Uncertainty to harmonic measurements with DFT techniques

Marcus Vinicius Viegas Pinto[1], Noara Foiatto[2], João Miguel Lac Roehe[3]
Marcos Augusto Stemmer[4]

[1] LABELO-PUCRS. Av. Ipiranga 6681, P 30, Office 210, Porto Alegre. RS. Brasil. CEP 90619-900.
Phone 55 51 3320 3551. Fax 55 51 3320 3901. mviegas@pucrs.br
[2] LABELO-PUCRS. Av. Ipiranga 6681, P 30, Office 210, Porto Alegre. RS. Brasil. CEP 90619-900.
Phone 55 51 3320 3551. Fax 55 51 3320 3901. noara@pucrs.br
[3] FENG-PUCRS. Av. Ipiranga 6681, P 30, Office 229, Porto Alegre. RS. Brasil. CEP 90619-900.
Phone 55 51 3320 3551. Fax 55 51 3320 3901. joaomiguel@pucrs.br
[4] FENG-PUCRS. Av. Ipiranga 6681, P 30, Office 313, Porto Alegre. RS. Brasil. CEP 90619-900.
Phone 55 51 3320 3551. Fax 55 51 3320 3901. stemmer@pucrs.br

Abstract

The harmonic content used by the electronics industry adopts the concept of total harmonic distortion, especially for line frequency (50Hz and 60Hz). Currently, for some services is necessary more information about harmonics, such as the amount of distortion on each order, to a large frequency spectrum. The aim of this work is the measurement uncertainty applied to a system developed for harmonics measurement, with traceability to DC voltage levels. The system is a software developed in visual basic language, called DIGITHARM, with a multimeter capable of sampling signals with rates up to 100k samples per second. The DIGITHARM, trough the Discrete Fourier Transform (DFT), determines the amplitude and frequency to the harmonic signals. Its measurement uncertainty stems from the multimeter specifications, like its analog digital converter, associated to DFT programming. As results, are highlighted the measurement capability trough the treatment of some uncertainty contributions, which are based on mathematical validations (for offset values), and its application range in terms of frequency values (up to 100kHz).

Keywords: Harmonics; Calibration and Measurement Capabilities; Uncertainty; Discrete Fourier Transform.

1. Introduction

This work was conducted under the covering of LABELO laboratories, located at the Pontifical Catholic University of Rio Grande do Sul (PUCRS), to describe the measurement capability through the system development for taking harmonic measurements. These measurements have traceability and uncertainty levels that are able to meet the demand of this kind of service. For this, was used the Discrete Fourier Transform (DFT), and the harmonic information was referenced to DC voltage levels.
Currently, some Institutions have done similar works in this area, like the Swedish National Testing and Research Institute, Sweden [1], the National Institute of Metrology, Quality and Technology (INMETRO) [3], in Brazil, the National Institute of Industrial Technology (INTI) [2], [13], in Argentina, and the Physikalisch-Technische Bundesanstalt (PTB) [4], in Germany. This paper describes the difference between those works, and its aim is to present the measurement capability according its new methods and uncertainty contributions.

2. Measurement method

The DFT was adopted to analyze the harmonic contents. The digital multimeter Agilent 3458A is able to digitize an arbitrary waveform with up to 100,000 samples per second [6]. With IEEE 488.2 protocol and the visual basic language, the application was developed in order to take the calculations and routines for data analysis, such as communication with the multimeter, the algorithms to sampling period and the length of the finite sequence, that will be performed the DFT. The developed system, was called DIGITHARM, and allows effective sampling rates until 100M samples/sec with a 12MHz bandwidth, as shown in Table 1.

