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Joint Classification and Parameter Estimation of Compressive Sampled FSK Signals

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Abstract – **This paper deals with the classification and parameter estimation of frequency-shift-keying (FSK) signals that are acquired using a compressive sampling approach. Such a technique allows reducing the sampling frequency needs, as the FSK signals are compressible in the frequency domain; the spectrum of FSK signal is sparse, being concentrated to a finite number of harmonics which depend on the modulation order. The classification and parameter estimation rely on the first-order cyclostationarity. The proposed method has been implemented in GNU Octave, and validated in simulation.**

I. INTRODUCTION

Modulation classification has been gaining greater interest in the scientific community, due to the new standards and proposals for spectrum allocation, which are trying to overcome the spectrum scarcity problem [1], [2], [3]. In both Europe and North America, new spectrum access policies are introduced in certain frequency bands, allowing a shared use of radio spectrum by multiple users [4]. In particular, even if the license of a specific spectrum band is owned by the primary/ incumbent users (PUs), it is allowed that secondary/ cognitive users (SUs) access the same band when the PUs do not actually use it. This access model is made possible through the emerging cognitive radio (CR) technology [5]. CR-based SUs monitor the spectrum in order to detect the presence of PUs along with identifying the PU physical layer parameters, such that the SU transmission does not generate harmful interference to the PUs. It is desired that the SUs monitor a wide frequency band in order to detect spectrum holes, which are unused by the PUs. Therefore, high sampling rates are required in the SU acquisition block.

Recently, compressive sampling has been proposed to reduce the sampling rate needs for signals which exhibit a sparse representation, e.g., in the frequency domain. For such signals, it is possible to reconstruct the original waveform or its frequency domain representation from a set of samples of a lower dimension than that required by the Shannon's theorem, by exploiting random demodulation, random filtering or random sampling [6]-[7].

Frequency-shift-keying (FSK) modulation continues to be commonly used, especially in the VHF and UHF frequency bands, due to its easy implementation and widespread usage in legacy communications equipment [8]. FSK is particularly suitable to be acquired using compressive sampling, as its spectrum is concentrated in a few frequency components, which correspond to the used tones. FSK signal classification and parameter estimation has been extensively studied in the literature following traditional methods, e.g., based on the instantaneous frequency [9] and the zero-crossing sequence [10]. Recently, methods exploiting signal cyclostationarity have been explored, due to their capability of working in fading scenarios [8].

The aim of this paper is to experimentally verify the application of classification and parameter estimation of FSK signals acquired using a compressive sampling approach. In particular, a random demodulation scheme will be used for compressive sampling, and the classification and parameter estimation based on the first-order cyclystationarity of the acquired signal will be considered.

The paper is organized as it follows. In Section II, the proposed method for compressive sampling, classification and parameter estimation is described, while simulation results are presented in Section III.

II. PROPOSED METHOD FOR COMPRESSIVE SAMPLING, CLASSIFICATION, AND PARAMETER ESTIMATION

The proposed method consists of two steps. In the former, the received signal is acquired by applying a compressive sampling technique based on random demodulation and its spectrum is obtained. Afterwards, the algorithm for modulation classification and tone spacing estimation is applied. Details on the two steps are subsequently provided.

A. Compressive sampling and spectrum reconstruction

The signal acquisition and reconstruction of its spectrum are carried out as shown in Fig. 1. Let $r(t)$ be the signal observed over Δt time in a frequency band B, whose spectrum is concentrated in a maximum of K frequency components. According to the Shannon's

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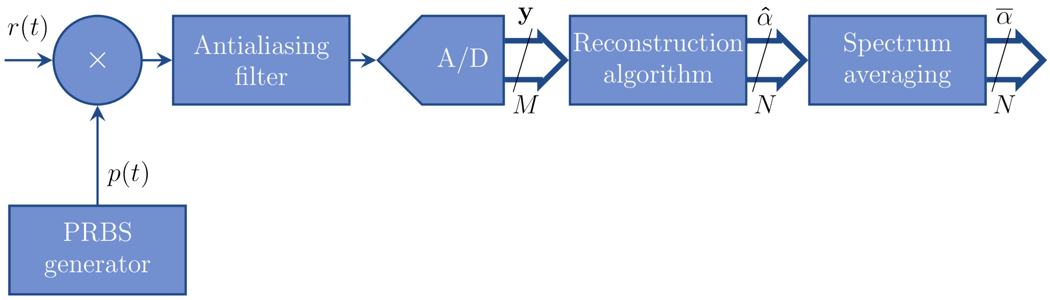

Fig. 1. Compressive sampling and reconstruction.

theorem, an analog-to-digital converter (ADC) with a sampling frequency greater than $2B$ would be required to sample $r(t)$, yielding an N-size vector $\mathbf{r}$ of samples. Instead of being directly acquired, $r(t)$ is mixed by a pseudo-random binary sequence (PRBS), with a bit-rate greater or equal to $2B$. The output of the mixer is passed through an antialiasing filter, and is then digitized using an ADC with a sampling frequency $f_s < 2B$; this results in a vector $\mathbf{y}$ of M samples, where $M < N$.

