions pertaining to current phenomenon of outlining and measuring Inca urban planning.

I. MACHU PICCHU – IMPERIAL INVESTMENT

During expansion of the Inca empire from about 1430 to the Spanish conquest in 1532 (chronology based on John Rowe [1]) Inca created infrastructure as well as small and large settlements like a read network, vast irrigation system, agricultural terraces, suspension bridges and many more to lunched a massive construction program which mark their presents across Andean region. One of these settlements was Machu Picchu, architectural investment generally associated with the Inca Pachacuti.

Beginning with Bingham publication [2] Machu Picchu has been identified as a fortified city, outpost into tropical forest, a sanctuary dedicated to the moon, a canter for "Chosen woman", the last refuge of Incas, a ceremonial canter. In spite of theoretical discussion about the function, a masonry of buildings in Machu Picchu are without doubts very fine and varied, without adobe bricks. These features and presents of temples and sacred places on Machu Picchu indicated that it was ceremonial centre of great importance. The state wanted to show imperial investment and firm an intention of their present in the area to the neighbouring people. Nonetheless, the settlement does not have distinctive characteristic for a city with probable population of one thousand inhabitants [3].

The settlement is divided into agricultural and urban zone with an entrance to residential area from the south (Fig.1). General layout of the settlement seems to be not achieve be chance but deliberately planned. It is visible in the space organisation of the complex where each sector has a specific border carefully arranged on *andenes* (agricultural terraces). Water system had to be planed before in order to create a water channel system to all sacred structure on Machu Picchu.

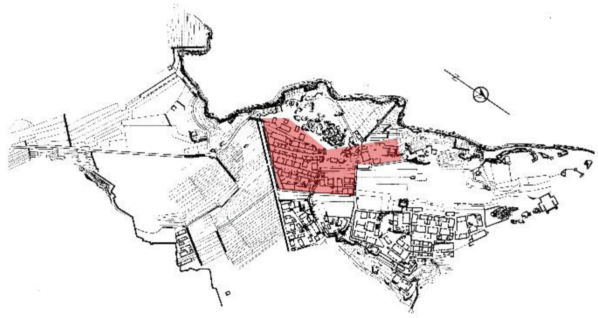

Fig. 1. Plan of Machu Picchu with a marked area of investigation, source: G. Gasparini, L. Margolies, Inca architecture, Indiana University Press, 1980, p. 89.

A. Study of basic unit - Imperial system of measurements or local tradition.

The purpose of this research is to look for a basic unit or units of measure (quantum), multiplication of which would help delineate the outline of Machu Picchu site. Did the Inca abided by the imperial measurement systems and construction rules in their urban planning or, quite the contrary, the ethnic diversity of people working on the Machu Picchu construction resulted in various metrological systems being used, which were based on the local traditions of those ethnic groups. Confirmation of this hypothesis will attest to the fact that architectural investment generally associated with the Inca Pachacuti were constructed based on metrological system imposed and supervised by imperial engineers. At the same time equally interesting is falsification of this hypothesis, where particular groups of working periodically (*mitayoq*) brought with them not only the local tradition of a stone work, but also local measures.

Inca architects/engineers and stonemasons were professionals exclude from labour tax. They were supported by the state and work on the building project but a scale of the participation in construction process of the settlements plans is not known [4]. Clay models which preserved could support the theory of previous plan of buildings and infrastructure with a standardized system of measure. However temporary working group of *mitayoq* which where use for this tape of work in great numbers could have significance influence in a process of delineating a building by using they own standard of measure.

B. Inca mathematic precision

As John Rowe said some years ago, "Information on Inca units of measurement is relatively abundant, but so scattered and unsystematised as to give the impression that the Inca had no very precise standards. Actually Inca skill in engineering works almost required a system of measurement at least as exact as that in use in 16th-century Europe" " [5].

