20th IMEKO TC4 International Symposium and
18th International Workshop on ADC Modelling and Testing
Research on Electric and Electronic Measurement for the Economic Upturn
Benevento, Italy, September 15-17, 2014

A Fast Maximum Likelihood Estimation for High-Resolution ADC Test

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***Abstract* –With the increasing resolution of analog to digital converter (ADC), the percentage of the test cost in the overall chip cost is improving gradually. Therefore, faster and more accurate test methods are required urgently for the high-resolution ADC nowadays. Compared with the least squares(LS) method, the maximum likelihood(ML) estimation method can extract more information from output codes, and estimators of measured parameters are consistent and asymptotically efficient. Due to the ADC resolution is increasing, the number of transition levels raise exponentially which causes huge test parameters space. The traditional ML estimation method can't tackle the situation of the high-resolution ADC. This paper focuses on the problems of how to reduce the computation complexity and implement high-resolution ADC test with ML estimation method. The main finding is that the proposed method reduced test time in comparison of the traditional ML method. And it makes the ML estimation method use in high-resolution ADC test.**

Keywords—ADC test, maximum likelihoodestimation, segmentation method, differential evolution (DE)

I. INTRODUCTION

Dynamic testing of analog-digital converters (ADC) is important in practice. The IEEE standard defines sine-wave fitting for ADC test, which recovers the sine signal from the output recordsusing the LS method [1]. In theory, LS fitting gives the best estimation under the condition that the observation noise is additive white Gaussian noise, which is independent, white, and normally distributed with zero mean [2]. However, there are many kinds of noise model in ADC testing [3-4]. LS fitting can't properly handle ADC nonlinearity or overload situation.

Recently researchers introduced ML estimation, which is an improvement to the usual three-parameter and four-parameter fitting. According to estimation theory, ML estimation has the advantage of solving complicated problems and obtaining practical estimators [5]. At present, researchers defined the proper ML function[6], which base probabilityon the transition levels of the ADC, and the key problem is the model of the noise; formulated the numerical method [7], which is based on a sine wave excitation and minimize it. What's more, the researcherscompared LS method with ML method [8]. They find that ML estimation has similar results with LS fitting in linear ADC test, but it is better than LS fitting in nonlinear ADC test. Because the number of transition levels increase with the number of the ADC resolution, excessive unknown parameters limit the application of the ML estimation in high-resolution ADC test.

Therefore how to reduce the complexity of estimating unknown parameters becomes especially important. Generally, there are the following solutions in academic field: In [7], thewhole set of parameters is divided into smaller numberof subgroups, using backtrackingline search to compute every subgroup. In [9], the paper usesLaplacian noise model to accelerate the algorithm. Properly approximations are applied which simplify the computation. Method introduced in [10] uses traditional histogram method to obtain transition levels, and Nelder-Mead methodis used to obtain signal parameters.Although methodsintroduced above solve the feasibility of ML estimation in the application of high-resolution ADC test. However, the test time of those methods is not short enough for the productionapplication.

Another problem in this paper is how to obtain transition levels accurately. The transfer function of the A/Dconverterformsamany-to-onemapping relationship between input signal values and digital codes.In general, researchers transfer the many-to-onemapping relationship to one-to-one mapping relationship by two methods. One use intermediate code points, another use code edge. In this paper, we need the transition point whichis a point of the analog signal value at which a digital code transitions from one code to the next code.Traditional histogram method compute the transition levels by statistics method, which needs many samples. In order to obtain the transition levels quickly, we use the fitting method to describe the transfer curve in the proposed method.

There are two methods to solve the ML function: gradient-based minimization and differential evolution. Due to the results of the two methods are the same approximately, the DE method implement easily when the unknown parameters is few.

In this paper, we proposea fast ML estimation method. The ML function bases on the probability of the transition levels which under the condition that the model of the noise is the gauss white noise. The transition levels are estimated by parameter spectral estimation, and segmentation technology is used to improve the precision of estimation. Then DE method is usedto estimate the rest of parameters.

ISBN-14: 978-92-990073-2-7

II. PROPOSED METHOD

In order to obtain accurate transition levels with fewer samples and accelerate test speed of high-resolution ADC characterization using ML method, we propose a transition levels estimation method using parametric spectral estimation. The segmentation of transfer curve is used to improve the accuracy of transition levels estimation. The DE method is chosen to solve ML function and obtain four sine signal parameters and noise standard deviation.

Fig.1 shows the test flow of proposed method. Firstly, the segment model of high-resolution ADC is established with optimal number of windows. Secondly, the transfer curve of each segment is obtained by parametric spectral estimation. The number of sample points in each segment is optimized in order to balance the accuracy of test results and the speed of the test method. The total transfer curve of ADC under test is obtained after dealing with the overlapping between adjacent segments. Then all transition levels used in ML estimation are obtained by transfer curve. After that, the DFT and LS method are used to acquire the initial values of sine parameters needed in ML estimation. Then DE method is selected to solve ML function, and approach the optimal solution by mutating and improving the candidate parameters from the initial ones. Finally, the dynamic characteristics of ADC are calculated from the fitting results of sine signal.

