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Quantum two-phase estimation inside a photonic integrated device

Mauro Valeri[1], Emanuele Polino[1], Martina Riva[4], Raffaele Silvestri[1], Davide Poderini[1], Ilaria Gianani[1,2], Giacomo Corrielli[3,4], Andrea Crespi[3,4], Roberto Osellame[3,4], Nicolò Spagnolo[1], Fabio Sciarrino[1]

[1]*Dipartimento di Fisica, Sapienza Università di Roma, Piazzale Aldo Moro 5, I-00185 Roma, Italy, fabio.sciarrino@uniroma1.it*
[2]*Dipartimento di Scienze, Università degli Studi Roma Tre, Via della Vasca Navale 84, 00146, Rome, Italy*
[3]*Istituto di Fotonica e Nanotecnologie, Consiglio Nazionale delle Ricerche (IFN-CNR), Piazza Leonardo da Vinci, 32, I-20133 Milano, Italy*
[4]*Dipartimento di Fisica, Politecnico di Milano, Piazza Leonardo da Vinci, 32, I-20133 Milano, Italy*

***Abstract* – Quantum Metrology aims at exploiting quantum resources to enhance the performances in measuring physical quantities. Photonic implementation easily allows the experimental investigation of these tasks. This work presents a photonic device able to demonstrate quantum simultaneous estimation of optical phases, surpassing results achieving by classical strategies. The chip realizes a reconfigurable three-mode interferometer, fabricated by using the advanced femtosecond laser writing technique. The high degree of tunability is given by the presence of optical phase shifters, which makes the device useful both for investigating multiparameter quantum metrology, and for studying how machine learning techniques can improve the learning process.**

I. INTRODUCTION

Quantum phase estimation is a benchmark scenario in quantum metrology. Indeed several physical systems can be mapped in a phase estimation problem [1]. For the single-phase estimation the goal is to measure a phase shift between two modes. In a more general and complex scenario the number phases (modes) can be more than one. A general scheme of multiphase estimation is reported in Fig.1. The estimation process can be represented by a cycle repeated N times, which consists of three main steps:

1. preparation of a probe states ρ_0 as sensitive as possible to the parameters $\boldsymbol{\phi}$ to be estimated. In phase estimation scenario a $(n+1)$-mode interferometer is able to encodes n independent relative phases $\boldsymbol{\phi} = (\Delta\phi_1, \Delta\phi_2, \dots, \Delta\phi_n)$ (one mode works as reference).
2. The interaction between the probe and the physical system for encoding unknown phases over the probe state: $\rho_0 \to \rho_{\boldsymbol{\phi}}$.

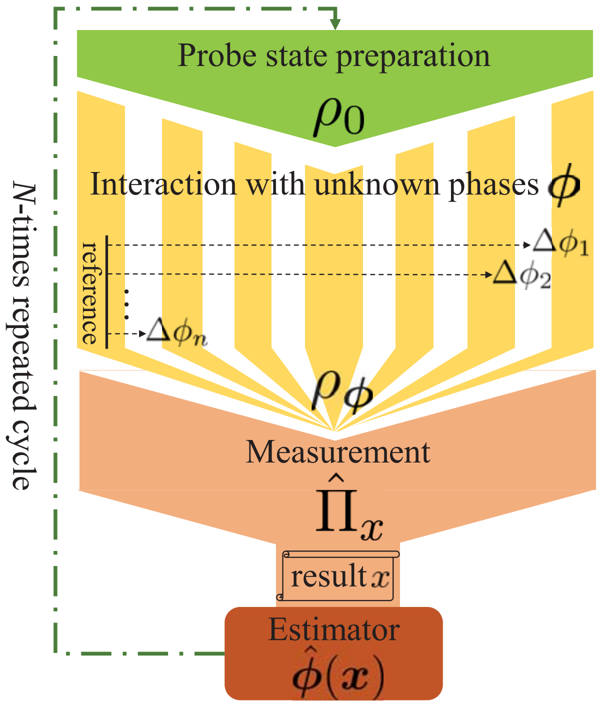

Fig. 1. Generic scheme of multiphase estimation. The learning process is the succession of several steps: interaction between a prepared state act as probe to catch information on unknown phases. Finally the estimator function provides an estimation of the parameters after last measuring result x, and depending by all the previous ones.

