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Distributed Gabriel Graph Construction and Meta-Information Gathering

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Abstract **– In this paper we provide a distributed way to transform the network topology underlying a set of agents embedded in a bi-dimensional space into a planar graph, and specifically into a Gabriel graph. Moreover, a distributed algorithm based on synchronized consensus methodologies is provided in order to gather some useful pieces of meta-information (number of faces, average size of the faces, size of the boundary, etc.) and to let each node in the Gabriel graph identify its "role" (i.e., node that links two subgraphs, node on a branch ending with a leaf, node belonging to a face, etc.). The insights obtained by means of the proposed approach may contribute to develop better geographic routing techniques as well as to improve the local decision capability of the agents in distributed environments.**

Keywords: Gabriel Graphs, Planar Graphs, Synchronized Consensus Algorithms

I. INTRODUCTION

Wireless sensor networks often implement routing protocols that are based on the node's position in order to deliver the messages; among them *face routing* is a mechanism that routes a message along the faces of a planar graph, and is often used as a recovery strategy when greedy algorithms (e.g., send the message to the neighbor which is closest to the target) fail [7].

In this and several other contexts (clustering, robot coordination, etc.) that require a high degree of autonomy for a network of interconnected agents, it is beneficial to derive a planar graph with few links that is optimal in some sense and to gain insights on the properties of such graph, such as the number of nodes, edges, faces, the average size of the faces and of the boundary, etc.

Moreover, it is beneficial to let each node understand its "role" in the network, e.g., leaf node, node that connects two densely connected components, etc.

In this paper we provide a methodology based on synchronized consensus algorithms aimed at transforming a unit disk graph into a planar graph, and specifically into a Gabriel graph, and at gaining insights on the aforementioned meta-information.

The structure of this paper is as follows: Section II describes Gabriel graphs and some of their properties; Section III outlines the proposed distributed methodology to construct a Gabriel graph; Section IV reviews the distributed consensus algorithms, while Section V and Section VI provide the proposed methodology and some conclusions, respectively.

A. *Preliminaries*

Let $G = \{V, E\}$ be a *graph* with n nodes $v_i \in V$ and e edges $(v_i, v_j) \in E$. A graph can be represented by an $n \times n$ *adjacency matrix* A, where $a_{ij} > 0$ if $(v_j, v_i) \in E$ and $a_{ij} = 0$ otherwise. The term a_{ij} represents the *weight* of the edge (v_j, v_i). A graph is said to be *undirected* if $(v_i, v_j) \in E$ whenever $(v_j, v_i) \in E$, and is said to be *directed* otherwise; in the following, where not explicitly stated, each graph is considered as undirected. Let the *degree* d_i of a node v_i be the sum of the weights of its edges, i.e., $d_i = \sum_{j=1}^{n} a_{ij}$.

A graph G is *connected* if for any $v_i, v_j \in V$ there is a path whose endpoints are in v_i and v_j. A graph is acyclic if it does not have cycles, i.e., a cyclic sequence of edges such as $\{(v_1, v_2), (v_2, v_3), (v_3, v_1)\}$. A *spanning tree* $T(V)$ on a set of nodes V is a connected acyclic graph covering all nodes in V; given a graph $G = \{V, E\}$, a *minimum spanning tree* $T(G)$ is a spanning tree over V such that the length of the path connecting any couple of nodes $v_i, v_j \in V$ is minimum with respect to any other path in G. Any graph that may be embedded in the plane such that distinct edges do not intersect except at vertices is termed a *planar graph* [2]. Let an *interior face* of a planar graph be a finite continuous region of the plane bounded by the vertices and edges of a cycle of the graph and having no vertices or edges in their interior. For a planar graph the *euler's formula* holds true, hence $f = e - n + 1$, where f is the number of interior faces.