Table 1. DIGITHARM digitizing methods [6]

Digitizing method	Maximum sampling	Bandwidth	Repetitive signal required
DCV	100 k/sec	DC - 150kHz[1]	No
Direct-Sampling	50 k/sec	DC - 12MHz	No
Sub-Sampling	100 M/sec[2]	DC - 12MHz	Yes

The choice for the sub-sampling method is based on the repetitive characteristic of the signal that is being measured, that allows a higher bandwidth and the use of the maximum sampling rate available. In the sub-sampling method, the multimeter uses one or more samples for each input signal period. Considering each successive period, the point where is done the first sample is delayed a bit, and then more samples are taken. After to complete the amount of periods required to complete the samples set, the multimeter reorganize the samples to ordinate them in the right sequence. The method advantage is that the input signal samples can be effectively spaced in a minimum interval of 10 nanoseconds, compared to a minimum of 10 microseconds for samples in the DCV method and 20 microseconds for direct sampling. The figure 1 illustrates the sub-sampling operation.

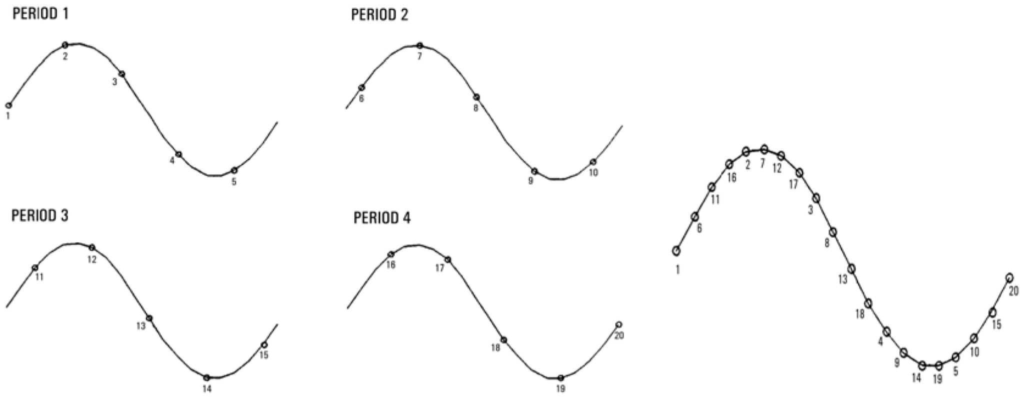


Figure 1. Representation of the sub-sampling method and the composite waveform [6]

The choice of the sub-sampling method causes an increase in the system uncertainty, because there is a reduction in its measurement resolution. However, with this method it's possible too, a substantial increase in the sampling rate, allowing an extension in the frequency range (up to 100kHz). For harmonic measurement, two methods are normally used, the synchronous and the asynchronous sampling method [4], [7], [8] and [9]. Every measurement taken by the multimeter was made by asynchronous mode. To calculate the DFT, is very important to detect the sampled waveform period. This period can be accomplished by a previous measurement of the fundamental frequency, from which shall be defined the effective sampling rate that is required to perform the sampling in a complete input signal period, or it can be done choosing the number of the samples large enough to make the error sufficiently small for the desired application. The latter method was implemented in DIGITHARM, that through the least squares method executes the detection period and apply the DFT on the detected period. Instead of 128 or 256 samples usually carried out in this kind of system, it has in one period, approximately 2000 samples, which provide a wider range of harmonics, as well as frequency higher than the usual (50Hz and 60Hz). This approach allows to use DIGITHARM for acoustic applications [14], providing the measurement of total distortion of the signal sound pressure in a frequency range up to 20kHz. In addition, the DIGITHARM makes possible the measurement of the each harmonic relative to the fundamental, for a number of harmonics up to samples half and the total harmonic distortion to one quarter of the samples number. In this case, for one period with 2000 samples, it has an individual distortion of 999 harmonic and a total distortion of 500 initial harmonics. In order to check these advantages, its measurement uncertainty was calculated, due to the need to adapt some of their contributions.

