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A CS method for DAC nonlinearity testing

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Abstract – **High resolution Digital-to-Analog Converters (DACs) are known to be characterized by static testing procedures with a remarkable duration, due to the huge number of employed codes. This paper proposes a method that allows to reconstruct the Integral Non-Linearity through the Compressed Sensing, by measuring the DAC output on a reduced number of codes. The proposed reduced-code test is evaluated for several compression ratios, turning out well performing in terms of Root Mean Square Error.**

I. INTRODUCTION

Over last years, research has been attempting to propose a new generation of data converters, to address traditional limits imposing high sampling rates counterbalanced by low accuracy and resolution. Such compromise between sampling frequency and resolution is reached through several prototypes of Digital-to-Analog Converters (DACs) [1]–[5].

Evaluating the performance of high accuracy DACs requires generally a long-term characterization. DAC testing methods can be either static or dynamic. Static testing provides specifications on DAC non-linearity [6], i.e. Integral Non-Linearity (INL) and Differential Non-Linearity, while figures of merit such as Spurious Free Dynamic Range or Total Harmonic Distortion are pointed out only by dynamic testing [7]. Especially in case of static testing, long times are needed, because of the number of codes to be tested, that is strictly related to the DAC resolution. Indeed, increasing the time required for testing means increasing also their cost of production.

Traditionally, in order to lower the duration of static testing, built-in self-test solutions have been proposed in literature [8], [9]. Unfortunately, built-in self-test cannot be applied to already existing DAC architectures. Less attention has been instead paid to devise new techniques for static testing. In [10] an algorithm based on a segmented model of the INL is proposed. In detail, the INL is approximated, for a given length of the input code, as sum of three errors associated with as many bit segments. The model consists, in fact, of three contributions associated with the most, intermediate and least significant bits of the input code, reducing the number of unknowns to be estimated. Moreover, testing time per sample to measure the DAC output is reduced by adopting, instead of a highly accurate voltmeter, an onboard digitizer, whose nonlinearity is compensated by the algorithm itself [10]. It should be noted that, although this method is independent of the DAC architecture to be tested, *a priori* knowledge of the architecture is required to segment the input code. In fact, in non-segmented DAC architectures the code segmentation limits the INL model segmentation.

In this paper, a method to reduce the duration of DAC static characterization is presented. Since the duration of linearity testing is related to the number of input codes, the proposed method is based on the idea of reducing the employed codes to characterize the DAC output. The INL curve is then reconstructed from the reduced codes by exploiting the Compressed Sensing (CS) technique. A considerable part of duration of the testing procedure is reduced by this way. For example, let linearity of a 16-bit DAC need to be tested. If testing requires $10\,\mathrm{s}$ for each of the $2^{16} = 65536$ input codes to be tested, the overall duration is about 182 hours, corresponding approximately to an one-week characterization procedure. Instead, by reducing the codes to be tested by a factor of 10, the overall duration is reduced to slightly below 18 hours for testing, with only additional $20\,\mathrm{s}$, required by the CS-based algorithm employed to reconstruct the INL curve. Compared to the algorithm in [10], the method in this paper is more general, as it does not need the knowledge of the DAC architecture before being implemented and it does not reduce testing time per sample through an onboard digitizer.

The paper is structured as follows. Section II describes the proposed method to test the INL curve. Section III reports the evaluation of the proposed method on a behavioral model of a commercial DAC and discusses the obtained results. Finally, conclusions and future developments are drawn in Section IV.

II. DAC STATIC TESTING

II.A. Conventional procedure

Static testing to evaluate the INL curve in DACs is based on an all-code characterization, since it requires to be implemented on all the input codes [11]. The conventional procedure for DAC static testing can be briefly summarized as follows. As depicted in Fig. 1a, the output voltage of an N-bit DAC is measured for the set of codes:

$$S = \{c_k\}_{k=0}^{K-1}, \tag{1}$$

with $K = 2^N$. Generally, the measured output voltage $U[k]$, corresponding to the input code c_k, deviates from

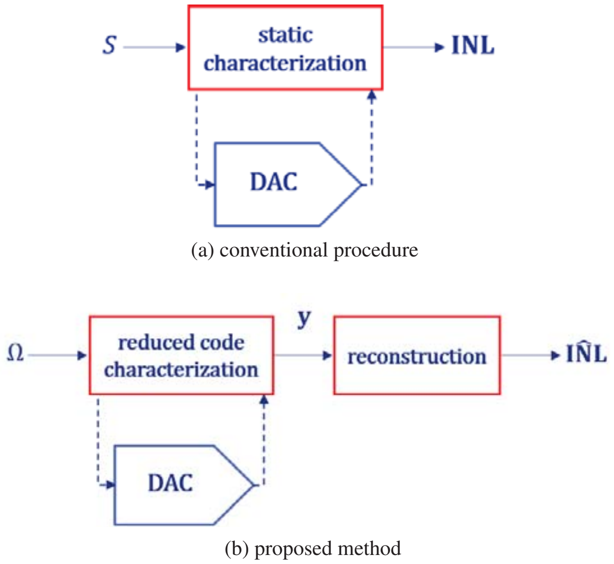


Fig. 1. (a) All–code method and (b) proposed method for DAC static testing.