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Figure 1 provides a taxonomy of the structures that compose a planar graph. Specifically, it holds $V = V_f \cup V_l \cup V_{c2}$, where:

1. the nodes in V_f belong to a face (red, black and purple nodes in the figure), and can be further decomposed as $V_f = V_j \cup V_{c1} \cup V_r$, where
   (a) the nodes V_j (in black) connect two or more faces;
   (b) the nodes V_{c1} (in purple) lie on a path that connects two faces;
   (c) the nodes V_r (in red) belong to a face but are not in V_j or V_{c1}
2. the *leaf* nodes V_l (in green) are actual leafs or are nodes on a path that connect just one face to a leaf node (we will refer to both cases as leaf nodes);
3. the *conenction* nodes V_{c2} (in white) that lie on a path connecting faces but do not belong to a face.

The edges of a planar graph can be decomposed as

$$E = E_p \cup E_i \cup E_c \cup E_l \qquad (1)$$

where: E_p are the *perimeter* edges, i.e., edges that lie on the boundary and belong to a face; E_i are the *internal edges*, i.e., edges belonging to two faces; E_c are the *connection edges*, i.e., edges on a path joining two faces; E_l are the *leaf edges*, i.e., the edges that have at least an endpoint which is in V_l.

Let the *boundary* γ of a planar graph be defined as the number of links on the boundary, where each link that does not belong to a face is counted twice, i.e.:

$$\gamma = e - e_i + e_c + e_l \qquad (2)$$

where e_i is the number of perimeter edges E_p, e_c is the number of connection edges E_c and e_l is the number of leaf edges E_l.

A *Unit disk graph* is a graph where the nodes are embedded in a metric space, such that there is a link between two nodes v_i and v_j whenever the distance between v_i and v_j is smaller than a threshold μ.

II. GABRIEL GRAPHS

Let a set V of n nodes and let $p_i = [x_i, y_i]^T \in \mathbb{R}^2$ be the position of node v_i and let a graph $G = \{V, E\}$.

A *Gabriel graph* is a graph $G_g = \{V, E_g\}$ with $E_g \subseteq E$, such that an edge $(v_i, v_j) \in E$ belongs to E_g if and only if, for each $v_h \in V, v_h \neq v_i, v_j$ it holds

$$||p_i - p_j||^2 < ||p_i - p_h||^2 + ||p_h - p_j||^2$$

where $||p_i - p_j||$ is the euclidean distance between node v_i and node v_j [1, 3]. Equivalently, a Gabriel graph is such that no node other than the endpoints of a link lies within the circumference centered in the midpoint of the link with diameter equal to the link length. A third equivalent definition is that an edge $(v_i, v_j) \in E$ belongs to E_g if and only if, for each $v_h \in V, v_h \neq v_i, v_j$ the angle $\angle v_i v_h v_j$ is acute.

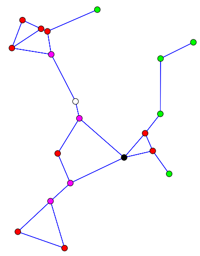

Fig. 1. Taxonomy of the structures that compose a planar graph: the green nodes represent leaf nodes N_l; the white nodes N_{c2} are nodes not belonging to a face that connect two subgraphs composed of faces; red purple and black nodes are nodes N_f belonging to faces, but the purple nodes N_{c1} are nodes that are connected to white nodes and the black nodes N_j are nodes that join two otherwise disconnected faces.

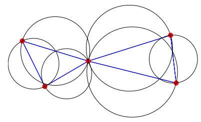

Fig. 2. Example of Gabriel Graph with $n = 5$ nodes. For each link, no node other than the endpoints lies in the corresponding black circumference centered in the midpoint of the link with diameter equal to the link length.

Note that a Gabriel graph G_g is connected whenever the original graph G is connected; moreover, it is a planar graph that contains every possible minimum spanning tree defined over G, although having a number of edges e that is very small compared with other planar graphs [3].

Let β be the average size of all faces of a planar graph G including the size of the boundary γ. In [3] it is shown that, for a Gabriel graph it holds $\beta = \frac{2e}{f+1}$.

