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Analysis and Measurement of Permittivity in Composite Materials at Low Frequencies

Bernardo Tellini[1], Mauro Bologna[2], Mirko Marracci[1], Roberto Micheletti[1]

[1]*Dipartimento di Ingegneria dell'Energia e dei Sistemi del territorio e delle Costruzioni, Università di Pisa, Largo L. Lazzarino, I-56122, Pisa, Italy. bernardo.tellini@unipi.it, mirko.marracci@unipi.it, roberto.micheletti@unipi.it*
[2]*Instituto de Alta Investigación, Universidad de Tarapacá, Casilla 6-D Arica, Chile, mauroh69@gmail.com*

***Abstract* – This paper focuses on the measurement of the effective permittivity $\varepsilon_{\rm eff}$ in composite structures. A reconsideration of some concepts related to the measurement of effective permittivity for dielectric media and composite materials at low frequencies is discussed. In particular, the possibility to measure the average electric displacement and electric field over a region of interest on the basis of the circulation law of the magnetic field by the use of properly shaped Rogowski coils is analyzed. A three-dimensional finite element analysis is carried out to simulate the effective magnetic permeability of a composite materials with the aim to find analogy between the two macroscopic electromagnetic parameters.**

I. INTRODUCTION

Several methods were developed during the years for the characterization of dielectric properties of materials [1]-[6]. At low frequencies, the parallel plate capacitor method is generally adopted, while at high frequencies transmission line (waveguide, coaxial probe and free-space) and resonant cavity techniqes are better applied. The parallel plate capacitor method implies an impedance measurement by a volt-amperometric method ($i - v$ measurement, LCR meter, impedance analyzer) of the bipole under test made by two plate electrodes filled by the investigated dielectric material. Transmission line techniques and in particular free-space methods reconstruct the dielectric properties of the sample material by the measurement of the reflected (S_{11}) and transmitted (S_{21}) signal. Resonant cavities can also be adopted taking into account that a dielectric sample affects the center frequency and the quality factor of the resonant cavity.

Stepping back to the theory, the electric permittivity ε as well as the magnetic permeability μ are macroscopic parameters and relate macroscopic fields in the matter. In particular, ε relates the electric displacement D and the electric field E. Strictly speaking, a relationship between D and H is possible even if in general this is a negligible effect, especially for slowly varying fields [7]. The concept of macroscopic field in the matter as average quantity of unobservable microscopic quantities over a representative spatial region was firstly introduced by H. A. Lorentz [8]. During the two last decades, the studies on metamaterials lead to revisit the concept of effective permittivity and permeability of a composite material having huge "atomic" scales if compared with the physical atomic dimensions. Basing on the theory developed by Veselago in 1968, Pendry et al. presented a practical way to make a lef-handed material with negative $\varepsilon_{\rm eff}$ and $\mu_{\rm eff}$ [9]-[13].

In previous papers two of the authors discussed the possibility of defining new magnetic materials with unusual magnetic properties at undustrial frequencies and revisited the measurement concept of magnetic permeability in composite resonator structures [14]-[17]. In particular, in Refs. [15] and [16] it is shown the possibility to obtain magnetic materials with negative permeability at the industrial frequency range maintaining rather compact the cell dimension of resonator elements. Then, it is discussed the operative meaning of the measurement of $\mu_{\rm eff}$ directly from the Lorentz's theory and it is shown the spatial region over which to average the fields of interest $B_{\rm ave}$ and $H_{\rm ave}$ through which to define $\mu_{\rm eff} = B_{\rm ave}/H_{\rm ave}$.

Following an analogous discussion to that provided in Refs. [15] and [16], we revisit here some concepts related to the measurement of effective permittivity for a dielectric medium and a composite material at low frequencies. The parallel plate capacitor configuration is adopted for the experimental analysis and the average quantities $D_{\rm ave}$ and $E_{\rm ave}$ are defined over the region between the two electrodes. Measurements of $D_{\rm ave}$ and $E_{\rm ave}$ are defined on the basis of the circulation law of the magnetic field along representative closed paths and they are obtained by the use of properly shaped Rogowski coils.

This paper is organized as follows. Section ii. describes the adopted finite element analysis to show the effective magnetic permeability $\mu_{\rm eff}$ of an array of cylinder resonators. Section iii. provides a field analysis of the parallel plate geometry and defines the spatial regions over which to average the quantities of interest to define the effective

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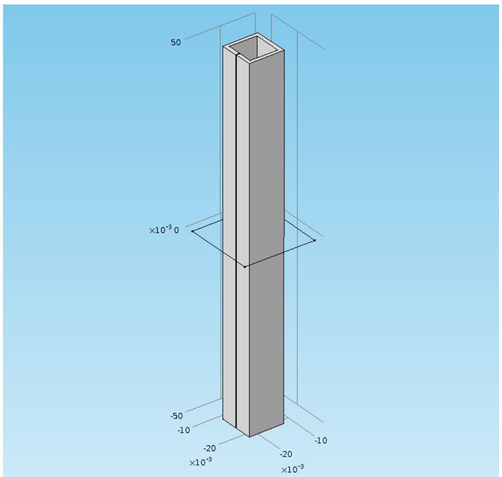

Figure 1: Cylinder resonator.