the ideal output voltage at the same input code of a residual error (after having compensated offset and gain errors), that represents the linearity error:

$$\varepsilon[k] = GQk + U[0] - U[k], \tag{2}$$

where G is the gain, and Q is the width of a code bin, that is the least significant bit. Then, the INL, corresponding to the input code c_k is evaluated according to:

$$INL[k] = \frac{\varepsilon[k]}{KGQ}, \tag{3}$$

that, in vector form, becomes:

$$\mathbf{INL} = \frac{1}{KGQ}[\varepsilon[0], \varepsilon[1], \ldots, \varepsilon[2^N - 1]]^T. \tag{4}$$

Therefore, in the conventional procedure, the INL curve is evaluated on each of the K input code levels.

II.B. The proposed method

The proposed method is intended to reduce the number of codes to be employed in DAC characterization for linearity testing. To this aim, the CS technique [12] is adopted to estimate the INL curve. Thus, let the proposed reduced-code method be described by following the paradigm of CS.

First of all, a reduced number of codes $M < K$ is randomly chosen (without repetition) for the DAC static characterization. As represented by the block diagram of Fig. 1b, the DAC output voltage $U[m]$ is measured in correspondence of the input code $c_m \in \Omega$, where Ω is a subset of S (1), consisting of the M randomly selected elements.

The INL evaluated at the only M codes can be modeled as the result of a linear transformation. Specifically, the M-size vector of the INL values at the selected codes is expressed as:

$$\mathbf{y} = \mathbf{\Phi} \cdot \mathbf{INL}, \tag{5}$$

where $\mathbf{\Phi}$ is the *measurement matrix*, i.e. an $M \times K$ rectangular matrix, that represents the selection process. The measurement matrix is in fact the following:

$$\mathbf{\Phi} = [\mathbf{I}]_{\Omega,:}, \tag{6}$$

where $\mathbf{I}$ is the $K \times K$ identity matrix and $[\cdot]_{\Omega,:}$ is a restriction operator that selects the rows of the matrix $\mathbf{I}$ according to the indices of the codes contained in Ω. In other words, the random selection reduces the rows of the identity matrix $\mathbf{I}$ by a factor K/M, named Compression Ratio (CR). The advantage of the proposed method consists in reducing the duration of static testing by directly reducing the codes employed for characterization. For example, in case of a 14-bit DAC, the conventional linearity testing requires $K = 16384$ codes. Instead, by considering the proposed method with a $CR = 8$, the number of codes to be employed to evaluate the INL curve is drastically reduced to $M = 2048$.

The proposed method rests on the assumption that, in a proper domain, the K-size vector $\mathbf{INL}$ is L-sparse, namely it is represented by a limited number $L \ll K$ of non-null elements. If the INL curve can be modeled by a small number of frequency components, similarly to the approximation proposed in [13] for ADCs, the assumption turns out verified in the Fourier Transform domain and the vector (4) can be expressed as:

$$\mathbf{INL} = \mathbf{\Psi c}, \tag{7}$$

where $\mathbf{\Psi}$ is the inverse Discrete Fourier Transform (DFT) matrix and $\mathbf{c}$ is a coefficient vector that is sparse in the basis $\mathbf{\Psi}$.

In accordance with the CS paradigm [12], the coefficient vector $\mathbf{c}$ can be reconstructed from the vector $\mathbf{y}$, containing the INL values corresponding to the reduced codes, the random measurement matrix (6) and the inverse DFT matrix $\mathbf{\Psi}$ chosen as sparsity basis. Thus, the sparsest solution for the coefficient vector $\mathbf{c}$ is calculated by solving the following ℓ_1-norm minimization problem:

$$\hat{\mathbf{c}} = \arg\min_{\mathbf{c}} \|\mathbf{c}\|_1 \quad : \quad \|\mathbf{\Phi\Psi c} - \mathbf{y}\|_2 \le \epsilon, \tag{8}$$

where ϵ is a positive threshold, that bounds the error committed in the ℓ_2-norm.