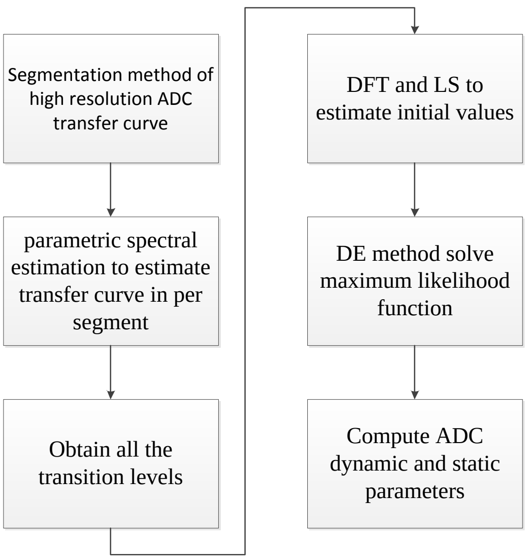

Fig. 1. Flow of Proposed Method

A. Parameter spectral estimation with segmentation

In theory, all the codes have the equal code widths. In fact, all code widths are not equal because of manufacturing imperfections. So the true transfer function is nonlinear which can be express by a set of basis functions. In this paper, we use *parameter spectral estimation*as the basis function. In order to improve the accuracy of parameter spectral estimation, we choose appropriate number of rectangular windows and dividehigh-resolution ADC's transfer curve into the appropriate segments. In each segment, parameter spectral estimation method introduced in [12-13] is applied to estimate the transfer curve. The method is based on Chebyshev polynomials to obtain transfer curve, which reduces sample points significantly. The total transfer curve of ADC under test is obtained after averaging the overlap section. The optimal number of segments depends on the sample points, the number of the ADC resolution and algorithm complexity [11].

B. ML estimation

The critical problem of ML method is how to define ML function for the probable ADC input signal. The probability density function is defined as

$$F(x,u,\sigma) = \frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{x} e^{-\frac{(x-u)^2}{2\sigma^2}}du \tag{1}$$

where *x* is the measured value of ADC output code, S is thestandard deviation of the Gaussian noise,*u* is the signal expression which contains unknown parameters. The noise samples are assuming to be independent. The ML function L(a) can be written as：

$$\max_{a,q,\sigma}(L(a)) = \max_{a,q,\sigma}\prod_{n=0}^{N-1} p(y(n) = Y_{a,q,\sigma}(n)) \tag{2}$$

where*q*is the vector of transition levels, *a*is a vector of its unknown parameters.

Because the nth ADC output sampley(n) is equal to the measured output valueY(n), which is a probability of outputcode distribution, we obtain:

$$P(y(n)=Y(n)) \tag{3}$$

We maximize likelihood function L(a) to solve equation. To simplify computation, we optimize the likelihood function by taking log-likelihoodfunctions.

$$\arg\max_{a,q,\sigma}(L(a)) \approx \arg\min_{a,q,\sigma}(-In(L(a))) = \arg\min_{a,q,\sigma}(-\sum_{n=0}^{N-1} InP(y(n) = Y_{a,q,\sigma}(n))) \tag{4}$$

The differentialevolution (DE) method is used to solve the ML function. Itis a kind of evolutionary algorithm based on real number encoding. The basic idea and overall framework is similar to genetic algorithms. The algorithm mainly involves the following parameters: the population size N, the individual dimension D, the variation factor F, and the crossover probability CR.The values of the four parameters directly affect the accuracy of the final result, so they need to beadjustedduring practical operation.Fig.2 is the flow of the DE method.