3. Final measurement $\hat{\Pi}_x$ and estimation of the parameters by exploiting a suitable estimator function $\hat{\boldsymbol{\phi}}(\boldsymbol{x})$, which at iteration k depends on all previous measurement results $\boldsymbol{x} = (x_1, x_2, \dots, x_k)$.

When we set the apparatus and probe state, the best estimator function provides an estimation with a precision, expressed in terms of covariance matrix $\mathrm{Cov}(\boldsymbol{\phi})$ of the unknown parameters, ruled by Cramér-Rao bound (CRB) [1]: $\mathrm{Cov}(\boldsymbol{\phi}) \geq \mathcal{F}^{-1}/N$, where $\mathcal{F}$ is the Fisher Information matrix of the system. The Fisher Information matrix depends on the likelihood function $p(x|\boldsymbol{\phi})$, describing the probability to measure a certain possible measurement outcome x, given the parameters $\boldsymbol{\phi}$. Maximizing estimation process over all possible quantum measurement, a tighter precision bound is given by the Quantum Cramér-Rao bound (QCRB) [1]:

$$\mathrm{Cov}(\boldsymbol{\phi}) \geq \mathcal{F}^{-1}/N \geq \mathcal{F}_Q^{-1}/N \tag{1}$$

where $\mathcal{F}_Q$ is called quantum Fisher Information matrix, which represents the amount of information encoded in the probe state. The QCRB depends only from the state of the probe $\rho_{\boldsymbol{\phi}}$ after the interaction and there is a crucial difference between quantum and classical states [1]. Quantum states can outperform classical ones, reaching the ultimate quantum limits. In this way there is a precision region achievable only by exploiting quantum probes, meaning that a quantum enhancement in the metrology task can be obtained. In single parameter scenario the inequality (1) represents a bound on the variance $\mathrm{Var}(\phi)$ of the estimation. Here, the ultimate precision achievable by using classical resources is the standard quantum limit (SQL), which reads $\mathrm{Var}(\phi) \geq 1/N$. Conversely, the ultimate quantum limit is represented by Heisenberg limit (HL), which scales as $\mathrm{Var}(\phi) \geq 1/N^2$, showing an enhancement by factor N respect to SQL. When more than one unknown parameter is involved, we are within the context of multiparameter metrology. In this case the QCRB is a matrix inequality, containing also correlation terms between parameters. A simplified version can be obtain by looking at the overall variance by tracing the inequality (1): $\mathrm{Tr}[\mathrm{Cov}(\boldsymbol{\phi})] \geq \mathrm{Tr}[\mathcal{F}^{-1}]/N \geq \mathrm{Tr}[\mathcal{F}_Q^{-1}]/N$. While single parameter estimation is well known both from a theoretical and experimental point of view, multiparameter estimation has still open questions [1]. Moreover very few experiments have been realized in estimating more than one parameters. Thus, it is really important to develop quantum sensors able to investigate multiparameter estimation scenario. In this work we present a novel platform that we use to experimental demonstrate the first simultaneous multiphase estimation with performance better than any classical approach. The implemented device is a multiarm interferometer, realized by femtosecond laser writing technique [2]. Here, an ultrafast laser is focused on a glass substrate and optical waveguides are written in the focus of the radiation by translating the substrate respect to the laser beam. Another important task is the realization of learning protocols allowing to reach the highest precision possible using few probes, or when we are constrained in using limited numbers of resources. Indeed, CRB represents the most common bound for the error of the estimate and is theoretically achievable only asymptotically, thus using in general a lot of resources. Here, machine learning techniques represent a useful tool to investigate how to improve the estimation process.