Finally, the INL is reconstructed as:

$$\hat{\mathbf{INL}} = \mathbf{\Psi}\hat{\mathbf{c}}. \tag{9}$$

Definitely, the method here proposed can be summarized in four main steps: (i) random selection of codes

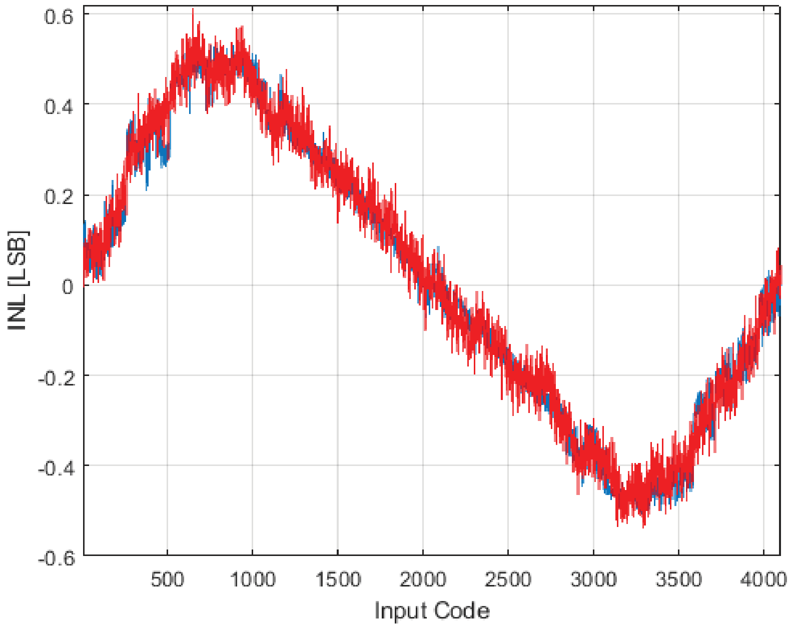


Fig. 2. Comparison between INL curves: in blue the INL evaluated at all the codes and in red the INL reconstructed from the codes reduced according to $CR = 16$.

according to a given CR from the all-code set and characterization of the DAC voltage outputs corresponding to the reduced number of codes, (ii) evaluation of the INL from the measured DAC outputs, (iii) recovery of the coefficients in the Fourier domain through the ℓ_1-norm minimization problem and (iv) reconstruction of the INL in the original domain from the computed coefficients.

III. NUMERICAL ANALYSIS

III.A. Test implementation

The proposed method for static testing based on random selection and reconstruction, as detailed in Subsection II.B, is compared to the all-code method described in Subsection II.A. The all-code method and the reduced-code method are both numerically implemented in MATLAB environment. The testing is carried out on the behavioural model of the 4–channel Analog Devices AD9144 DAC, available online from the Analog Devices website [14]. Specifically, among the 16 bits of resolution of the behavioural model, only the most significant 12 bits are considered.

Firstly, the all–code method, illustrated by Fig. 1a, is implemented. For this static characterization, a constant value corresponding to each DAC code is given as input to the behavioural model. Then, the output of the behavioural model is recorded for all the input codes and the vector $\mathbf{INL}$ is evaluated according to Eq. (4).

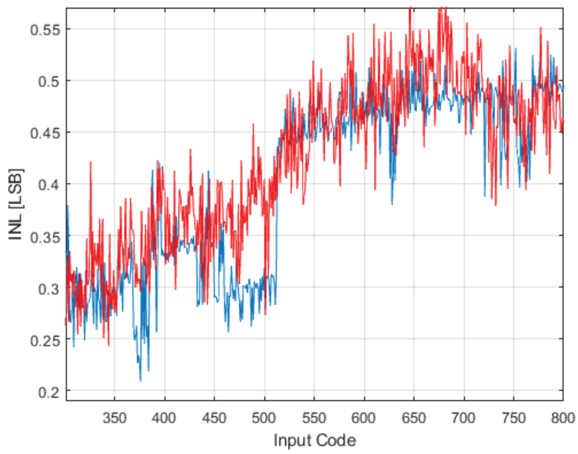
Secondly, the proposed method is implemented as shown in Fig. 1b. In this phase, the codes are randomly reduced of a given CR and the vector $\hat{\mathbf{INL}}$ is reconstructed according to (9). In detail, the algorithm employed to solve the minimization problem (8) is the Orthogonal Matching Pursuit [15], where the threshold $\epsilon = 0.005$ is set on the basis of an experimental analysis.