Let $\hat{\beta}$ be the average size of the interior faces and let γ be the size of the boundary. By some algebra, it is easy to show that $2e = \hat{\beta} f + \gamma$.

In the following, we will consider Gabriel graphs G that satisfy the following hypothesis.

Hypothesis 1. *The Gabriel graph G is such that any connected subset $V_{c3} \subseteq V_{c2}$ links no more than 2 otherwise separate components of G that contain each at least one node in N_f.*

III. DISTRIBUTED GABRIEL GRAPH CONSTRUCTION

The following Lemma provides a distributed way to obtain a Gabriel graph G_g by substituting the terms a_{ij} of the adjacency matrix of the original graph G with the terms $\hat{a}_{ij}$.

Lemma 1. *Let a network of n agents, each with a known position $p_i = [x_i, y_i]^T \in \mathbb{R}^2$. Suppose the graph G that underlies the network is a connected unit disk graph, and let*

$$\phi_{ijh} = sgn\Big(a_{ij}\Big) sgn\Big(||p_h - \frac{1}{2}\delta_{ij}^+|| - \frac{1}{2}||\delta_{ij}^-||\Big),$$

where $\delta_{ij}^+ = p_i + p_j$ and $\delta_{ij}^- = p_i - p_j$.

If each agent i calculates the terms $\hat{a}_{ij}$ for all $i = 1, \ldots, n$, $j \neq i$ as follows:

$$\hat{a}_{ij} = sgn\Big[1 + \min_{h=1,\ldots,n,h\neq j}\Big(\phi_{ijh}\Big)\Big] \tag{3}$$

then the network represented by the adjacency matrix $\hat{A} = \{\hat{a}_{ij}\}$ is a Gabriel graph with unitary weights.

Proof. Let us consider a generic edge (v_j, v_i) belonging to G. For the edge to belong to the Gabriel subgraph, it must be verified that no other point p_h lies in the Gabriel circle that intersects p_i and p_j; in other words for each $v_h \neq v_i, v_k$:

$$(x_h - x_{ij}^c)^2 + (y_h - y_{ij}^c)^2 > r_{ij}^c$$

where

$$p_{ij}^c = [x_{ij}^c, y_{ij}^c]^T = \frac{1}{2}\delta_{ij}^+(k), \quad r_{ij}^c = \frac{1}{2}\delta_{ij}^-.$$

The term ϕ_{ijh} summarizes the above condition, in that $\phi_{ijh} = 1$ when the condition is fulfilled and $\phi_{ijh} = 0$ otherwise. Since G is a unit disk graph, the Gabriel circle that intersects nodes v_i and v_j is contained in the unit circle centered in v_i, hence the Gabriel condition has to be verified only for the neighbors of v_i. Such condition is verified for all $v_h \neq v_i, v_k$ if it is verified for the node v_h which is closest to the Gabriel Circle, hence the statement is proved. □

Notice that the above method is based just on local information (the position of neighboring nodes). Moreover, agents i and j reach independently the same conclusion on the edge (v_i, v_j), i.e., $\hat{a}_{ij} = \hat{a}_{ji}$ and the resulting graph is undirected.

IV. CONSENSUS ALGORITHMS

Suppose each node in a graph G represents an agent with an initial condition $x_i(0) \in \mathbb{R}$; at each iteration t the nodes update their state as

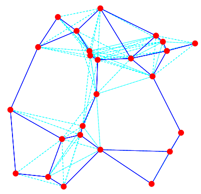

Fig. 3. Example of Gabriel Graph obtained from a unit disk graph with $n = 25$ nodes in random position in the unit square and $\mu = 0.35$. The cyan dotted edges are dropped using the approach in Lemma 1, resulting in a Gabriel graph.