The compressive sampling can be modeled as a matrix multiplication [11]:

$$\mathbf{y} = \mathbf{\Phi r}, \tag{1}$$

where $\mathbf{\Phi}$ is obtained starting from the samples of the PRBS and the impulse response of the filter. Furthermore, with $\mathbf{x}$ as the vector of the discrete Fourier transform (DFT) coefficients of $\mathbf{r}$, and with $\mathbf{\Psi}$ the matrix representing the inverse DFT, the following expression can be written:

$$\mathbf{r} = \mathbf{\Psi x}. \tag{2}$$

If $M > K$, an estimate of the DFT coefficients $\mathbf{x}$ can be obtained by finding the most sparse solution such that:

$$\mathbf{y} = \mathbf{\Phi \Psi x} + \mathbf{e}, \tag{3}$$

with $\mathbf{e}$ as the additive noise. It has been demonstrated that the problem of finding the most sparse solution can be approximated by the ℓ_1-norm minimization:

$$\begin{aligned} &\hat{\mathbf{x}} = \arg\min \|\mathbf{x}\|_1 \\ &\text{subject to:} \quad \|\mathbf{\Psi^T \Phi^T}(\mathbf{y} - \mathbf{\Phi \Psi x})\|_\infty < \varepsilon, \end{aligned} \tag{4}$$

where $\|\cdot\|_1$ and $\|\cdot\|_\infty$ are the ℓ_1-norm and the ℓ_∞-norm, respectively, and ε is a small positive constant. Finally, the obtained DFT coefficients are averaged over R data batches in order to reduce the probability of identifying frequency components that are not actually present in the signal.

B. *Classification and parameter estimation*

The flowchart of the proposed algorithm is shown in Fig. 2 [8]. The normalized DFT coefficients obtained from the reconstruction algorithm are processed by executing the following steps:

Step 1: Selection of candidate frequencies. From the DFT coefficients, an estimate of the first order cyclic moment (CM) is obtained, as reported in [12]. Since the CM magnitudes at cyclic frequencies (CFs) take significant values, the frequencies for which the CM magnitude exceeds a cutoff value V_{co} are selected as candidates. If the number of selected candidates, N is below 2 (i.e., the minimum FSK modulation order), an adaptive procedure for setting V_{co} based on the bisection method is triggered [8]. This procedure ends when a desired number of candidates is selected, which equals the maximum expected FSK modulation order, Ω_{max}.

Step 2: Local maximum refinement. The adjacent candidates which are located at a distance greater than a minimum allowable distance, d_l are retained. For adjacent candidates separated by a distance lower than d_l, the candidate with the lower CM magnitude is ignored.

Step 3: Integer Multiple Relationship (IMR) based refinement. The distances between every two candidate frequencies selected in Step 2 are calculated, and the IMR is verified. If the IMR is satisfied, the corresponding frequency candidates are retained. Furthermore, as the CFs are equally spaced, the positions of any missing frequencies are inferred, and these are included as candidates. If the IMR property is satisfied by none of the candidate frequencies, Step 3 is skipped.

Step 4: Application of a cyclostationarity test. The cyclostationarity test proposed in [13] is used to check whether the previously selected candidate frequencies are CFs. A first-order CM based statistic is estimated for each candidate CF and compared to a threshold, Γ, which is determined from the probability of declaring that a candidate is a CF when it is not. If the estimated statistic exceeds the threshold, the corresponding candidate is declared a CF.

Step 5: Modulation order identification. The decision on the FSK modulation order is based on the number and position of the first-order CFs. For example, the received signal is decided to be 2-FSK when two first-order CFs appearing on different sides of the central frequency are