III.B. Test results

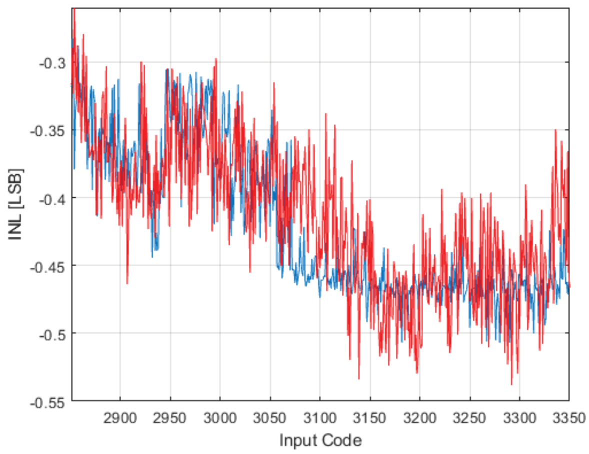
The performance of the proposed method is analyzed in MATLAB environment by comparing the $\mathbf{INL}$ obtained by the all-code testing, with the reconstructed vector $\hat{\mathbf{INL}}$, by means of the Root Mean Square Error (*RMSE*):

$$RMSE = \sqrt{\frac{1}{K}\sum_{k=0}^{K-1}[INL(k) - \hat{INL}(k)]^2}. \qquad (10)$$

Fig. 2 illustrates the obtained results. The INL curve evaluated at all the 2^N codes, as the traditional static testing imposes, is depicted in blue. The INL curve reconstructed through the proposed method from a number of codes reduced according to $CR = 16$ is, instead, depicted in red. By comparing the two curves, the INL evaluated at all the codes is well overlapped by the INL reconstructed through the proposed reduced–code method. Fig. 3a and Fig. 3b show then two enlargements respectively on code ranges $[301 - 800]$ and $[2851 - 3350]$, where the deviation of the reconstructed INL curve from the original INL curve is most notable. The good performance of the reconstruction is confirmed by the $RMSE$ value equal to 0.044.



(a)



(b)

Fig. 3. INL enlargement (a) on codes $[301 - 800]$ *and (b) on codes* $[2851 - 3350]$.

The performance of the proposed method is more deeply examined in terms of $RMSE$ for different lengths M of the vector $\mathbf{y}$ of the reduced INL. Specifically, the analysis is carried out depending on the CR values equal to 2^b, with $b = \{2, 3, \ldots, 7\}$. Moreover, since the selection process of the codes in the measurement matrix (6) is random, the reconstructed vector $\hat{\mathbf{INL}}$ is evaluated for 200 different random trials. Therefore, the $RMSE$ is computed at each random iteration. Then, both the average and the standard deviation of the obtained $RMSE$ values are computed.

The results of this analysis are presented in Fig. 4, where the markers represent the averages, while the error bars indicate twice the standard deviations. As expected, either the mean and the standard deviation increase with the CR. In particular, the average value of $RMSE$ exhibits an almost linear trend. The standard deviation exhibits a similar trend, increasing of the same order of magnitude of the average, up to the maximum value of 0.011. It should be underlined that, on the basis of the obtained results, the proposed reduced–code method generally proves to be very performing. Very small values of $RMSE$ are actually obtained also depending on high values of CR. Such results are particularly encouraging for the purpose of the static characterization. In fact, just an $RMSE = 0.058$ is committed on average if the number of codes employed to evaluate the INL curve is reduced by $CR = 128$.

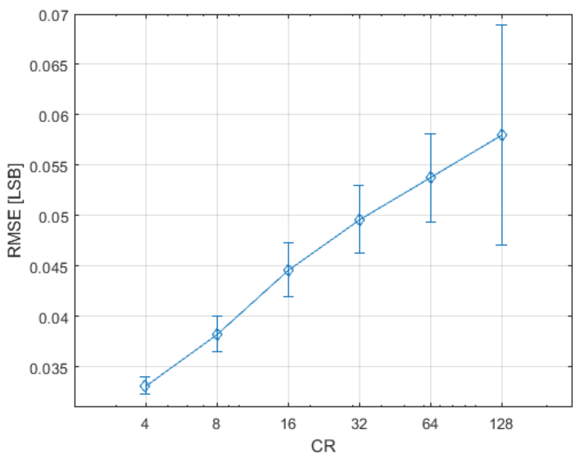


Fig. 4. $RMSE$ values versus CR values obtained by 200 *random trials: the marker and the error bar indicate the average plus/minus the standard deviation.*

IV. CONCLUSIONS

In this paper, a method to reduce the duration of static testing to evaluate the DAC linearity is presented. The method reduces the number of codes of static characterization and then exploits the CS technique to reconstruct the INL curve on all the codes.

Thus, a random matrix and a DFT matrix are proposed, respectively, as measurement matrix and sparsity basis. The reconstructed INL curve is also compared to the original curve through the $RMSE$, for several values of CR. The numerical results, obtained by averaging 200 different random trials, show generally small values of $RMSE$ also depending on high values of CR. The proposed method demonstrates thereby promising to reduce the overall duration of DAC static testing.

Future work will be devoted to implement a more complete model for the numerical analysis, by considering non-idealities and noise affecting the process of static characterization. Furthermore, the reduced–code method will be extended to static testing of ADCs. Finally, the method will be also implemented on hardware, in both the cases of DACs and ADCs, evaluating the actual impact on the duration required to characterize the INL curve and its accuracy in the case of an actual DAC.

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