$$x_i(t+1) = \mathcal{U}_i(\{x_j(t) : v_j \in \mathcal{N}_i \cup \{i\}\}) \tag{4}$$

where $\mathcal{U}_i$ is a function of the current state of the node i and his *neighborhood*

$$\mathcal{N}_i = \{v_j \in \mathcal{V} : (v_i, v_j) \in \mathcal{E}\}$$

Let $\chi(x_1(0), \ldots, x_n(0)) \in \mathbb{R}$ be any function of the initial conditions of all the nodes; the *χ-consensus problem* consists in finding a function $\mathcal{U}_i(\cdot)$ such that

$$\lim_{t\to\infty} x_i(t) = \chi(x_1(0), \ldots, x_n(0)) \quad \forall i = 1, \ldots, n.$$

Let us now discuss the max-consensus problem, where the nodes have to converge to the maximum of the initial conditions, i.e., $\chi(\cdot)$ is the maximum of its arguments. The problem, for a connected and undirected graph, is known to have a solution in finite time [4] choosing

$$\mathcal{U}_i(\cdot) = \max_{h\in\mathcal{N}_i\cup\{i\}} x_h(t). \tag{5}$$

In the *average-consensus* problem the nodes are required to converge to the average of their initial conditions, i.e.

$$\chi(\cdot) = c^T \begin{bmatrix} x_1(0) & \ldots & x_n(0) \end{bmatrix}^T$$

where $c^T = \frac{1}{n}1_n^T$ and 1_n is a vector with n components all equal to 1.

Let each node choose

$$\mathcal{U}_i(\cdot) = w_{ii}x_i(t) + \sum_{j=1}^{n} w_{ij}x_j(k) \tag{6}$$

where $w_{ij} = 0$ if $(v_i, v_j) \notin \mathcal{E}$. The update strategy for the entire system can be represented as $x(k+1) = Wx(k)$, where the $n \times n$ matrix W contains the terms w_{ij}.

According to [5], this choice of $\mathcal{U}_i(\cdot)$ yields a solution if and only if : (I) W has a simple eigenvalue at 1 and all other eigenvalues have magnitude strictly less than 1; (II) the left and right eigenvectors of W corresponding to eigenvalue 1 are 1_n and c^T, respectively.

A possible choice for W is $w_{ij} = \tau a_{ij}$ if $v_i \neq v_j$ and $w_{ij} = 1 - \tau \sum_{l=1}^{n} a_{il}$ otherwise [4], where $\tau < 1/\min_{i=1,\dots,n} \sum_{j=1}^{n} a_{ij}$ is a constant parameter which is the same for all the nodes.

V. META-INFORMATION GATHERING

Let us show how to obtain meta-information such as the value of n, e, f, β of a Gabriel graph in a distributed manner. Later in this section we will also describe an algorithm to identify the role of each node in the Gabriel graph (see Figure 1, where the nodes are labeled using the proposed methodology), which is a precondition for the distributed calculation of $\hat{\beta}$ and γ.

A. *Meta-information Gathering*

Combining the max-consensus and the average consensus algorithms, it is possible to calculate the number of agents n in the network in a distributed way [6].

Specifically, suppose a *leader* is elected via max-consensus (e.g., the nodes each have a unique identifier and the node with maximum identifier is elected as leader via max-consensus). Now, let the nodes execute an average consensus algorithm with $\bar{x}_i(0) = 1$ if node v_i is the leader and $\bar{x}_i(0) = 0$ otherwise: the average-consensus yields

$$\lim_{t\to\infty} \bar{x}_i(t) = \frac{1}{n}.$$

As for e, if each agent chooses $\hat{x}_i(0) = d_i/2$ it is easy to show that the average consensus yields

$$\lim_{t\to\infty} \hat{x}_i(t) = \frac{\sum_{i=1}^{n}\sum_{j=1}^{n} \hat{a}_{ij}}{2n} = \frac{e}{n}.$$

The number of faces can be obtained as

$$\lim_{t\to\infty} \frac{\hat{x}_i(t) + \bar{x}_i(t) - 1}{\bar{x}_i(t)} = e - n + 1 = f,$$

while β can be obtained as

$$\lim_{t\to\infty} \frac{2\hat{x}_i(t)}{\hat{x}_i(t) + \bar{x}_i(t)} = \frac{2e}{f+1} = \beta.$$

Notice that, the values of $\hat{\beta}$ and γ can not be easily determined.

