

Compressed sensing with model based reconstruction

Imrich Andráš, Ján Šaliga, Linus Michaeli,

*Department of Electronics and Multimedia Telecommunications,
Faculty of Electrical Engineering and Informatics,
Technical University of Košice,
Letná 9, 042 00 Košice, Slovak Republic
{imrich.andras, jan.saliga, linus.michaeli,}@tuke.sk*

Abstract – This work presents a way of reducing the power consumption of the existing wireless sensor network used for monitoring of water quality. The improvement of sensor nodes energy efficiency is achieved by utilizing compressed sensing and lowering the average sampling rate. A novel continuous model based reconstruction method is proposed, utilizing a water parameter signal model rather than a discrete dictionary. No trained dictionary and hence no signal database is required a priori to compressed sensing application.

Keywords – compressed sensing, sparse signal, nonuniform sampling, model based reconstruction, water quality monitoring

I. INTRODUCTION

Water quality of surface and ground waters continues to be an issue of public concern throughout the developed world. The quality of all water bodies and resources has direct and indirect impact on the environment, the agriculture and related industries and in the end on human health. Water quality is often monitored and forecasted in rivers flowing through country borders, near settlements, industrial and agricultural plants, in water bodies designated for drinking, irrigation, fish farming etc. A wireless sensor network (WSN) designed for river water quality monitoring was proposed by [1] as a versatile automated tool for WQM. The sensor nodes measure multiple water parameters such as temperature, salinity, dissolved oxygen, pH, total dissolved solids etc. [2]. Sensor nodes were realized as solar-powered buoys fixed in the Ipel river, sending data wirelessly to gateways [3]. The developed system was tested in pilot operation spanning over a year, covering periods with extreme weather and flash floods. Measured water parameter signals (WPS) were acquired by on-board sensors, transferred to a central database and stored [4].

The sensor nodes used solar power for charging the integrated battery. In order for the sensor node to remain

operational, the long-term average power consumption must be lower than the average solar power available. This proved to be of particular concern during winter months, when only a limited amount of solar power is available, the solar panel may be covered in snow or white frost and the battery performance is reduced due to low temperatures. The WSN node power consumption comprises of two main parts: the transmitter that draws power when transmitting data, and the probes that draw power upon triggering of the measurement process. Significant reduction in power consumption could therefore be achieved either by lowering the sampling frequency or by compression of data that are being transmitted [5]. The use of compressed sensing (CS), a newly emerged method of signal compression and reconstruction, is proposed. CS allows to directly lower the sampling frequency without information loss, thus reducing the power consumption of both the probes and the transmitter at the same time.

This work demonstrates that CS can be used for WQM and efficiently implemented in an existing WSN. A novel reconstruction method based on general WPS model is proposed, which does not require any a priori available WPS database or trained dictionary. The article is organized as follows: Section II provides an overview of general CS theory and related works. Description of the concerned WSN is provided in section III, as well as the proposed sampling and reconstruction method. Experimental results are shown in section IV, where the performance of proposed methods is evaluated. Discussion in section IV compares the proposed method to the established dictionary based reconstruction and lists advantages and disadvantages of each.

II. COMPRESSED SENSING

Compressed sensing was originally proposed by [6][6], causing a sensation in the signal processing community in the years after. CS allows acquisition of certain signals with lower sampling frequency than dictated by the Nyquist-Shannon sampling theorem. The main and distinctive feature of CS is that the compression is

performed directly by the sampling process. Very little (if any) digital signal processing is required, the computational and power requirements are thus greatly reduced.