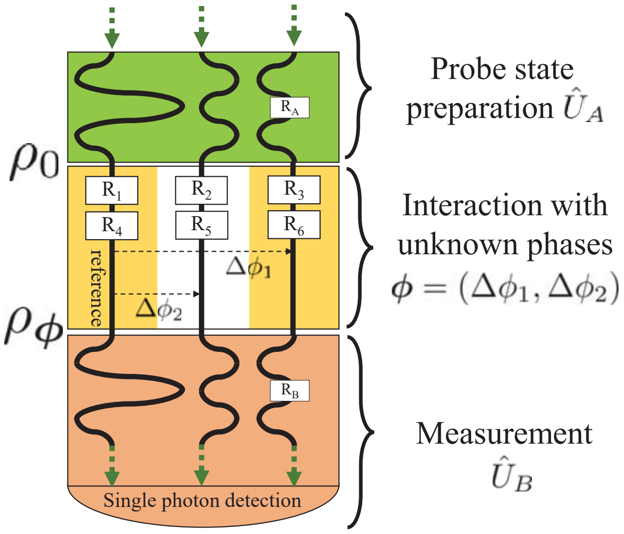

Fig. 2. Scheme of the circuit of the employed photonic device. Two external tritter operators realize preparation and measurement steps of the learning scheme. The tritter is realized with a two dimensional decomposition combining three directional couplers. The reconfigurability of the chip is achieved thanks to the presence of thermo-optics phase shifters: the dissipated power on resistors along the internal arms allow regulation of two independent phase shifts, with respect to the reference arm.

II. PHOTONIC DEVICE

Interferometers represent a natural framework for studying phase estimation problems. The standard scheme to optimally measure a single phase is the Mach-Zehnder interferometer, where it is possible to achieve the ultimate limit in estimation precision [1]. In presence of multiple phases the natural generalization is represented by multimode interferometers. In our case, we adopt an integrated device which realizes an estimation scheme suitable for two phase-estimation composed by two cascaded tritter operators and internal phase shifts (see Fig. 2). Each tritter consists of a tunable phase-shifter and three directional couplers, two of which are balanced and one is unbalanced. The external operators represent the preparation of the probe and the final measurement, respectively. Considering one arm as reference, two independent differences of phases are encoded by the photonic states passing through three internal arms, namely $\boldsymbol{\phi}^{(tot)} = (\Delta\phi_1^{(tot)}, \Delta\phi_2^{(tot)})$. The optical device is high reconfigurable thanks to the presence of ohmic resistors placed above the waveguides. When the power P_{R_i} is dissipated by resistence R_i, the heat propagates around the waveguide and the optical phase shift ϕ_{R_i} is changed depending on the dissipated power. In a detailed characterization of this response, linear and non linear dependence on P_{R_i} can be considered, together with cross-talk contributions by couple of resistors ($\propto P_{R_i} P_{R_j}$) [3, 4]. Our chip has several independent phase shifters, which allow us to set not only the unknown phases, but also to tune each part of the learning scheme. Consider now, the quantities $\boldsymbol{\phi}^{(L)}(\{R\})$ and $\boldsymbol{\phi}^{(NL)}(\{R\})$, representing respectively the linear and non linear contributions of $\boldsymbol{\phi}^{(tot)}$ introduced by the power dissipation on the set of resistors $\{R\}$. The resulting phase shift caused by multiple action of all resistors reads $\boldsymbol{\phi}^{(tot)} = \boldsymbol{\phi}^{(L)}(\{R\}) + \boldsymbol{\phi}^{(NL)}(\{R\})$. This relation is composed of several contributions and in particular, it is possible to divide the global phases coming from dissipated powers in two groups. We associated a first layer to the unknown phases $\boldsymbol{\phi} = (\Delta\phi_1, \Delta\phi_2)$, while the remaining other terms as control phases $\boldsymbol{\Phi} = (\Delta\Phi_1, \Delta\Phi_2)$, thus giving $\boldsymbol{\phi}^{(tot)} = \boldsymbol{\phi} + \boldsymbol{\Phi}$. The additional terms $\boldsymbol{\Phi}$ can be exploited as feedback phases to be regulated according to a particular heuristic method, in order to improve the estimation of the unknown phases $\boldsymbol{\phi}$.