B. *Node Role Identification*

Let us provide a brief description of the proposed algorithm for the node role identification, under Hypothesis 1. First of all, the leaf nodes are identified by iteratively letting each node v_i with $d_i = 1$ drop his link and be labeled as leafs (the green nodes in figure 1). The number of leaf nodes and edges is counted using an average consensus over the Unit disk graph where nodes labeled as leaf select $x_i(0) = 1$ and all other nodes select $x_i(0) = 0$; knowing n it is easy to obtain the number of leaf nodes and edges. White and purple nodes in Figure 1 represent connection nodes that are on a path connecting subgraphs composed of faces (the white nodes V_{c2} do not belong to a face, while the purple nodes V_{c1} belong to a face). After removing a link among two non leaf nodes, a distributed connectedness test is executed: if the Gabriel graph without the removed link is disconnected, then the link is a connection link and the nodes are connection nodes.

If this case is verified, removing the link might create a new branch ending with a leaf where the nodes in this branch are all labeled as connection nodes, in a way similar to the approach used for leaf nodes.

As for the connectedness test, it can be executed in a distributed way by letting one of the endpoints of the link choose $x_i(0) = 1$ and all other nodes choose $x_i(0) = 0$; if computing the number of nodes n_{after} after the link removal it holds $n_{after} \neq n$, then the network is disconnected. Note that the number of paths ψ that involve nodes in V_{c1} and V_{c2} can be easily obtained by letting the first node identified on each connection path select $\tilde{x}_i(0) = 1$ and all other nodes select $\tilde{x}_i(0) = 0$, and executing an average consensus it holds

$$\lim_{t\to\infty} \tilde{x}_i(t) = \frac{\psi}{n}.$$

The joint nodes V_j (i.e., nodes belonging to more than one face, but such that each of their edges belong to just one face, see the black node in Figure 1) are identified by iteratively letting each node drop all its links and calculate the number of faces in the graph before and after the link removal: the removal of the links of a joint node is the only case that results in a network disconnection in 3 components (including the node itself) and such that the number of lost faces is smaller than $d-1$ (removing the black node in Figure 1, 2 faces are lost, but the degree of the black node is 4). More in detail, to calculate the number of faces after node v_i drops all its links, the leaf nodes are also removed. Then, the two disconnected components G_{g1} and G_{g2} (not considering node v_i and the leaf nodes) calculate their value of $n_{g1}, n_{g2}, e_{g1}, e_{g2}$ and f_{g1}, f_{g2}. Eventually, node v_i calculates the sum of the values f_{g1} and f_{g2} by choosing the value associated to two neighbors that have different values of f and that are not labeled as leaf nodes.

C. *Distributed Calculation of $\hat{\beta}$ and γ*

First, note that $e = e_p + e_i + e_c + e_l$, where e_p is the number of links belonging to a face that lie on the boundary, e_i is the number of links with at least an endpoint which is an internal node (i.e., nodes belonging to a face that are not on the boundary), e_l is the number of links with an endpoint labeled as leaf node and e_c is the number of edges with both endpoints labeled as connection nodes. Let us further denote by n_j and n_{c2} the number of joint nodes (the black node in Figure 1) and the number of connection nodes that do not lie on a face (the white node in Figure 1).