In order for CS to be applicable it is assumed that the (discrete) input signal vector $\mathbf{f} \in \mathbb{R}^{N \times 1}$ can be expressed as a linear combination of s discrete basis functions as

$$\mathbf{f} = \mathbf{\Psi}\mathbf{x}. \quad (1)$$

The basis matrix $\mathbf{\Psi} \in \mathbb{R}^{N \times L}$ contains all the possible basis functions $\psi_l \in \mathbb{R}^{N \times 1}, 1 \leq l \leq L$ that the input signal can be composed of. Vector $\mathbf{x} \in \mathbb{R}^{L \times 1}$ conducts the linear combination and has s non-zero entries, with $s \ll L$. The signal \mathbf{f} is then denoted as s -sparse, with its sparsity defined by the ℓ_0 pseudo-norm

$$\ell_0 : \|\mathbf{x}\|_0 = \sum_{i=1}^n |x_i|^0. \quad (2)$$

A. Analog-to-information conversion

The sampling process in CS aims to acquire all the information contained in the signal while taking as few samples as possible. This process is often referred to as analog-to-information conversion (AIC) and can be performed in various ways [7]. In general, the input signal is correlated with $M < N$ measurement signals defined by the measurement matrix $\mathbf{\Phi} \in \mathbb{R}^{M \times N}$, yielding the information signal

$$\mathbf{y} = \mathbf{\Phi}\mathbf{f} \in \mathbb{R}^{M \times 1}. \quad (3)$$

Note that the information signal vector has fewer entries than the original signal vector; hence the sampling frequency has been reduced. In order to avoid aliasing, the measurement matrix usually has random attributes and matrices $\mathbf{\Psi}$ and $\mathbf{\Phi}$ must be mutually incoherent. The mostly studied hardware methods of AIC are random demodulation [8], random modulation and pre-integration [9] and non-uniform sampling (NUS) [10], each with their own structure of measurement matrix. Hardware (analog) AIC implementation is promoted mainly in high frequency applications where it allows surpassing the bandwidth limit of current analog-to-digital converters (ADC). Digital AIC implementation requires a Nyquist ADC front-end, but it is a much simpler encoding alternative to conventional compression methods.

B. Reconstruction

During reconstruction only the bases $\mathbf{\Psi}$, $\mathbf{\Phi}$ and the information signal \mathbf{y} are known. The original signal \mathbf{f} can be recovered provided that the conditions from II are met and $s < M \ll N$. Reconstruction is performed as

$$\hat{\mathbf{f}} = \mathbf{\Psi}\mathbf{x} \quad (4)$$

after finding \mathbf{x} . Let us for convenience denote the reconstruction matrix

$$\mathbf{A} = \mathbf{\Phi}\mathbf{\Psi} \in \mathbb{R}^{M \times L} \quad (5)$$

and insert (1) into (3):

$$\mathbf{y} = \mathbf{\Phi}\mathbf{\Psi}\mathbf{x} = \mathbf{A}\mathbf{x}. \quad (6)$$

Finding \mathbf{x} requires solving an undetermined system of M equations and L unknowns, which is made possible by the sparsity prerequisite. If a correct optimal solution exists, it can be found by solving

$$\min \|\mathbf{x}\|_0 \text{ subject to } \mathbf{A}\mathbf{x} = \mathbf{y}. \quad (7)$$

[11] has shown that in sparse systems ℓ_p solutions are equivalent for small p , which allows to find $\hat{\mathbf{x}}$ as

$$\min \|\mathbf{x}\|_2 \text{ subject to } \mathbf{A}\mathbf{x} = \mathbf{y}. \quad (8)$$

(8) is much more convenient to solve than (7) as it allows to use efficient optimization approaches, e. g. utilizing a matrix pseudoinverse

$$\hat{\mathbf{f}} = \mathbf{\Psi}(\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T \mathbf{y}. \quad (9)$$

The general CS framework is summarized in Fig.1.

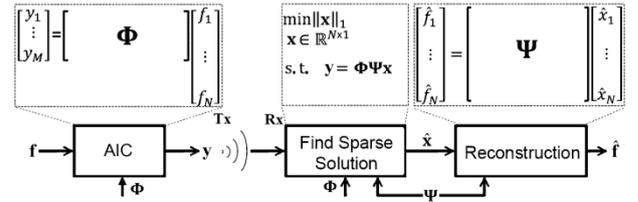


Fig.1 General CS framework

C. Challenges

There are several problems that need to be solved when utilizing CS. In a practical application the acquired signal parameters are unknown and only a priori estimated during CS system design. There will also be deviations from the mathematical descriptions when physical components are concerned.