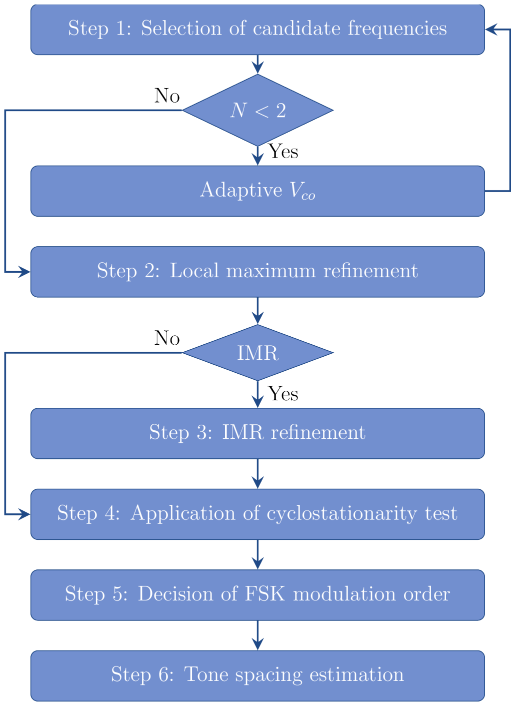

Fig. 2. Block scheme of the algorithm for signal classification and parameter estimation [8].

detected. Further, the received signal is considered to be Ω- FSK ($\Omega = 2^m, \Omega \geq 4$) if at least $2^{m-1}+1$ out of the Ω first-order CFs are detected and the corresponding distances satisfy the IMR property. The output is either the modulation order or *"Cannot decide"*. The latter decision is made if the conditions on either the CF number or position are not satisfied.

Step 6: Tone frequency spacing estimation. The minimum distance between adjacent CFs is calculated. Note that for M-FSK ($\Omega = 2^m, \Omega \geq 4$) signals, although some CFs may be missed, since at least $2^{m-1}+1$ CFs are detected, the minimum distance between adjacent CFs provides the tone frequency spacing.

III. SIMULATION ANALYSIS

Simulations were carried out in GNU/Octave, with FSK signals corrupted by additive white Gaussian noise (AWGN). The symbol rates were $\{\frac{1}{16}, \frac{1}{32}\}f_s$, and the tone spacing was equal to $0.078 f_s$, with f_s as the sampling frequency. Observation lengths of 4096, 8192, and 16384 samples were used, being grouped in batches of $N = 256$ samples. Each batch was reduced by compressive sampling using an undersampling rate (USR), equal to 4 or 8. Then, the estimation of the DFT coefficients was carried out by averaging over R data batches, where $R = \frac{N_T}{N}$, with N_T as the total number of acquired samples.

Fig. 3a provides the correct classification percentage versus the signal-to-noise ratio (SNR) for $\Omega = 2, 4, 8$.

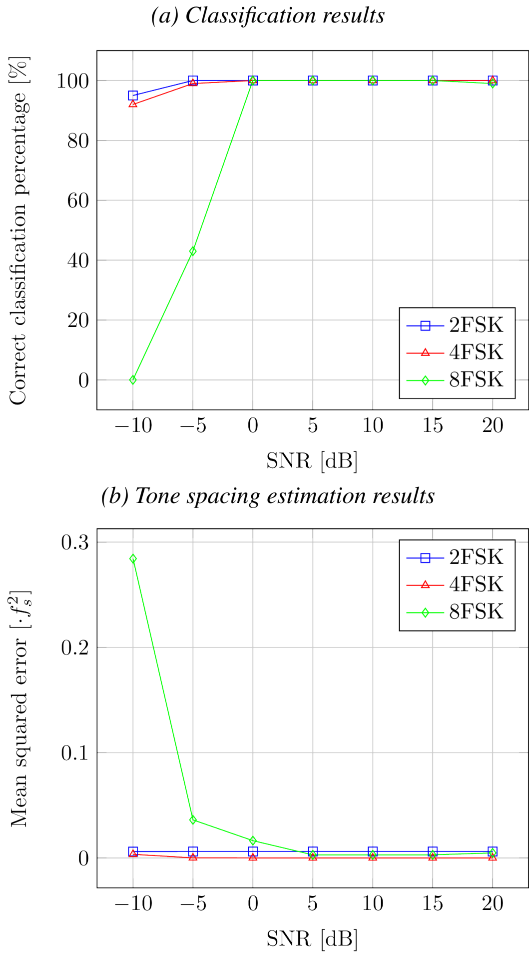

Fig. 3. Correct classification percentages (a) and tone spacing estimation mean square error (b) versus SNR for $\Omega = 2, 4, 8$.

It can be seen that a performance approaching 100% is obtained for $\Omega = 2, 4$ at SNRs above -5 dB, while more than 0 dB SNR is required for $\Omega = 8$. Furthermore, Fig. 3b depicts the mean square error for the tone spacing estimation versus SNR. A very good performance is achieved for $\Omega = 2, 4$ even at -10 dB SNR, while SNR>5 dB is needed for $\Omega = 8$ to reach a similar performance. In Figs. 4a and 4b, classification and estimation results are respectively presented versus SNR for $\Omega = 8$ and different observation lengths. As expected, an improved performance is obtained as the observation length increases.