3. Measurement uncertainty

A measure isn't complete without the expression of its uncertainty. The methodology to calculate the measurement uncertainty was based on [10], from the equation 1 [11], where the contributions considered were factors from the multimeter, like its construction and settings, as well the measurement method (Table 2).

$$F(n) = \frac{1}{N}\sum_{k=0}^{N-1} f(k)\left(\cos\frac{n\pi k}{N} + j\sin\frac{n\pi k}{N}\right) \quad (1)$$

Where:

- N: total samples digitized
- k: sample index
- n: DFT index

Table 2. Resume of uncertainty contributions

Component	Unity	
UMP resolution	[V]	$\boldsymbol{RES_{UMP} = f_{MM} \times 0,0001\%}$
Specification for voltage measurement [6]	[V]	$ESP_{UMP} = f_{MM} \times 0{,}02\% + Offset_{2\sigma}$
ADC Quantization error (analog digital converter) [1]	[V]	$EQT_{UMP} = \frac{2 \times Span_{range}}{2^{16}}$
Loading error (multimeter input capacitance) [6]	[V/V]	$ELD_{UMP} = \frac{\left(f_{MM} - \frac{f_{MM}}{Z_{UMP}+50} \times Z_{UMP}\right)}{f_{MM}}$ Where: $Z_{UMP} = \left(\frac{10M\Omega \times \frac{1}{2\pi f 140pF}}{10M\Omega + \frac{1}{2\pi f 140pF}}\right)$
Quantization noise [1]	[V/V]	$ERQ_{UMP} = \frac{1}{6 \times N \times 2^{2(n-1)}}$
Bandwidth error [1] and [5]	[V/V]	$EBW_{UMP} = \frac{1}{\sqrt{1+\left(\frac{f}{f_g}\right)^2}} - 1$
Aperture time error [5]	[V/V]	$ETA_{UMP} = \frac{\sin \pi f T_a}{\pi f T_a} - 1$
DFT Quantization Standard Deviation [1]	[V]	$EQT_{UMP} = \sqrt{\frac{\left(\frac{Span_{range}}{2^n}\right)^2}{(6N)}}$
DFT Jitter Error [1] e [6]	[V]	$EJT_{UMP} = \sqrt{\left(\frac{2\pi f \Delta t \times Span_{range}}{12}\right)}$

Where:
UMP: reference standard
RES: resolution
ESP: manufacturer specification
f_{MM}: average measurements to fundamental harmonic
f: analyzed harmonic frequency
f_g: maximum frequency for the multimeter range
N: samples for a period waveform
n: ADC resolution (bits)
T_a: ADC aperture time
Δt: jitter time interval

ISBN-10: 84-616-5438-2 | ISBN-13: 978-84-616-5438-3

Some aspects related on table 2 should be highlighted, such as the multimeter manufacturer's specification, through the characterization process of its offset. This process was conducted considering 100,000 samples for each voltage range used by DIGITHARM. The multimeter operation manual informs its performance for the functions of Sub-Sampling Direct Current (SSDC), Sub-Sampling Alternating Current (SSAC), Direct Sampling Direct Current (DSDC), Alternating Current and Direct Sampling (DSAC) for 16 bits resolution. This specification is defined as 0.02% of reading plus an offset value defined for each range. To evaluate the effective offset contribution, the multimeter after the warm up time, was submitted to routine self-calibration (ACAL) in all of its functions, as recommended by its operation manual. Through its input terminals short-circuited with a rigid copper wire and SSDC mode, the measurement was triggered by its external trigger input, with a square waveform in 50 Hz, 5Vpp and 20% pulse width. The measurements were taking considering a setting of effective sampling rate of 10µs, with 100k digitized samples. The measures were evaluated according to two standard deviations for the total sample. To the average offset, was adopted the equation 2 (for 95.45% confidence level) and the results obtained were used for the uncertainty measurement (Table 3).