Sparsity and bandwidth of the signal that will be acquired has to be known in advance. In order to construct the measurement matrix, M and N must be known, that are in direct relation to input signal sparsity and bandwidth. In a real world application estimating signal sparsity is not a trivial task – any real signal is not ideally sparse, contains noise, is subject to distortions etc. Input signal also may run out of its bounds of freedom defined by $\mathbf{\Psi}$.

Constructing a suitable dictionary $\mathbf{\Psi}$ is crucial in order to achieve the desired compression ratio while maintaining low reconstruction error. In many applications the ideal $\mathbf{\Psi}$ is a set of arbitrary waveforms. This set can be obtained if a database containing sufficient number of possible signal (1) variations is available. Extracting a set of basis functions can be done by well-known methods such as principal component analysis [5]. The downside is that the training database has to be acquired with Nyquist sampling, which can be time-consuming and costly.

The current state of the art is focused on improving the performance of the CS framework by novel approaches that can deal with all the nonideal conditions. The pinnacle

would be an adaptive algorithm that can recover any sparse signal with no prior knowledge of its properties like sparsity or Ψ basis ([12], [13]). Advances are being made in sensing matrix construction ([14], [15], [16]), reconstruction procedures ([17], [18], [19]) as well as hardware platforms that facilitate CS ([20], [21], [22]).

III. MATERIALS AND METHODS

A. Existing water quality monitoring system

The WSN used to collect data during its pilot operation was deployed near the Slovak-Hungarian border in the Ipel river. WSN was organized as two sub-networks, each containing 5 buoys carrying multiparameter Ponsel probes and sending data to onshore gateways. The general WQM system structure is shown in Fig.2.

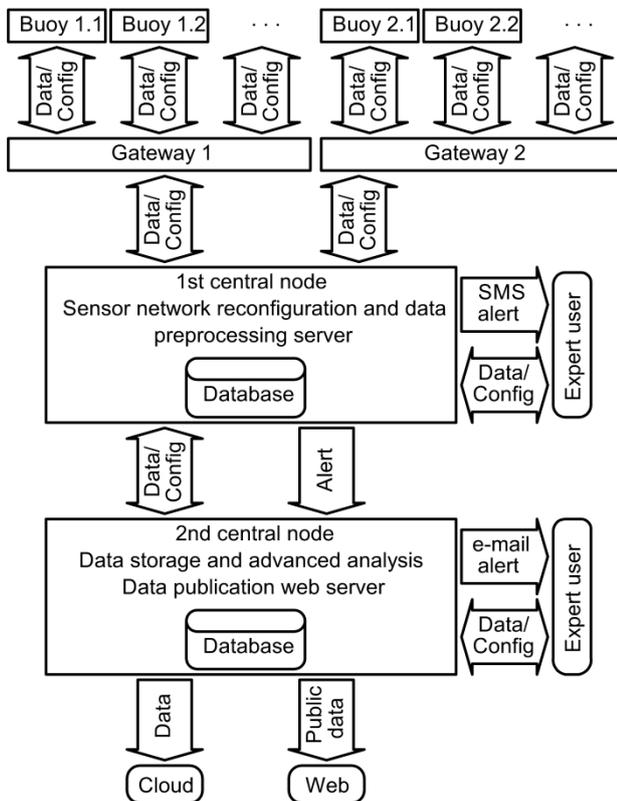


Fig.2 Structure of the deployed WQM system.

Buoys and gateways communicate via encoded multi-hop communication protocol with automated node discovery, ensuring energy-efficient and secure data transfer. Buoys are in power-save mode until awoken by a message/command from the gateway.

The gateways collect data received as replies from buoys and pass them through the Internet to the 1st central node in Budapest. Here the data are temporarily stored and compared to preset limits in real time. In case a measured value exceeds the preset limit, the 1st central node generates SMS alerts that are sent to expert users. Alerts

together with data are also sent through web services to the 2nd central node in Kosice. All the measured data are transferred every 24 hours from the 1st to the 2nd central node, where they are stored in the database [4].

The sensor nodes – buoys (Fig.3) are designed as self-powered units with on-board probes, microcontroller and 433MHz short range wireless transceiver. Buoy design allows for modularity and additional probes can be added in the future if needed [2].