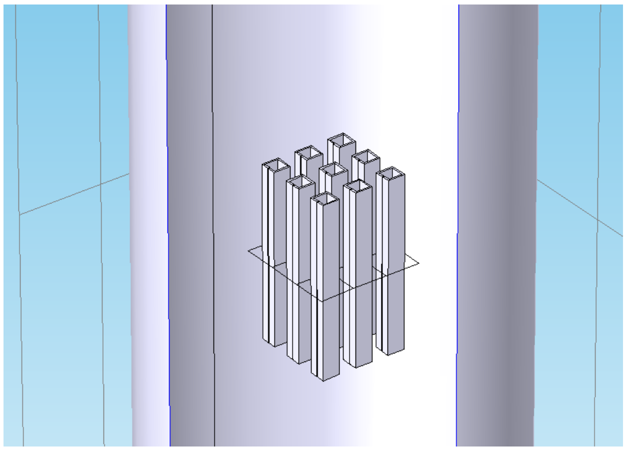

Figure 2: Simulated array of resonators.

ε_{eff} in analogy to what discussed for μ_{eff}.

II. FREQUENCY DOMAIN-FINITE ELEMENT ANALYSIS OF COMPOSITE MATERIALS

We developed here a frequency domain-finite element analysis to describe the effective permeability behavior of an array of resonator solenoids, as it was analytically and experimentally discussed in [15] and [16]. Each single resonator is a hollow cylinder (height of 100 mm) with square cross section (external and internal edge of 10 mm and 8 mm, respectively). A dielectric slice (thickness of 0.2 mm) cuts the cylinder along its axis. All this is shown in Fig. 1.

A 3×3 array of resonators was adopted for our simulations. An external cylinder of 1 m length and 100 mm radius, and axis parallel to that of the resonators is adopted to simulate an uniform magnetic field over the array. This was obtained by applying a tangential magnetic field directed along the cylinder axis on the lateral surfaces of the external cylinder. In Fig. 2 and Fig. 3, the complete simulated geometry and the tetrahedral relevant mesh are shown. According to the discussion developed in [15] and [16], the calculation of the effective magnetic permeability can be easily performed by evaluating the average quantities of the macroscopic magnetic flux density B and magnetic field H over the region of interest represented by the unit cell of the array.

Assuming for simplicity uniformity of the fields along the axis direction, the average calculation of such quantities can be reduced in practice to the evaluation of the their flux through corresponding cross-section surfaces of the unit-cell. Such surfaces are reported in Fig. 4 as S_1 (red surface $-$ left plot of Fig. 4) and S_2 (red surface $-$ right plot of Fig. 4).

In more detail, the effective B_{eff} is given by the B flux

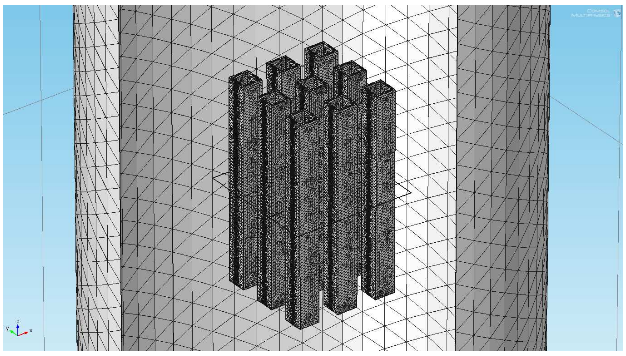

Figure 3: Adopted mesh elements.

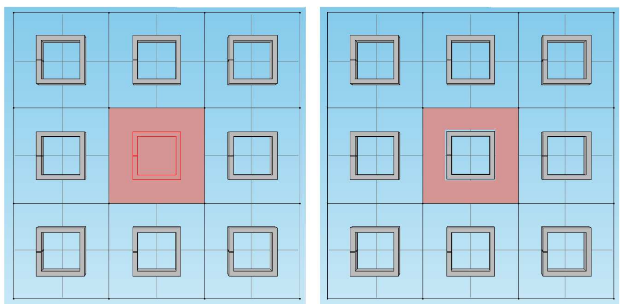

Figure 4: Representative surfaces of the unit-cell: S_1 - red surface left plot; S_2 - red surface right plot.