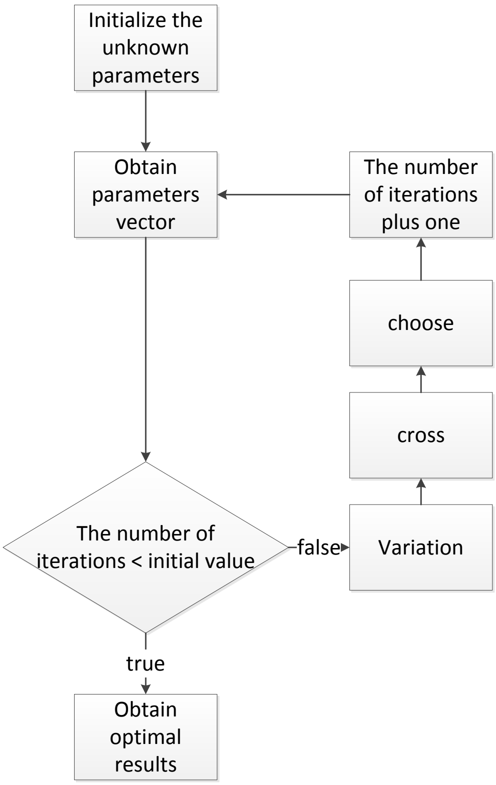

Fig. 2. Flow of DE algorithm

Although DE does not need initial values of unknown parameters, weestimate the initial values to improve the convergence of algorithm and reduce the number of iterations. The initial frequency is obtained by DFT method, and the initial values of signal parameters and the standard deviation ofthe noise are obtained by LS fitting.

III. NUMERICAL RESULTS

In this section, simulation resultsare presented to illustrate the improved speed of the proposed method. The comparison of the proposed method and 4PSF method proved the accuracy and precision. The samples of the proposed method are less than the traditional method vastly which show the rapidity of the proposed method.

The main difference between the ML estimation based on histogram method and the proposed ML method is the estimation of the transition levels.This paper use parameter spectral estimationand segmentation method to obtain transition levels, while the traditional ML method use histogram to obtain transition levels which needs many samples. In this paper, a 14-bits ADC is used to perform the simulation.

The number of segmentation is configured to 4, and 2048 samples in each segment are used to estimate the transfer curve. The total number of samples is 8192, which is much less than the number used in histogram method.

The INL of the ADCwhich obtained from theparametric spectral estimationand segmentation shows in Fig. 3. The experimental results show that the combination of segmentation method and parameter spectral estimation improves the accuracy of transfer curve estimation.

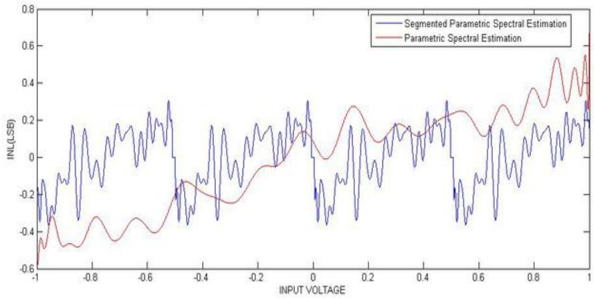

Fig. 3. The comparisonbetween the INL of the proposed method and the INL of the parametric spectral estimation method

According to the transfer curve, the transition levels are obtained, which is used as the known parameters in ML function. Before the whole iterative procedure, the rest of the initial values used in DE method are obtained by DFT and LS fitting.

In order to proof the accuracy and feasibility of the proposed method, the simulation results are compared to the results obtained by four-parameter sine(FPSF) fitting.The signal to noise ratio is configured to 80dB to perform the simulation.

Our goal is to proof the accuracy and feasibility of the proposed method. Therefore this simulation compared the results of the proposed method and of the four parameter sine fitting, which contain the fitting sine wave, the residual error of the fitting function and output codes. Also, this paper calculated the values of ENOB and SINAD of this two methods, which are typical performance parameters of an ADC.

The signal from the ADC output codes can be expressed as follows:

$$y(n) = s(n) + w(n), n = 1,2,...,N \quad (5),$$

where the s(n) is the sine wave of the ADC output codes, w(n) is the noise which cotains the error introduced in the input signal and the ADC. Morever, the fin is chosen to satisfy the ratio between fin and fs as follows:

$$\frac{fin}{fs} = \frac{M+\delta}{N} \quad (6),$$

where (M+ δ) is the number of cycle.

The expression of residualerrorpresents as follows, which is important to calculate dynamic parameters.

$$\eta^2_{rms} = \frac{1}{N}\sum_{n=1}^{N}(y[n]-x[n])^2 \quad (7),$$

where x[n] are the values from recovered signal.

Therefore, the SINAD can be caculated as follows:

$$SINAD = 10\log_{10}\frac{\frac{A^2}{2}}{\eta_{rms}^2} \quad (8)$$

where A is the amplitude of fitting sine-wave.

The ENOB represents the accuracy of the ADC output under the given excitation, which is one of the most performance parameters of an ADC. In order to estimate the value of the ENOB, this paper use the definition given in the ADC testing standard as follows,

$$\hat{ENOB} = N - \log_2\frac{\eta_{rms}}{\sigma_q} \quad (9),$$

where σ_q is the ideal root-mean-square value of the quantizationerror, which is evaluated under the assumption of auniformly distributed error. The expression represents as $\sigma_q = Q/\sqrt{12}$, in which Qis the ideal code binwidth of the ADC under test.

Fig.4 shows the comparison of the fitting result obtained by the proposed method and the four-parameter fitting method. They are almost same.