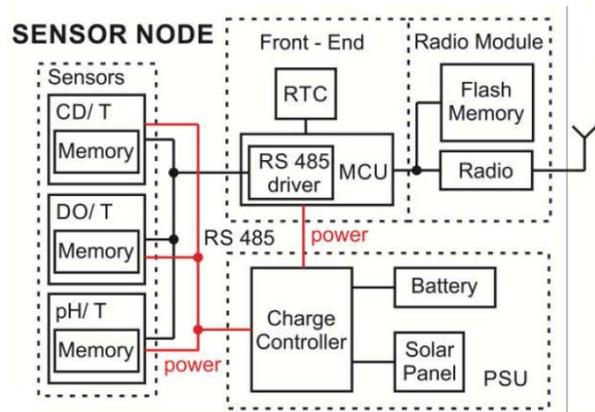


Fig.3 Block diagram of the sensor node.

Due to the intended versatility of the described WSN a high sampling frequency was chosen. The buoys start measurements and transmit the measured data only in response to command from gateway. The original sampling period $T_s=1$ hour was therefore not set by buoys themselves, but rather the gateways. Implementation of CS with NUS is possible with no changes to the buoys themselves. Software change only is required at the application level of the 1st central node and gateways, which makes CS a simple-to-implement solution to issues discussed in the introduction.

B. Proposed sensing and reconstruction procedure

Data acquired by the described WQM system show that WPS of all the measured parameters correlate and display the same basic pattern. A typical WPS contains a large slowly changing DC component, a faster AC component with relatively small amplitude and a small amount of noise [23]. Examples of WPS are shown in Fig.4. The AC component is a distorted sine wave with fundamental frequency $D=1.157 \times 10^{-5}$ Hz (1 per day). Frequency analysis shows that there are only up to 4 higher harmonics of appreciable magnitude. The WPS thus carries only limited information content and can be classified as frequency sparse, meeting the criteria for NUS application.

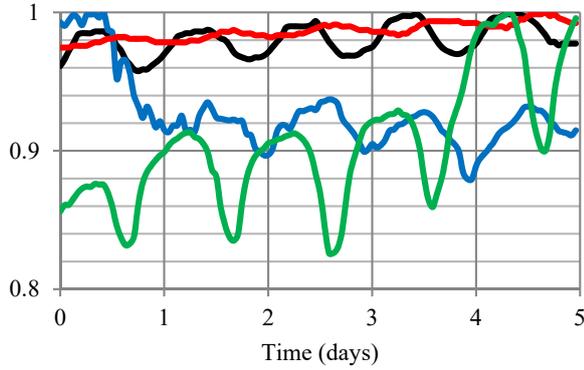


Fig.4 Examples of water parameter signals.

The following model is proposed for WPS representation:

$$f_n = {}^\Lambda P_n + {}^K S_n, n = 1, 2, \dots, N. \quad (10)$$

$${}^\Lambda P_n = \sum_{i=0}^{\Lambda} {}^\Lambda B_i (nT_S)^i \quad (11)$$

is a polynomial of order Λ with coefficients

$${}^\Lambda \mathbf{B} = [{}^\Lambda B_0 \quad {}^\Lambda B_1 \quad \dots \quad {}^\Lambda B_{\Lambda-1}] \quad (12)$$

that represents the slowly changing DC component. The AC component is represented by a sum of sinusoids, from fundamental to the K -th harmonic

$${}^K S_n = \sum_{j=1}^K {}^K C_j \sin(2\pi j D n T_S + {}^K \theta_j), \quad (13)$$

defined by their amplitudes

$${}^K \mathbf{C} = [{}^K C_1 \quad {}^K C_2 \quad \dots \quad {}^K C_K] \quad (14)$$

and phases

$${}^K \boldsymbol{\theta} = [{}^K \theta_1 \quad {}^K \theta_2 \quad \dots \quad {}^K \theta_K]. \quad (15)$$