Finally, in Fig. 5a, a comparison of the correct classification results versus SNR and for $\Omega = 8$, when the USR has been set to 4 and 8, respectively. As expected, for USR= 8 a slight decrease of the performance is shown; very high classification rates have been obtained starting form 0 dB. A similar decrease in performance can be observed in the estimation of the tone spacing,

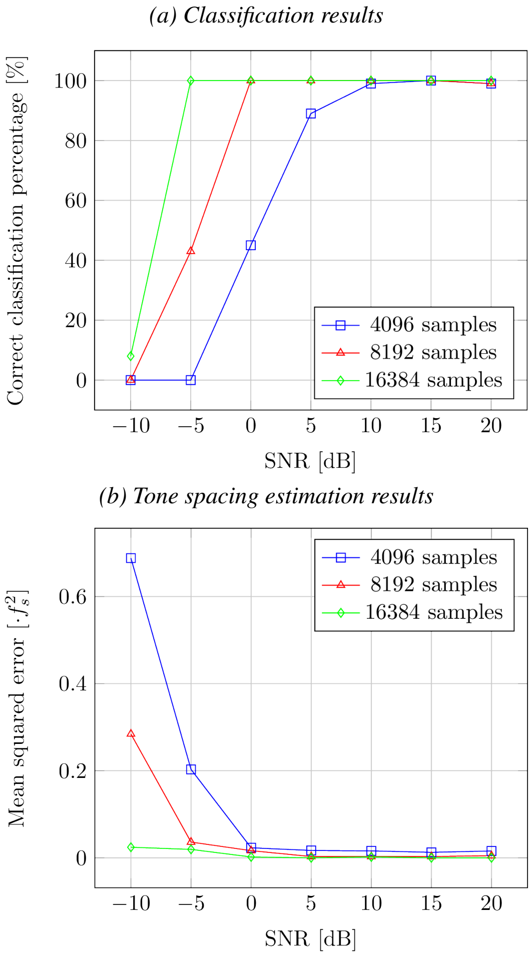

Fig. 4. Correct classification percentages (a) and tone spacing estimation mean square error (b) versus SNR for various numbers of acquired samples.

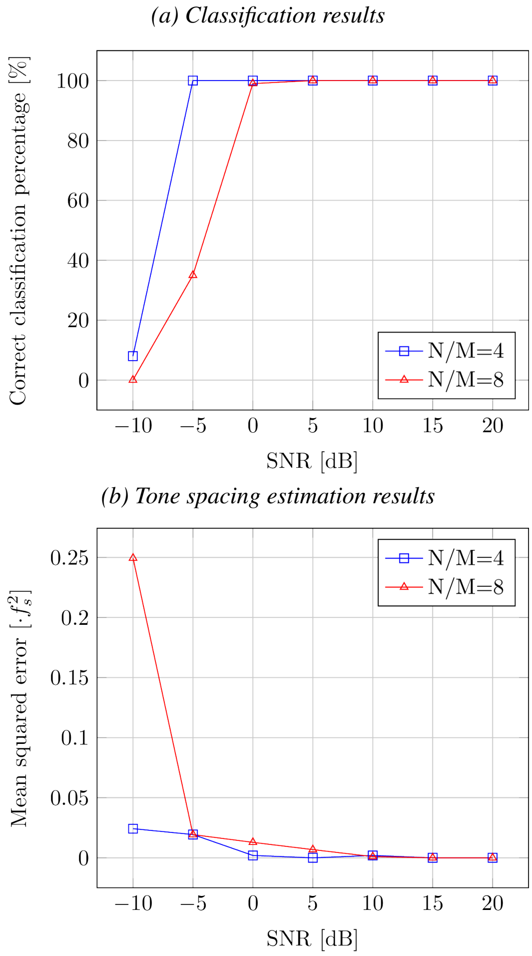

Fig. 5. Correct classification percentages (a) and tone spacing estimation mean square error (b) versus SNR and for different undersampling rates, N/M.

shown in Fig. 5b. In this case, the two curves show similar behaviors starting from -5 dB. It is worth noting, however, that this slight performance decrease observed for higher undersampling rates can be balanced by a greater compression that can lead either to the a reduction of the sampling rate or to an increase of the analyzed bandwidth.

IV. CONCLUSION

In this paper, a method is proposed for the classification and parameter estimation of frequency-shift-keying (FSK) signals that are acquired using a compressive sampling approach. Such a method will allow to successfully identify FSK signals even with a reduction of the sampling rate of the acquisition section. A preliminary validation phase of the proposed method, performed by simulations in the presence of AWGN, has been presented, and showed good results, even for low SNR values. Further work goes in the direction of a complete characterization of the method in presence of multipath fading and to an experimental characterization employing off-the-air signals.

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