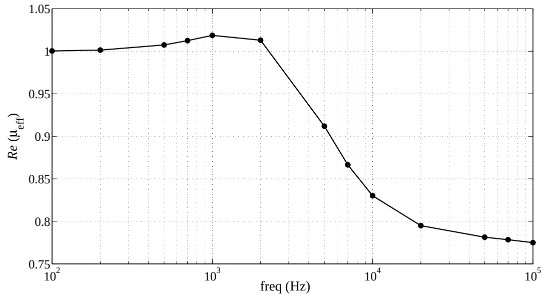

Figure 5: Real part of $\mu_{\rm eff}$.

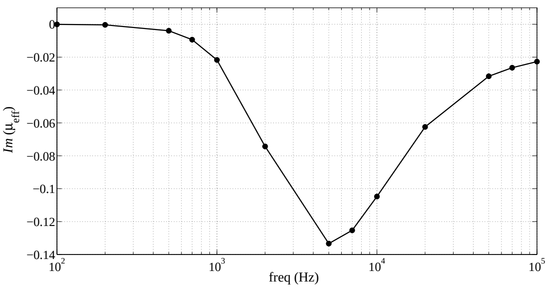

Figure 6: Imaginary part of $\mu_{\rm eff}$.

through S_1 divided by S_1, while the effective $H_{\rm eff}$ is given by the B flux through S_2 divided by S_2. Indeed, in the region represented by S_2 the relationship between the magnetic fields is $B = \mu_0 H$. As a consequence, the flux through S_1 is related to $\mu_0 H_{\rm eff}$. In such a way, the ratio between the two fluxes is representative of the effective permeability $\mu_{\rm eff} = B_{\rm eff}/\mu_0 H{\rm eff}$. Being S_1 and S_2 of different areas a normalizing factor F has to be taken into account, as discussed in [15] and [16].

In order to obtain resonant conditions at industrial frequencies around a few kHz, the relative electrical permittivity of the dielectric slice was chosen with the huge value of 10^{12}.

The simulation was carried out selecting 14 different frequencies in the range 100 Hz $-$ 100 kHz. Results are reported in Fig. 5 and Fig. 6 in terms of real and imaginary part of $\mu_{\rm eff}$. Although the number of adopted frequencies is not so high the resonant effect is clearly visibile around 5 kHz.

III. THEORETICAL ANALYSIS OF EFFECTIVE PERMITTIVITY IN COMPOSITE MATERIALS

In analogy with the previous section and [15] and [16] for $\mu_{\rm eff}$ we derive here a similar theoretical discussion for the effective permittivity $\varepsilon_{\rm eff}$. Assuming a linear relationship between the field D and E over the investigated material, the most general local relationship between D and E for an isotropic dielectric material can be written as [7]:

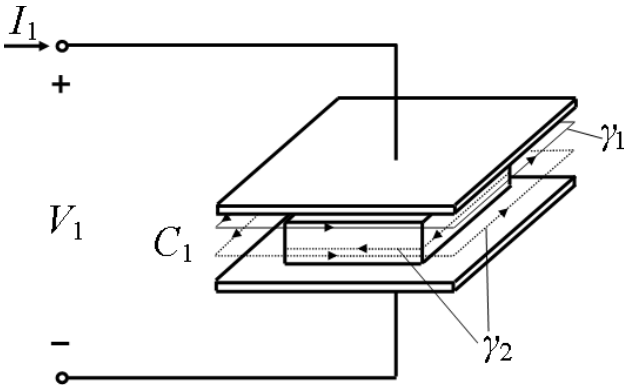

Figure 7: "Single cell" parallel capacitor configuration. Circulation path γ_1 and γ_2 are also shown.

$$D(\mathbf{x},t) = \varepsilon_0 \left(E(\mathbf{x},t) + \int_0^\infty g(\tau) E(\mathbf{x}, t-\tau) d\tau \right) \quad (1)$$

Defining the Laplace - Fourier Transform as $\hat{f}(\omega) = \int_{-\infty}^{\infty} f(t) \exp[-\imath\omega t] dt$ and applying it to (1) we obtain:

$$D(\omega) = \varepsilon(\omega) E(\omega) \quad (2)$$

where the complex permittivity is:

$$\varepsilon(\omega) = \varepsilon_0 \left(1 + \int_0^t g(\tau) e^{\imath\omega\tau} d\tau \right) \quad (3)$$

As written in the Introduction, D and E are macroscopic fields resulting from a spatial volume average of the unobservable field in the matter over a representative region. As better discussed in [15] and [16] such a spatial region is represented by the "single cell" itself, under the assumption that the maximum linear cell dimension is small compared to the field wavelength.