NUS is proposed as WPS sampling method, as it is simple to implement on existing WQM system and compatible with frequency sparse signals. Let us consider a Nyquist ADC at the sensor node acquiring (1) with $T_S=1$ hour, but only a limited number M of all the samples is randomly selected and transmitted as (3). Let Ξ contain indices of the selected samples

$$\Xi = \{\xi_1, \xi_2, \dots, \xi_M\}. \quad (16)$$

Entries of Ξ are random numbers, following the rules

$$\forall m \in \mathbb{N}, 1 \leq m \leq M : \xi_m \in \mathbb{N}, 1 \leq \xi_m \leq N, \quad (17)$$

$$\forall i, j \in \mathbb{N}, 1 \leq i < j \leq M : \xi_i < \xi_j. \quad (18)$$

Elements of measurement matrix representing NUS AIC can then be obtained as

$$\varphi_{mn} = \begin{cases} 1 & \text{if } n = \xi_m \\ 0 & \text{otherwise} \end{cases} \quad (19)$$

and the information signal (3) elements become

$$y_m = f_{\xi_m}. \quad (20)$$

In any form of CS reconstruction (16) needs to be passed along with the samples. This could be done in form of a synchronization mark or as the seed value for random

sequence generator. The WQM system described in sect. III.A uses a timestamp within the data packet, which is fully acceptable for the purpose.

[19] proposed a CS reconstruction method that does not rely on a fixed dictionary Ψ , but rather employs a time domain signal model and differential evolution parametric estimation (DEPE). This was demonstrated on general frequency sparse signals with NUS. DEPE approach could be seen as having a non-constant Ψ signal base with just s basis functions [24]. The basis functions are model defined and varied in their parameters until a condition is met for successful reconstruction. It was demonstrated that in certain situations this approach yields better precision and lower computational cost than dictionary based methods. DEPE can make use of the WPS model (10) and reconstruct the original WPS after NUS. (8) in this particular case will take the form

$$\{{}^\Lambda \hat{\mathbf{B}}, {}^K \hat{\mathbf{C}}, {}^K \hat{\boldsymbol{\theta}}\} = \arg \min_{{}^\Lambda \mathbf{B}, {}^K \mathbf{C}, {}^K \boldsymbol{\theta}} \|\hat{\mathbf{y}} - \mathbf{y}\|_2, \quad (21)$$

where $\hat{\mathbf{y}}$ are samples of the reconstructed signal at the sampling instances (16)

$$\hat{y}_m = \sum_{i=0}^{\Lambda} {}^\Lambda B_i (\xi_m T_S)^i + \sum_{j=1}^K {}^K C_j \sin(2\pi j D \xi_m T_S + {}^K \theta_j), m = 1, 2, \dots, M. \quad (22)$$

Parameters ${}^\Lambda \hat{\mathbf{B}}, {}^K \hat{\mathbf{C}}, {}^K \hat{\boldsymbol{\theta}}$ can be used with model (10) for original signal reconstruction at arbitrary time instances, analog to (22).

A possible issue with DEPE is ensuring the model's orthogonality and convergence to one single sparse solution. DEPE can search large solution spaces with no regards to the model's linearity. This is advantageous, but only if the model reliably meets the mentioned criteria. It is proposed that the parameters ${}^\Lambda \hat{\mathbf{B}}$ are estimated via polynomial fit prior to DEPE and then fed into (22) as a starting point.

IV. RESULTS

The performance of proposed sensing and reconstruction method was tested on data obtained during WPS pilot operation. Testing was performed via simulations that sufficiently represent implementation on the physical WQM system. 200 WPS segments with length $N=120$ (5 days) were chosen randomly from the available WPS database for testing.

DEPE needs to have the solution space bounds defined prior to reconstruction. [19] has shown that the algorithm is not particularly sensitive to correctly set bounds, but doing so aids computational cost and reconstruction success rate. The maximum number of harmonic components in (14) was set to $K=4$ as suggested by the WPS analysis discussed in sect. III.B. The polynomial order of (12) was set to $\Lambda = 3$. This number was found by

investigation of inflection points of WPS after filtering out the AC component. Both K and Λ could also be found by an educated guess or by trial and error, since they are small numbers by the nature of model (11).

A study [5] was conducted on WPS acquisition via CS with reconstruction based on extensive trained dictionary. This study was conducted using the same test signals and the same sampling method, but with direct compressed sensing reconstruction (DCSR) according to (9). The results of this study will be provided here as reference.