III. CHARACTERIZATION OF THE QUANTUM SENSOR

The precise calibration is a fundamental step in order to exploit the sensor for the quantum phase estimation. The complete characterization of the response of each internal resistor is made by injecting single photons in the input i and measuring output probability distribution $P(i \to j)$ along the output arm j ($i, j = 1, 2, 3$). During these measurements the single resistor is studied independently, while tuning-off the other resistors of the chip. In this way any input-output combination is recorded as function of the power dissipated by the single resistor. The acquired data are fitted by a complex model which consists of several independent parameters, according to the relation between the power dissipated and the phase shift generated, considering the expected unitary operator of the chip. Among them can be distinguished static and dynamical relevant parameters. The former are given by the offset phases $\Delta\phi_{10}^{(tot)}, \Delta\phi_{20}^{(tot)}$ obtained when no power is dissipated by resistors and the transmittivities of the six directional couplers presented in the external operators. The latter are the coefficients at first and second order associated respectively to the linear and non-linear phase dependence from the power dissipated by resistors. Some results are reported in Fig.3. The goodness of the calibration is certified first from the χ^2-value, which is comparable with the number of experimental data. Then, the model of the circuit, extracted from the fitting process, is employed to correctly predict the experimental output probabilities also when two resistors are simultaneous active. Indeed, a

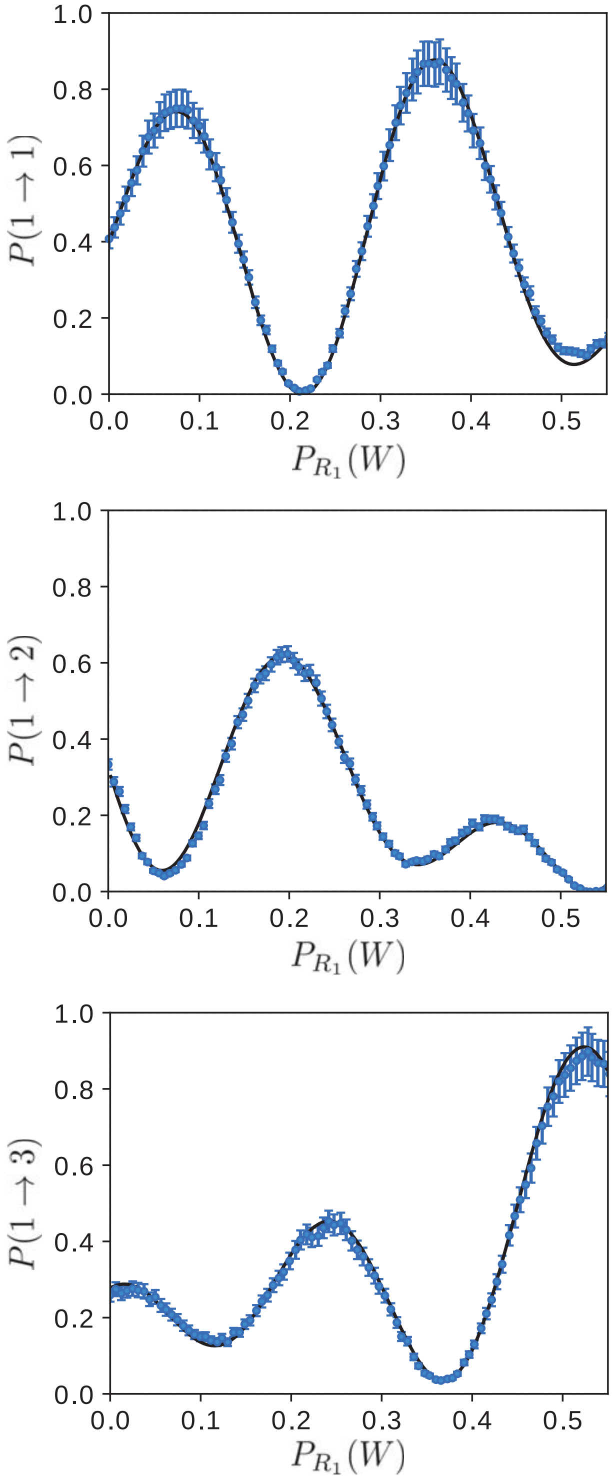

Fig. 3. Examples of single-photon input analysis by dissipating power P_{R_1} over a single internal resistor R_1. The fitting curve (lines) show an optimal agreement with the experimental data (dots).

good description using only one resistor does not guarantee the correct response when voltages over multiple resistors are applied. Notably, this check is made also using two indistinguishable photons as input, in order to guarantee the right functionality also in quantum regime, preserving quantum properties. In this way, we demonstrated the quality of our calibration, reaching and certifying a full control of the reconfigurability of the chip and its suitability for quantum metrology tasks.