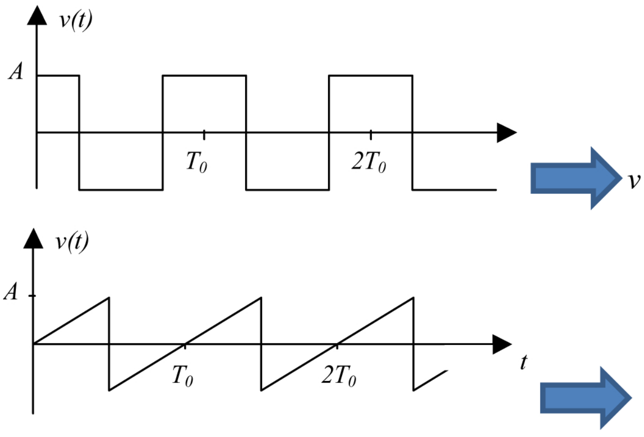
$$\boldsymbol{Offset} = \sqrt{\boldsymbol{\mu_{MM}^2} + \boldsymbol{MM_{2\sigma}^2}} \qquad (2)$$

Where:
μ_{MM}: Standard deviation
$MM_{2\sigma}$: Average (2σ)

Table 3. DIGITHARM offset values

Multimeter range (UMP)	**100mV**	**1V**	**10V**	**100V**	**1000V**
Offset 1σ	3µV	14,5µV	110µV	3,9mV	77,9mV
Offset 2σ	8µV	36,4µV	240µV	11mV	185mV
Offset [6]	<90µV	<800µV	<8mV	<80mV	<800mV

The DIGITHARM validation was also considered in its capability measurement. For this, was adopted the waveforms with high distortion, mathematically defined, such as the square and sawtooth waveforms (Figure 2). The sawtooth waveform has amplitudes of harmonics, composed by 1 / n, where n is the harmonic index (with n = 1,2,3...n) for the odd and even numbers [12]. Similarly, is composed the harmonic amplitude of the square waveform, however, only for an odd number.



$$v(t) = \frac{4A}{\pi}\left[\cos(\omega_0 t) - \frac{1}{3}\cos(3\omega_0 t) + \ldots\right]$$

$$v(t) = \frac{2A}{\pi}\left[\operatorname{sen}(\omega_0 t) - \frac{\operatorname{sen}(2\omega_0 t)}{2} + \ldots\right]$$

Figure 2. Mathematical definition for waveforms with high distortion

To the system validation was used numerical simulation and a function synthesizer, to generate two waveforms, square and sawtooth, with 5ns edge time. As results to the DIGITHARM measurements, the highest error found was about 0.024% and 0.025%, respectively, for square and sawtooth waveforms. Another step in this validation was taken by comparing the measurements from the DIGITHARM with the values from the equipment calibrated by INMETRO. In this case, the results obtained were lower than limits applied to uncertainty contributions considered by DIGITHARM.

4. Results

The uncertainty contributions were estimated for amplitudes at the beginning and the end of each measurement range, for 100Hz up to 100kHz. These values were calculated in reference to the fundamental amplitude and were linearized in order to obtain the Calibration and Measurement Capabilities (CMC) to DIGITHARM (table 4). The V_{RMS} in table 5 is the root mean square value of the fundamental given in volts. This form of presentation allows to know the measurement capacity according to the main variables applied to harmonic measurement.

Table 4. DIGITHARM CMC

Measurement range	**CMC (V)**
Distortion: 0% to 100% - 1V to 10V (Harmonic frequency range: 100Hz to 1kHz)	-0,0040% V_{RMS} + 0,00053
Distortion: 0% to 100% - 1V to 10V (Harmonic frequency range: 1kHz to 3kHz)	-0,0043% V_{RMS} + 0,00053
Distortion: 0% to 100% - 1V to 10V (Harmonic frequency range: 3000Hz to 7788Hz)	-0,0041% V_{RMS} + 0,00053
Distortion: 0% to 100% - 1V to 10V (Harmonic frequency range: 7788Hz to 31152Hz)	-0,0028% V_{RMS} + 0,00052
Distortion: 0% to 100% - 1V to 10V (Harmonic frequency range: 31152Hz to 70092Hz)	-0,00047% V_{RMS} + 0,00051
Distortion: 0% to 100% - 1V to 10V (Harmonic frequency range: 70092Hz to 100000Hz)	-0,0011% V_{RMS} + 0,00052