Preliminary results show that under ideal conditions the proposed DEPE reconstruction method underperforms in comparison to dictionary based DCSR. A trained dictionary obtained from an extensive WPS database is capable of near-perfect-reconstruction. A simple-by-design model like (11) cannot do this, as can be seen in Fig. 5, leading to higher reconstruction error.

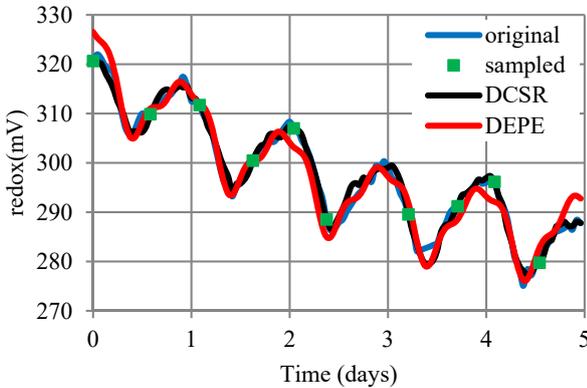


Fig.5 Example of WPS reconstruction under ideal conditions with compression ratio $N/M=6$.

Near-perfect-reconstruction capability may be counterproductive if the conditions are not ideal. Fig.6 shows that the DCSR algorithm performs worse than DEPE under conditions discussed in sect. II.C. Even a small amount of noise causes the proposed DEPE method to perform better. The question for the end user is whether recovering all the intricate details of the original WPS is practically needed.

DCSR has too many options when trying to solve (9), which causes it to “invent” its own noise and artifacts to fit the distorted samples. DEPE by adhering to the semi-analytical signal model yields the most likely fit, resulting in both lower reconstruction error and subjectively more realistic output. Model based reconstruction is also naturally denoising, similar to methods used in biomedicine ([25], [26]). Situation is similar when quantization noise is degrading the samples.

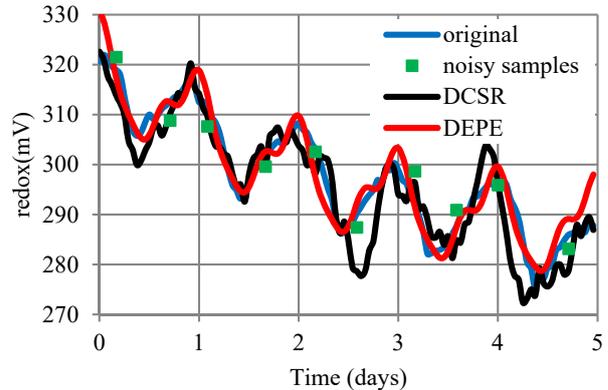


Fig.6 Example of WPS reconstruction from noisy samples with compression ratio $N/M=6$.

It can be concluded that DCSR trades robustness off for near-perfect reconstruction, while the proposed DEPE offers modest but stable performance in challenging conditions. A big disadvantage of DCSR may be the need for Nyquist WPS database. If the WPS database was not available, DEPE would be much easier to implement than DCSR. The limits of model (10) required to be known for implementation of DEPE can be observed in just a few days, possibly by sample grabbing. Obtaining a training database for DCSR would require deploying the WSN itself since hundreds of days’ worth of Nyquist sampling are needed [5].

V. CONCLUSIONS

A novel CS reconstruction method based on water parameter signal model and differential evolution parametric estimation has been presented. Preliminary results show that this method is robust and offers stable performance with varied number of samples or samples that are degraded. The proposed method is compatible with nonuniform sampling, which is simple to implement in general and particularly on the WSN analyzed in this work.

The authors propose to set the compression ratio to 6 which would reduce the buoys’ power consumption to app. 25% of the original. Implementation of nonuniform sampling in the analyzed WSN would require no hardware changes at all and no changes of the buoys’ firmware. Further reduction of power consumption could be achieved by rescaling and quantizing the high resolution data, although this step would require a firmware revision. 12-bit quantization would provide the necessary dynamic range excess while further reducing the data payload by over 60%, making overall power consumption less than 10% of the original.