On the basis of such considerations, we can revisit the operative meaning of the measurement of ε through the measurement of the macroscopic fields D and E at low frequencies. To this purpose, let us consider a parallel plate capacitor configuration shown in Fig. 7 for which the investigated composite material is inserted between the two parallel plate electrodes. The average electric displacement over the composite material $D_{\rm ave}$ can be estimated measuring the circulation of $\mu_0 H$ along the external perimeter of the cell as shown in Figs. 7 and 8.

In other terms $D_{\rm ave}$ can be defined on the basis of the circulation law of the magnetic field along the circulation path γ_1 shown in Figs. 7 and 8 and measured by the use of a γ_1-shaped Rogowski coil. Without loss of generality,

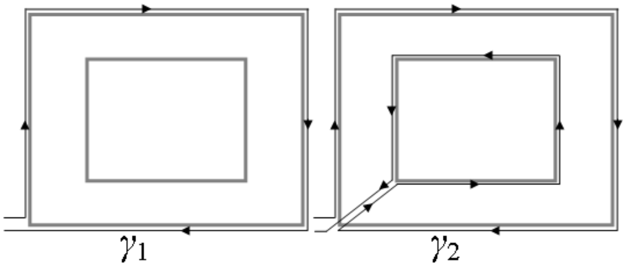

Figure 8: 2D representation of the "single cell" parallel capacitor configuration. Circulation path γ_1 and γ_2 are clearly represented.

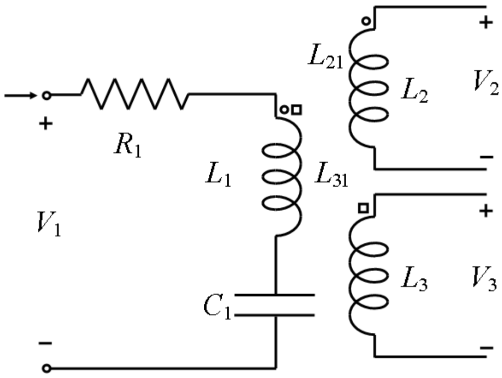

Figure 9: Equivalent lumped parameter model of the defined "single cell" setup.

a multiple turn configuration for the Rogowski coil can be adopted during the experimental validation of the proposed approach in order to mediate the obtained results along the direction perpendicular to capacitor plates (i.e. the region between the two electrodes).

On the other hand, the measurement of the average electric field E_{ave} can be obtained on the basis of the circulation law of the magnetic field along the circulation path γ_2 shown in Figs. 7 and 8 and measured by the use of a γ_2-shaped Rogowski coil. In more detail the circulation of $\mu_0 H$ along the so shaped path (γ_2) is related to the flux of the quantity $\varepsilon_0 E$.

This finds analogy with what discussed in Section ii. for the magnetic permeability. The equivalent lumped parameter model of the "single cell" setup is shown in Fig. 9.

Before concluding this section it is worth to observe the field equations and field distribution over the region between the electrodes. In particular, for a parallel plate capacitor with a given charge distribution $\sigma(t)$ the driving field equations over the region between the two electrodes are:

$$\nabla \times \mathbf{E} = -\partial_t \mathbf{B}, \tag{4}$$

$$\nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \partial_t \mathbf{E}. \tag{5}$$

Applying the Fourier Transform and solving with the condition that the fields take finite value at the origin, we obtain:

$$\hat{E}_z = \frac{\hat{\sigma}(\omega)}{\varepsilon_0} J_0\left(\frac{r\omega}{c}\right), \tag{6}$$

$$\hat{B}_\theta = \frac{\imath\hat{\sigma}}{c\varepsilon_0} J_1\left(\frac{r\omega}{c}\right). \tag{7}$$

Approximating the field for $r\omega/c \ll 1$ and inverting the Fourier Transform, we have

$$E_z(r,t) = \frac{\sigma(t)}{\varepsilon_0}, \tag{8}$$

$$B_\theta(r,t) = \mu_0 \frac{\dot{\sigma}(t)}{2} r. \tag{9}$$

where $\dot{\sigma}(t) \equiv d\sigma(t)/dt$.

IV. CONCLUSIONS

The main purpose of the paper has been to discuss the measurement of the effective permittivity ε_{eff} in composite resonator structures. Starting our analysis from the basic theory of the field in the matter we revisited some concepts related to the measurement of effective permittivity for dielectric media and composite materials at low frequencies. A simple measurement methodology to characterize the effective permittivity in composite resonator structures has been presented based on the measure of the average electric displacement and electric field over a region of interest by the use of properly shaped Rogowski coils. A three-dimensional finite element analysis was carried out to simulate the effective magnetic permeability of a 3×3 array of resonators to discuss the analogy between the measurement of ε_{eff} and μ_{eff} in composite materials.

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