The art of cutting stones is without doubt very astonishing achievement of precision. It gives an impression that all Inca buildings were built of cut and fitted stonemasonry, but most of the Inca building were constructed of semi cut or unworked fieldstones laid up in an argillaceous mortar. Of course finely worked stones shows an important function of a building but it not determine an interpretation. In Machu Picchu case most of the building are constructed with mortared fieldstone masonry. In this construction masonry is quite varied. On the end of a wall or for a framed of doors or niches stones were partially worked to fit into their position as well in a corners of buildings where headers and stretchers are alternated providing excellent stability. Fieldstone masonry may have been coursed or uncoursed which depends of the shape and size of stones used. In coursed masonry stone are roughly the same height. They have usually 65-90 cm thick, but most of them measure close to 80 cm [6]. The precision in Inca masonry and civil engineering was not only presents in this filed of Inca material culture. Knotted string device: *quipu* and an *abacus* – calculating device express Inca mathematical knowledge. Construction and use of *quipu* were mention in many Spanish sources as an extremely accurate accounting device. Data were encoded there using decimal system indicated by the size and position of the knots. Marcia and Robert Ascher [7] provided some ideas about Inca mathematical abilities based on their studies of quipus: arithmetic ideas used by Incas must have included at minimum: addition, division into equal parts, division into simple unequal fractional parts, multiplication of integers by integers and fractions.

II. ARCHITECTURE AND METHODS OF BUIDING LAYOUT

Area of the empire cover many different ecological zones so to conquer this varied landscape Inka introduce architectural practice which could adept to these diverse regions and climates. Alexander von Humboldt, the German naturalist from early nineteenth century described Inka architecture as: "uniformity of construction… One should think that a single architect has built this large number on monuments"[8]. In Inka architecture exist a basic form of settlement which is: single-room structure. Based on older central Andean building tradition this form was used to create a diversity of settlement types across the Inka Empire. They created variation of form by making small but important changes which allowed to enrich a function and makes an architecture highly standardized. Structural elements of Inca architecture are constantly repeated. Interior of buildings is occupied with a niches or windows arranged in a row in specific intervals, trapezoidal in shape with a shortest side on the top. They place them in all kinds of buildings not only in temples like other Andean cultures. Niches are usually situated on 120 - 150 above a floor level and their function can be different depends of their location and size. Similar shape have doorways. On both these elements Incas located a monolithic stone or wooden lintel. Last architectural detail visible inside and outside of a building structure are pegs. Outside they were used to tie a roof to a gable wall and inside probably as a hangers located symmetrically between the niches on the same high.

Distinctive characteristic of Inca walls which obviously has an influence on architectural planning is construction of battered walls. Walls have a sloping or trapezoidal cross section (from 3 to 5 degree batter) which is also often visible in a plan of a building. Function and purpose of this feature is based on statics of a building, where the thrust of the roof beams with a thick thatch roof could be more easily absorbed. Based on Wedge-stones found in situ on Ollantaytambo we can assumed that some of walls were constructed simultaneously from both ends and a working groups reached a point where was a gap to put only one more stone from the front. This type of wedge stones were set in regular way on sites like: Qorikancha, Saqsaywaman [9]. That is not a rule of wall construction, careful observation of construction features is needed for each type of wall.

According to know features of Inca architecture building layout exist at least on three levels: a foundation, a level of niches or windows and occasionally floor level where is a doorstep *in situ.* These studies are concentrate on non-invasive technics of documentation so it is not possible to reach foundation level or even establish an assume level because of not regular bedrock of the mountain. In the buildings, where the floor level is preserved by a doorstep, layout of the building is reconstructed but most of the cases it is not visible. However known level of construction remain on the elevation of niches and windows. They are wrapped around all four walls with a symmetrical distance between them. Measurements of niches and windows can be different from building to building but their arrangement on the wall is standardize. They are always oriented symmetrically on the wall with a door on the middle or two doors on equal distance, so it means that a length of the wall has to be known previously. They are two theories of a wall construction process with niches. Niles [10] presented a theory that the walls were built up to the desired height of the sills of the niches. Then a frame of niches were constructed at a desired intervals. When all the frames were placed a gaps between them were filled and additional courses of masonry laid on top **(**Fig .2**).**

For that reason types of length measurements included in the dataset were on the level of niches which most likely to have standardized measurements due to their uniformity in buildings and in some cases layout of floor level.