5. Conclusions

The DIGITHARM is a tool for expanding the scope of services provided by LABELO for its metrology services. Its uncertainty levels are sufficiently low to attend the Brazilian industrial demand, but they are about ten times greater than the uncertainty obtained by measurement systems such as INMETRO or PTB. These values are in order to 0.0040% for INMETRO and 0.040% for LABELO. The main difference between those is that the INMETRO [3] and PTB [4] systems are optimized for frequencies close to 60Hz, which the multimeter configurations have lower error bounds, but that limits its bandwidth. The DIGITHARM is applied to a broader frequency spectrum than INMETRO [3] and PTB [4], with harmonic measurements up to 100kHz. For this, others multimeter configurations were considered, which made its measurement uncertainty bigger than INMETRO and PTB, but taking the advantage of measurement in a wider frequency range, that enable acoustic applications.

6. References

[1] SVENSON, S. *Power measurement techniques for non-sinusoidal conditions – The significance of harmonics for the measurement of power and other AC quantities.* Technology University of Chalmers. Götteborg, Sweden, 1999.
[2] KAMPIK, M.; LAIZ, H.; KLONZ, M. *Comparison of three accurate methods to measure AC voltage at low frequencies*. Instrumentation and Measurement Technology Conference, 1999. IMTC/99. Proceedings of the 16th IEEE , vol.3, no., pp.1472,1477 vol.3, 1999.
[3] FRANCO, A. M. R.; TÓTH, E.; DEBATIN, R. M.; PRADA, R. *Development of a power analyzer.* 11th IMEKO TC-4 Symposium on Trends In Electrical Measurements and Instrumentation" pp. 168-172, Lisbon 2001.
[4] KYRIAZIS, G.A.; IHLENFELD , K.W.G. *Considerations on the comparison of two most accurate sampling methods for measuring harmonic parameters at low frequencies.* Conference on Precision Electromagnetic Measurements (CPEM). Torino, 09-14 July 2006.
[5] FRANCO, A.M.R., TÓTH, E., DEBATIN, R.M., OLIVEIRA, P C., CYRILLO, M.F.B. *Osciloscópio Virtual de alta exatidão*. III METROSUL. Curitiba, 2002.
[6] *3458A Multimeter User's Guide. Edition 4*. Agilent Technologies, Inc. USA, 2000.

[7] KYRIAZIS, G. A.; CAMPOS, M.L.R. *An approach based on asynchronous sampling and fast hartley transforms for evaluating harmonics of periodic signals with negligible leakage.* Conference on Precision Electromagnetic Measurements (CPEM). Torino, 09-14 July 2006.

[8] KYRIAZIS , G. A. *Bayesian estimation of harmonic components using digital voltmeters with asynchronous sampling*, Ph.D. Thesis, Federal University of Rio de Janeiro (COPPE/UFRJ), (www.pee.ufrj.br/teses/textocompleto/2005062451.pdf). Rio de Janeiro, Jun. 2005**.**

[9] FRANCO, A.M.R., TÓTH, E., DEBATIN, R.M. *Influência dos harmônicos na medição da potência (Influence of Harmonics in Power Measurement.* 5o Seminário Internacional de Metrologia Elétrica, Rio de Janeiro, 2002.

[10] Guia para Expressão da Incerteza de Medição (GUM). Terceira edição brasileira em língua portuguesa. 120 p. ABNT/INMETRO. Rio de Janeiro, 2003.

[11] DINIZ, P. R. R. *Processamento digital de sinais: Projeto e análise de sistemas*. Bookman. Porto Alegre. 2004.

[12] VASCONCELLOS, R. T. B. *Análise de erros em sistemas de amostragem aplicados a medições de alta exatidão.* Tese (Doutorado em Engenharia Elétrica). COPPE, da Universidade Federal do Rio de Janeiro. Março de 2012.

[13] DI LILLO, L., LAIZ, H., YASUDA, E. , GARCÍA, R. *Sampling wattmeter at INTI*. VIII Semetro. João Pessoa, Brazil, 2009.

[14] International Electrotechnical Commission. IEC 60942:2003-01. *Electroacoustics: Sound calibrators*. Genebra, Suiça. 2003

ISBN-10: 84-616-5438-2 | ISBN-13: 978-84-616-5438-3