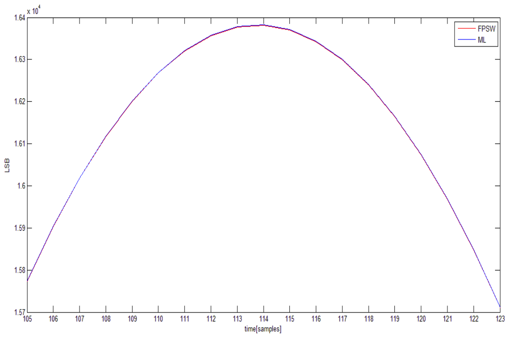

Fig. 4. The fitting results of the two methods

Fig.5 and Fig.6 show the residual error obtained by the two different methods. Under the same condition, the residual error of the proposed method is about half of the residual error obtained by the FPSF fitting.

Finally, the comparison of SINAD and ENOB calculated by definition is presented in Table 1. The result shows that the proposed method is more accurate than four-parameter sine fitting.

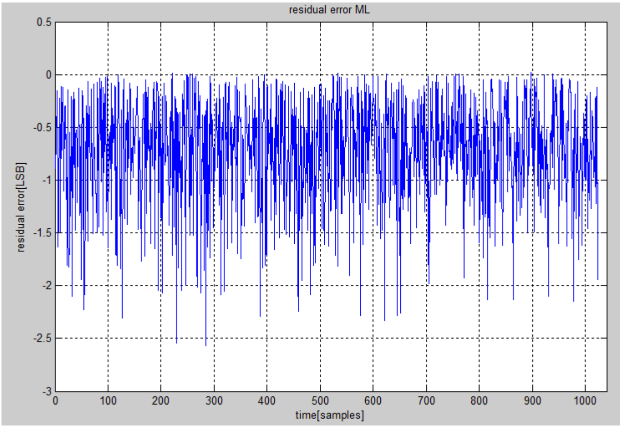

Fig. 5. The residual error of the proposed method

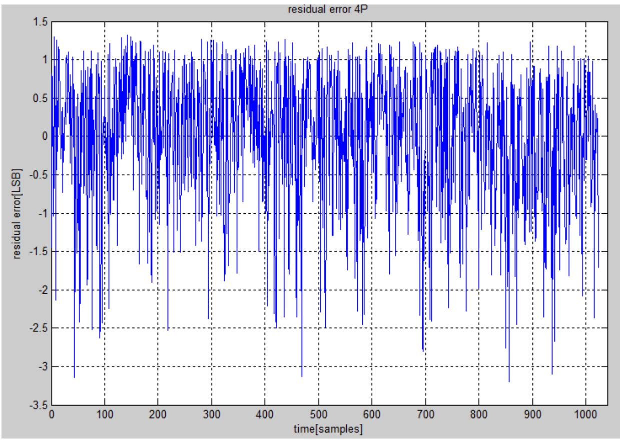

Fig. 6. The residual error of the FPSW fitting

Table 1 The comparison ofSINAD and ENOB results

	SINAD/dB	ENOB/bits
Proposed method	76.53	12.42
FPSF method	73.53	11.92

The key question is how to obtain the transition levels correctly. This paper obtains the transition levels by the transition curves which obtain by the parameter spectral estimation with segmentation. We can't obtain the transition levels from the transition curves directly. So we fix the digital codes, obtain the transition levels from the transition curves which extracting from the output information of the ADC. Due to the nonlinear of the transition curves of the ADC, each section of the transition curves can express by a kind of math functions. In theory, this is a good idea. However, the difficult job is that how to find each math function of these sections. In this paper, we use parameter spectral estimation in each section. One of the follow jobs is that we can find kinds of math functions which are more better to show the transition curves, like the least squares fitting.

We also tackle the data from the ADS62P43 by the proposed method. However, we can't obtain the results which we expected. So we think the proposed method has some questions in practical testing.

Although the LS method is easy to apply and obtain the values of the parameters which are more better, ML method show the performance of the ADC more accurately. In fact, the values of the parameters which obtain from the ML method maybe are worse than that of the LS method. In this paper, in order to prove the accuracy of the proposed method, we compared the ML method with the LS method. Due to the results are obtained from simulation, the values of SINAD and ENOB are better than that of the LS method. The traditional ML method uses the histogram method to obtain the transition levels which needs many samples. Obviously, the proposed method has the high speed.

IV. CONCLUSIONS

A fast ML estimation method for high-resolution ADC test is proposed in this paper. Compared with traditional ML estimation method, this method reduces the time of sampling significantly. And it also provides more accurate results. Moreover, this method makes it's possible for the ML estimation application of high-resolution ADC.

V. ACKNOWLEDGMENT

This work was supported by National Science and Technology Major Project(No.2013ZX03001011-003).

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