VI. ACKNOWLEDGMENT

The work is a part of the project supported by the Science Grant Agency of the Slovak Republic (No. 1/0722/18).

REFERENCES

- [1] Macekova, L., Ziga, M., (2014), The wireless sensor network concept for measurement of water quality in water streams, *Acta Electrotechnica et Informatica*, Vol. 14, No. 2, 2014, 60–67, DOI: 10.15546/aeci-2014-0020
- [2] P. Galajda, M. Drutarovsky, J. Saliga, M. Ziga, L. Macekova, S. Marchevsky, D. Kocur, (2015), Sensor node for the remote river water quality monitoring, MEASUREMENT 2015, Proceedings of the 10th International Conference, Smolenice, Slovakia.
- [3] Ján Šaliga, Matej Žiga, Pavol Galajda, Miloš Drutarovský, Dušan Kocur, Ludmila Maceková, (2015), Wireless sensor network for river water quality monitoring, XXI IMEKO World Congress “Measurement in Research and Industry”, August 30 - September 4, 2015, Prague, Czech Republic
- [4] Ján Šaliga, Dušan Kocur, Pavol Galajda, Miloš Drutarovský, Ludmila Maceková, Imrich Andráš, Linus Michaeli, (2017), Multi-parametric Sensor Network for Water Quality Monitoring, IMEKO TC19 Workshop for metrology for the sea, Naples, Italy, 11 – 13 October, 2017
- [5] Andráš, I., Dolinský, P., Michaeli, L., Šaliga, J., (2018), Sparse Signal Acquisition via Compressed Sensing and Principal Component Analysis, *Measurement Science Review*, vol. 18, no. 5, pp. 175 – 182, ISSN 1335-8871
- [6] Donoho, D. L., (2006), Compressed Sensing, *IEEE Trans. On Informtion Theory*, vol. 52, no. 4, pp. 1289 – 1306, ISSN 1557-9654
- [7] Daponte, P., De Vito, L., Rapuano, S., Tudosa, I., “Analog-to-Information Converters in the Wideband RF Measurement for Aerospace Applications: Current Situation and Perspectives,” in *IEEE Instrumentation & Measurement Magazine*, 2017. Vol. 20, no. 1, pp. 20 – 28. ISSN 1094-6969
- [8] Zhuo Pang, Mei Yuan, Michael B. Wakin, (2018), A random demodulation architecture for sub-sampling acoustic emission signals in structural health monitoring, *Journal of Sound and Vibration*, Volume 431, Pages 390-404, ISSN 0022-460X, <https://doi.org/10.1016/j.jsv.2018.06.021>
- [9] Candes, E., Becker, S., (2013), Compressive sensing: Principles and hardware implementations, 2013 Proceedings of the ESSCIRC, 16-20 Sept. 2013, ISSN 1930-8833
- [10] M. Wakin et al., (2012), A Nonuniform Sampler for Wideband Spectrally-Sparse Environments, *IEEE Journal on Emerging and Selected Topics in Circuits and Systems*, vol. 2, no. 3, pp. 516-529, doi: 10.1109/JETCAS.2012.2214635A
- [11] Fung, G. M., Mangasarian, O. L., (2011), Equivalence of Minimal ℓ_0 - and ℓ_p -Norm Solutions of Linear Equalities, Inequalities and Linear Programs for Sufficiently Small p , *Journal of Optimization Theory and Applications*, vol. 151, no. 1, pp. 1 – 10, ISSN 15732878, DOI: <https://doi.org/10.1007/s10957-011-9871-x>
- [12] Ramon Fuentes, Carmelo Mineo, Stephen G. Pierce, Keith Worden, Elizabeth J. Cross, (2019), A probabilistic compressive sensing framework with applications to ultrasound signal processing, *Mechanical Systems and Signal Processing*, Volume 117, Pages 383-402, ISSN 0888-3270, <https://doi.org/10.1016/j.ymsp.2018.07.036>.
- [13] Angshul Majumdar, (2018), An autoencoder based formulation for compressed sensing reconstruction, *Magnetic Resonance Imaging*, Volume 52, Pages 62-68, ISSN 0730-725X, <https://doi.org/10.1016/j.mri.2018.06.003>