Fig.2 View inside the Inca building on Machu Picchu. Construction level of niches marks with a line of dots, photo: author.

III. METHODOLOGY

A. Quantum model and Monte Carlo simulations

Statistical model of cosine quantogram, has successfully been employed during the analysis of architectural sites of Mediterranean culture as well as European medieval urbanism [11]. Written sources from Greek, Roman (e.g. Isidore of Charax, Sextus Julius Frontinus, Lucius Junius Moderatus Columella) or Egyptian times gives an opportunity to proof a usage of metrological system in urbanism or architectural design. Due to the survival of measuring devices like rods is possible to determine the fundamental quantum underlying the design and construction. However for the cultures or regions where these kind of knowledge is not establish a solution is to deriving basic unit of length from an architectural dimensions and process these data using a statistical methods.

The method: cosine quantogram used in this study was developed by D. G. Kendall [12] for detecting a quantum of an unknown size from a set of data. In the case of Inca architecture each building dimensions of Machu Picchu y can be described as integer M multiples of the basic unit: q, plus error: ε.

$$y = qM + \varepsilon \tag{1}$$

The error could appear due to ancient building execution or modern method of measure. In the equation (2) ε which is significantly smaller then q is analysed and then formula calculate an amount which cluster around q. Results of the right candidate for a quantum (q) are present on a line graph as the highest peak (Fig.3). N in the formula is a number of dimensions.

$$f(q) = \sqrt{2/N}\sum_{i=1}^{n}\cos\left(\frac{2\pi\varepsilon_1}{q}\right) \tag{2}$$

Fig.3 Graph presents a quantum for sector 2B with a score 6.7

For a purpose of the research on Machu Picchu site cosine quantogram formula was implemented by Paweł Kościuk into a JavaScript to simplify calculation. All collected data in .txt format are imported with an information of a range of searching and a value of division.

Works of other scholars proof that cosine quantogram method encountered a problem if more than one quantum exist in a set of data [13]. For this reason Machu Picchu site was divided into architectural groups with the same function or construction and calculate separately.

B. The sample- Length measurements from 3d point cloud.

All the data were collected from 3D point cloud generated by 3D laser scanners and Multistation during last years and reach a final stage in this year. However for a purpose of this paper around 50% of the scanned site was investigated and another half is currently during a processing (Fig.1). In spite of this a set of data should be consider as one integral part because the area of 3D point cloud constitutes an upper part of architectural complex of Machu Picchu where lower part is on the opposite site of the Main square (*Plaza principal*). This division in Inca urbanism was present in many settlements (*hanan*-upper part and *hurin*-lower part) [14] so this studies gives an opportunity to check if a division in metrological system of this two sectors exist for this particular settlement.

3D point cloud obtain form 3D laser scanning or LiDAR as a type of documentation which gives at that moment in archaeological studies the most precise results of measure especially if an object is not regular and simple in his form. Furthermore allowed to create directly from obtain data section and geometry reconstruction when some areas are not accessible. Based on these features for a purpose of 3D documentation and these studies, Multistation Leica Nova TS60 and 3D scanner Leica ScanStation P40 was used by a team from 3D Scanning and Modelling Laboratory (Wroclaw University of Technology) and The National Archaeological Park of Machu Picchu.

Data set collected from the 3D point cloud was first transformed into vectorised plans and sections. Each plan contain desired measurements was created as a line from the pick points (Fig. 4). Data from upper part of urbanised site of Machu Picchu were divided into separate architectural groups, based on subdivision of modern site organisation (Fig.5**).**

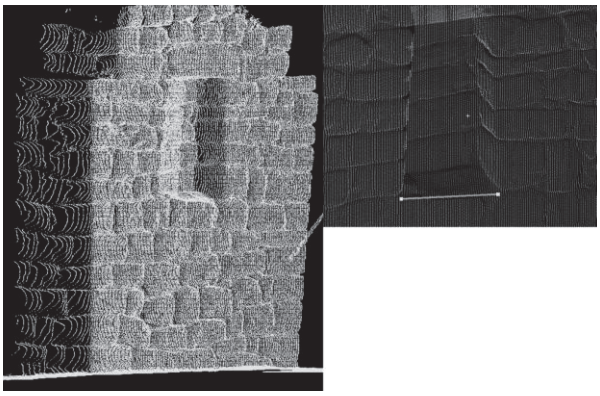

Fig. 4 Example of the 3D point cloud during a preparation of data in Leica Cyclon.