IV. EXPERIMENTAL RESULTS

Reaching the bound (1) in an estimation process, requires the knowledge of the correct likelihood function of the whole apparatus seeded by photonic probes. For this reason it is of paramount importance a fine characterization of the device as a function of the voltages applied to different resistors, as discussed in the above paragraph. From the characterization results we computed the reconstructed Fisher Information matrix $\mathcal{F}_{\text{exp}}$ of the system as function of the internal set pair of phases $\phi^{(tot)}$, and thus the best precision achievable according to (1). The resulting Fisher shows regions of phases space where achievable precision can be improved by quantum resources. These regions can be extended to the overall phase interval $[0, 2\pi] \times [0, 2\pi]$ by exploiting adaptive protocols. In a first work [3], we demonstrate quantum enhanced performances of the system by injecting two indistinguishable photons as input states. Here, after setting $\mathbf{\Phi} = \{0, 0\}$, we simultaneously estimated the pair of phases using a non-adaptive technique based on maximum likelihood estimator. The maximum likelihood provides the estimation $\hat{\phi}$ which maximizes the total probability of having the two-photon events measured during the estimation process. The results are reported in Fig.4, showing that CRB below any classical strategies is achieved. Then we studied an online adaptive estimation scenario, exploiting a Bayesian estimator. The Bayesian framework is a natural solution for adaptive estimation approaches, due to its capability to update knowledge depending on previous obtained information. Here, the posterior distribution $p(\phi|\mathbf{\Phi}, x)$ resulting of the Bayes' rule, contains the information necessary to give an estimation of the parameters. A suitable estimator function is represented by expected value of the posterior: $\hat{\phi} = \int d\phi\, \phi\, p(\phi|\mathbf{\Phi}, x)$. This estimator is efficient and unbiased, able to asymptotically achieve the CRB [1]. In the adaptive protocol the circuit is tuned, controlling $\mathbf{\Phi}$, after each estimation cycle according to certain heuristic. Using an optimized heuristic, supported by a machine learning technique, we demonstrated in [4] the adaptive estimation of pairs of phases, saturating the CRB with few adopted resources. The adaptive strategies show improved performances, setting at each step the sensor in the optimal working point to estimate the parameters, thus resulting the optimal choice in limited data regime.

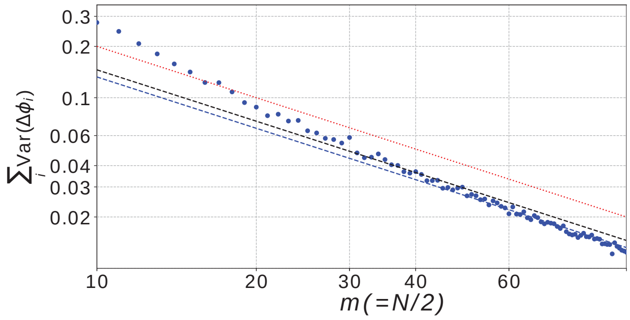

Fig. 4. Experimental non-adaptive simultaneous two-phase estimation. The phases pair is chosen in a region of quantum enhancement, that is achievable when two indistinguishable photons probe the system. Resulting measured overall variance (blue dots) is plotted as function of injected couples of photons m. The quantum limit (blue dashed line) is asymptotically reached and, after injection $N = 80$ photons ($m = 40$ pairs of indistinguishable photons), both classical bounds are surpassed, corresponding to simultaneous (black dashed line) and separated (red dashed line) estimation scenario with distinguishable photons.

V. OUTLOOK

Our investigations demonstrate the capability of an integrated multiarm interferometer for studying quantum multiphase estimation. Firstly we demonstrated simultaneous quantum enhanced estimation of two phases, considering as employed resource, the number of detected coincidences. Then we realized an optimal adaptive Bayesian protocol, able to approach the ultimate bound on the estimation of two phases, using only few single photons. In this way, we demonstrated how machine learning techniques are able to improve the estimation processes also in a multiparameter scenario. Furthermore, we recently demonstrated an other adaptive strategy which shows optimal performances in the estimation problem of a single phase [5]. Also here the machine leaning tool has represented an optimal support for achieving the phase estimation task. The next step of this study aims at demonstrating quantum enhanced performances in online adaptive scenario, injecting multiphoton probe states [4]. Other improvements of CRB are related to increasing the number of involved modes, by exploiting an interferometer with a higher number of arms [3].

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