Lemma 2. *Let a Gabriel graph $G = \{V, E\}$ and let d_i^* be the degree of a node without considering the links E_l. Suppose that G satisfies Hypothesis 1.*

It holds

$$\sum_{i \in V} (d_i^* - 1) = e - e_l + e_i + n_j + \psi. \qquad (7)$$

Proof. Let us assume, for the moment, that there are no leaf nodes, hence $e_l = 0$. Let us first show that Eq. 7 holds true for any graph composed of a single face; in this case $d_i^* = 2$ for each node hence

$$\sum_{i \in V} (d_i^* - 1) = e.$$

Now suppose that two graphs $G_1 = \{V_1, E_1\}$ and $G_2 = \{V_2, E_2\}$ are composed each of just one face, and have e_1 and e_2 edges, respectively. Let us consider a graph $G_{12} = \{V_{12}, E_{12}\}$ which is the fusion of G_1 and G_2, such that an edge (v_{1a}, v_{1b}) of G_1 is merged with an edge (v_{2a}, v_{2b}) of G_2 (the node v_{1a} is merged with v_{2a} and v_{1b} is merged with v_{2b}). The degree of each of the 4 nodes to be merged is equal to 2, while the degree of each of the 2 merged nodes is equal to 3, hence:

$$\sum_{i \in V_{12}} (d_i^* - 1) = \sum_{i \in V_1} (d_i^* - 1) + \sum_{i \in V_2} (d_i^* - 1) = e_1 + e_2$$

and G_{12} has $e_{12} = e_1 + e_2 - 1$ edges; as a result

$$\sum_{i \in V_{12}} (d_i^* - 1) = e_{12} + 1.$$

The above reasoning can be extended to any number of merged links, yielding

$$\sum_{i \in V} (d_i^* - 1) = e + e_i.$$

Let us now consider the case where two graphs G_1 and G_2 such that each link belongs to at least one face are merged by fusing a node $v_{1a} \in V_1$ with a node $v_{2a} \in V_2$. The degree of v_{1a} and v_{1b} are d_{1a} and d_{1b}, respectively, while the degree of the merged node is $d_{1a} + d_{1b}$, therefore

$$\sum_{i \in V_{12}} (d_i^* - 1) = \sum_{i \in V_1} (d_i^* - 1) + \sum_{i \in V_2} (d_i^* - 1) + 1$$

in this case $e_{12} = e_1 + e_2$, hence it holds

$$\sum_{i \in V_{12}} (d_i^* - 1) = e_{12} + e_i + 1.$$

Generalizing to several merged nodes we get

$$\sum_{i \in V} (d_i^* - 1) = e + e_i + n_j.$$

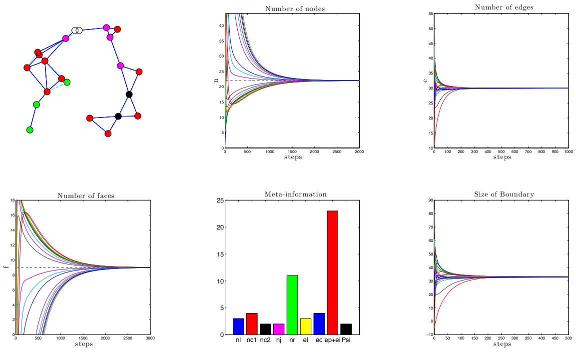

Fig. 4. Example of execution of the proposed methodology.

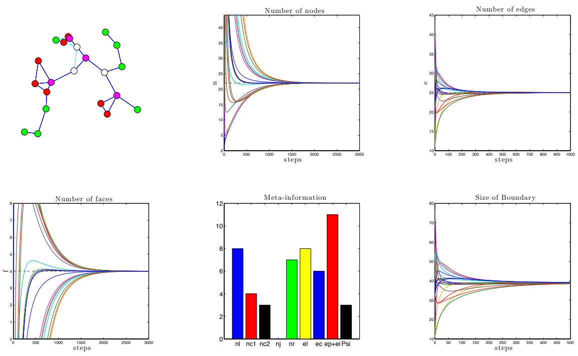

Fig. 5. Example of execution of the proposed methodology when Hypothesis 1 does not hold true.