III. RESULTS

A. Comparison of Quanta between Buildings.

Residential area was divided in three groups: 2A, 2B, 3 with a fallowing numbers of measurements: 50, 84, 34. Sector with a temple of the sun was created separately and included 128 measurements. Building of so called: Royal estate has 60 numbers of length measurements.

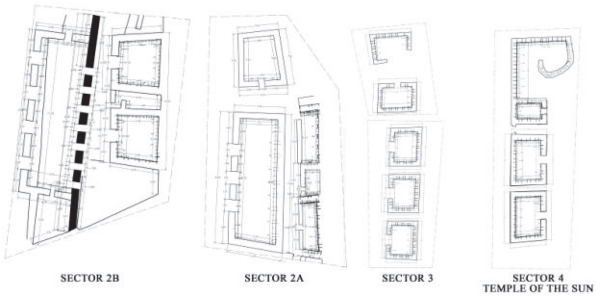

Fig.5 Architectural groups of building measurements, acc to author.

The distribution of all measurements suggests that two quanta may exist in investigated data set : 0,27 m and 0,41-0,44 m. The estimate of the quantum q for residential sector: 2A, 2B and 3 is 0.41-0,43 m (Fig.6). The estimate of q for the temple of the sun and for the building of sacred square is 0, 27 m (Fig.7). Accompanying buildings of the temple has quantum :0,42 m. For a royal estate area estimate *q* is 0,44 m.

There is no significant difference of quantum between the residential buildings or storage buildings: 0,41-0,44 m. However for sacred buildings quantum is completely different and precise: 0,27 m.

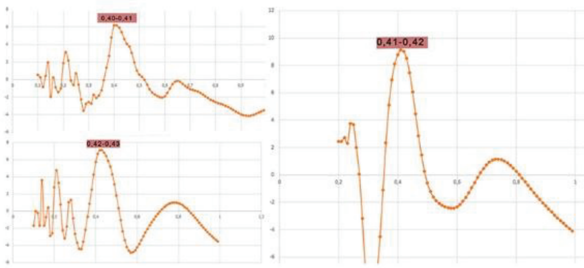

Fig.6 Results of quantum for residential sector.

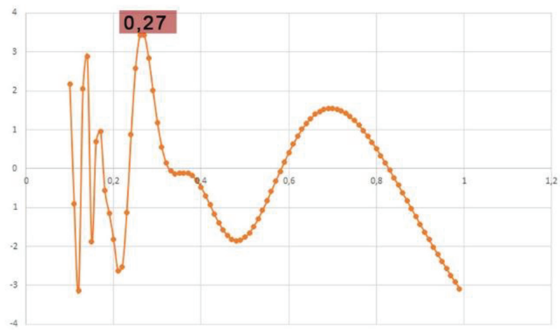

Fig.7 Results of quantum for sacred square and the building of the sun temple.

B. Anthropological measurements and quantum results

In order to exclude senseless quanta, lower and upper bounds were placed into a program to limited value of quantum :0,02- 1,60 metres. Measurement systems based on body proportions are still common in Latin America. The smallest unit of measure is a finger: between 0,05-0,06 m and upper bounds capture the estimate value of fathom: 1,62 m [15].

Result of quantum could be compared with body proportion of cubit (quechua: *khococ*) [16]. as a distance around 0,45 m. For 0,27 m does not exist equivalent in human proportion in Latin America, because length of foot is usually 0,25 m. It is possible that information about this unit of measure does not exist in Hispanic sources but it fits as a part of fathom measure:6*0,27=1,62 m.

Throughout history of civilization, the system of urban planning defined the level of development. The search for a basic unit of measure is inextricably connected to human existence on earth, as we construct the surrounding environment on the base of our body proportions. This phenomenon shall complement our knowledge of Machu Picchu, as well as other sites on Inca territory, in terms of function and the process of creation.

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