- [14] Charles J. Colbourn, Daniel Horsley, Violet R. Syrotiuk, (2018), A hierarchical framework for recovery in compressive sensing, *Discrete Applied Mathematics*, Volume 236, Pages 96-107, ISSN 0166-218X, <https://doi.org/10.1016/j.dam.2017.10.004>
- [15] Hongping Gan, Song Xiao, Yimin Zhao, (2018), A large class of chaotic sensing matrices for compressed sensing, *Signal Processing*, Volume 149, Pages 193-203, ISSN 0165-1684, <https://doi.org/10.1016/j.sigpro.2018.03.014>
- [16] Jiajun Ding, Donghai Bao, Qingpei Wang, Xiongxiang He, Huang Bai, Sheng Li, (2018), A novel multi-dictionary framework with global sensing matrix design for compressed sensing, *Signal Processing*, Volume 152, Pages 69-78, ISSN 0165-1684, <https://doi.org/10.1016/j.sigpro.2018.05.012>
- [17] Guowei You, Zheng-Hai Huang, Yong Wang, (2017), A theoretical perspective of solving phaseless compressive sensing via its nonconvex relaxation, *Information Sciences*, Volumes 415–416, Pages 254-268, ISSN 0020-0255, <https://doi.org/10.1016/j.ins.2017.06.020>
- [18] Yu Zhou, Hainan Guo, (2019), Collaborative block compressed sensing reconstruction with dual-domain sparse representation, *Information Sciences*, Volume 472, Pages 77-93, ISSN 0020-0255, <https://doi.org/10.1016/j.ins.2018.08.064>
- [19] Imrich Andráš, Pavol Dolinský, Linus Michaeli, Ján Šaliga, (2018), A time domain reconstruction method of randomly sampled frequency sparse signal, *Measurement*, Volume 127, Pages 68-77, ISSN 0263-2241, <https://doi.org/10.1016/j.measurement.2018.05.065>
- [20] M. Safarpour, R. Inanlou, M. Charimi, O. Shoaebi and O. Silvén, (2018), ADC-Assisted Random Sampler Architecture for Efficient Sparse Signal Acquisition, in *IEEE Transactions on Very Large Scale Integration (VLSI) Systems*, vol. 26, no. 8, pp. 1590-1594, doi: 10.1109/TVLSI.2018.2821696
- [21] Miguel Marquez, Henry Arguello, (2019), Coded aperture optimization for single pixel compressive computed tomography, *Journal of Computational and Applied Mathematics*, Volume 348, Pages 58-69, ISSN 0377-0427, <https://doi.org/10.1016/j.cam.2018.08.034>
- [22] Xudong Zhang, Jianan Xie, Chunlai Li, Rui Xu, Yue Zhang, Shijie Liu, Jianyu Wang, (2018), MEMS-based super-resolution remote sensing system using compressive sensing, *Optics Communications*, Volume 426, Pages 410-417, ISSN 0030-4018, <https://doi.org/10.1016/j.optcom.2018.05.046>
- [23] WSN-AQUA Wireless Sensor Network for wATER QaUALity Monitoring (2015). <http://husk.fei.tuke.sk>, Online: 2018/21/08
- [24] Michaeli, L., Šaliga, J., Dolinský, P., Andráš, I., (2019), Optimization paradigm in the signal recovery after compressive sensing, *Measurement Science Review*, vol. 19, no. 1, pp. 35 – 42. ISSN 1335-8871
- [25] Y. Lu, J. Yan and Y. Yam, (2009) "A Generalized ECG Dynamic Model with Asymmetric Gaussians and its Application in Model-Based ECG Denoising," 2009 2nd International Conference on Biomedical Engineering and Informatics, Tianjin, pp. 1-5. doi: 10.1109/BMEI.2009.5305699
- [26] Dolinsky, P., Andras, I., Michaeli, L., Grimaldi, D., (2018), Model for generating simple synthetic ECG signals, *Acta Electrotechnica et Informatica*, Vol. 18, No. 3, pp. 3 – 8, ISSN 1335-8243