Let us now consider a case where two graphs G_1 and G_2 such that each link belongs to at least one face are merged by means of a connection path composed of m additional nodes in V_c. In this case the node in G_1 and the node in G_2 that are connected by means of the path belong to V_{c1} and increase their degree by one, while each other node in the path has $d_i^* = 2$, hence

$$\sum_{i \in V_{12}} (d_i^* - 1) = \sum_{i \in V_1} (d_i^* - 1) + \sum_{i \in V_2} (d_i^* - 1) + 2 + m$$

in this case $e_{12} = e_1 + e_2 + m + 1$, hence it holds

$$\sum_{i \in V_{12}} (d_i^* - 1) = e_{12} + e_i + n_j + 1$$

Generalizing to several paths we get

$$\sum_{i \in V} (d_i^* - 1) = e + e_i + n_j + \psi$$

where ψ is the number of such paths. If $e_l \neq 0$, then Eq. 7 is proven. □

Theorem 1. *Let a Gabriel graph $G = \{V, E\}$ and suppose that G satisfies Hypothesis 1. It holds*

$$\gamma = 2e + e_c + n_j - n_{c2} + \psi - \sum_{i \in V} (d_i^* - 1) \quad (8)$$

Proof. Summing Eq. 2 and Eq. 7, the unknown term e_i disappears, and rearranging the terms, Eq. 8 is obtained. □

Remark 1. *Let us show that γ and $\hat{\beta}$ can be computed in a distributed fashion. In order to calculate Eq. 8 in a distributed way, we need just to show how to calculate $\sum_{i \in V}(d_i^* - 1)$, beacuse e can be calculated as described above and e_c, n_j, n_{c2} can be calculated by executing the node role identification and then letting the nodes share information via average consensus (for instance n_j can be calculated if each agent chooses an initial opinion equal to 1 if it is a joint node and 0 else). If each node executes an average consensus algorithm choosing $\underline{x}_i(0) = d_i^* - 1$ if it is not a leaf node and $\underline{x}_i(0) = 0$ otherwise, then it holds*

$$\lim_{t \to \infty} \frac{\underline{x}_i(t)}{\overline{x}_i(t)} = \sum_{i \in V} (d_i^* - 1).$$

Using the above result, both γ and $\hat{\beta}$ can be calculated, as well as the term $e_i + e_p = e - e_l - e_c$; note that the terms e_i and e_p can not be determined, while knowing them would be a first step in the identification of internal nodes. Figure 4 is an example of execution of the proposed methodology: specifically, the distributed calculation of the nodes n, of the edges e and of the faces f is reported, together with the result of the meta information gathering reported in the bar plot and the distributed calculation of γ, which is given in the right-lowermost plot. The roles identified for the nodes are reported on the graph with the same color code of Figure 1.

D. About Hypothesis 1

The above results are limited to Gabriel graphs G that satisfy Hypothesis 1. Figure 5 provides an example when Hypothesis 1 does not hold. In this case $\gamma = 38$ but the agents estimate $\gamma = 39$. It can be noted in this case that there are 3 components with nodes in V_f that are linked via a connected path with nodes in V_{c2}, and the algorithm proposed fails to identify the nodes that connects the paths (the central purple node), which is considered as a node in V_{c1} instead of V_{c2}. The cases when Hypothesis 1 is not verified seem to be quite rare. In a first experiment we consider a network with $n = 20$ nodes with random uniform positions in the unit square and $\mu \in [0.2, 0.5]$ with sampling steps of 0.05 and we generate 1000 graphs for each value of μ: each graph generated respects Hypothesis 1. Figure 5 suggests that graphs that violate Hypothesis 1 might be quite sparse, hence in a second experiment we focus on low-connectivity networks, considering a network with $n = 20$ nodes with random uniform positions in the unit square and $\mu \in [0.2, 0.3]$ with sampling steps of 0.01 and generating 1000 graphs for each value of μ: again, each graph generated respected Hypothesis 1.

VI. CONCLUSIONS

In this paper we provide a distributed way to obtain a planar graph and calculate some useful metainformation as well as detect the role of each node.

Future Work will follow two main directions. First, we will extend the proposed methodology to Gabriel graphs that violate hypothesis 1. Then we will use the metainformation obtained in order to design more effective georgraphic routing techniques for wireless sensor networks.

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