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A Novel Blind Adaptive Correction Algorithm for 2-Channel Time-Interleaved ADCs

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Abstract- A Time-Interleaved ADC (TI-ADC) enables effectively sampling at integer multiples of the single ADC's sampling frequency but its performance is limited by the mismatches between the individual ADCs. In this contribution, a novel fully blind adaptive compensation structure for 2-channel Time-Interleaved ADC frequency response mismatch correction is proposed. The proposed method overcomes the existing methods in a sense that the TI-ADC mismatch identification can be performed without reducing the bandwidth e.g. by allocating an interleaving mismatch spur band (IMSB) in the spectrum via oversampling. The performance of the proposed approach is verified using actual hardware measurements of an RF sampling ADC.

I. Introduction

In various technical applications today, digital signal processing (DSP) is utilized instead of analog signal processing due to the former being more powerful, precise, flexible and robust. An Analog-to-Digital-Converter (ADC) is used to sample the analog signal at discrete-time intervals and its sampling rate is, among others, a main bottleneck factor that determines the system performance. Typically an ADC has either a high resolution or a large bandwidth, each obtained at the cost of the other. Developing a single core ADC with a faster sampling rate, while maintaining the effective resolution, becomes increasingly difficult due to technological limitations.

An alternative to continuously developing single core ADCs with higher sampling rates is to have multiple ADCs operating in parallel [1]. Each ADC samples at a different point in time and these samples are then interleaved to achieve a higher sampling rate. Theoretically, an arbitrary number of ADCs can be interleaved to achieve a higher sampling rate while maintaining the effective resolution. However in practice, the ADC's analog bandwidth, which is unchanged, limits the number of ADC that can be interleaved. Also the mismatches between the interleaved ADCs distort the signal and cause spurs in the spectrum [2]. This limits the ADC's spurious free dynamic range (SFDR) and therefore its effective resolution. Fortunately correction methods can be used to mitigate the mismatch between the interleaved ADCs [3]-[7]. Therefore the TI-ADC's lost effective resolution is recovered. This contribution deals with a novel digital correction method for 2-channel TI-ADC mismatches. The first step is to identify the mismatch followed by its compensation. In this contribution, the mismatch modeling and correction of the real-valued TI-ADC output signal is done using complex signal processing methods.

II. Modeling of a 2-Channel TI-ADC Mismatches

Consider a single tone complex signal $x_c(t)=\exp(j\Omega_a t)$ being sampled by a complex sampling ADC, which samples both the real and imaginary part, at integer multiples of $2T_s$ to give $x_{1c}(t)$ as shown in Eq. (1). This results in $X_{1c}(j\Omega)$, where the frequency spectrum of $X_c(j\Omega)$ repeats at integer multiples of $\Omega_s/2$, with $\Omega_s=2\pi/T_s$, as shown in Eq. (2). Since $x_c(t)$ can also be expressed as $\cos(\Omega_a t)+j\sin(\Omega_a t)$, if only $\mathrm{Re}\{x_{1c}(t)\}$, $x_1(t)$, is taken, it is shown in Eq. (3) that this introduces the conjugate frequency component $\exp(-j\Omega_a t)$.

$$x_{1c}(t)=x_c(t)\sum_{k=-\infty}^{\infty}\delta(t-k2T_s)=\sum_{k=-\infty}^{\infty}x_c(k2T_s)\delta(t-k2T_s)=\sum_{k=-\infty}^{\infty}e^{j\Omega_a k2T_s}\delta(t-k2T_s) \qquad (1)$$

$$X_{1c}(j\Omega)=\frac{1}{T_s}X_c(j\Omega)*\sum_{k=-\infty}^{\infty}\delta(\Omega-k\frac{\Omega_s}{2})=\frac{1}{T_s}\sum_{k=-\infty}^{\infty}X_c\left(j\left[\Omega-k\frac{\Omega_s}{2}\right]\right) \qquad (2)$$

$$x_1(t)=\sum_{k=-\infty}^{\infty}\cos\left(\Omega_a k2T_s\right)\delta(t-k2T_s)=\sum_{k=-\infty}^{\infty}\left[\left(e^{j\Omega_a k2T_s}+e^{-j\Omega_a k2T_s}\right)/2\right]\delta(t-k2T_s) \qquad (3)$$

For clarity, a single tone example was used above, but the same also applies if $x_c(t)$ is a band limited complex signal, whose bandwidth B is such that $(k-1)\Omega_s/2\leq B<k\Omega_s/2$, e.g. $0\leq B<\Omega_s/2$ for a baseband sampling case. (Note: the same symbol $x_c(t)$ is re-used for a single tone case as well as for the band limited case and the mathematical modeling is carried out in continuous-time form to simplify notation).

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A complex sampling ADC model is used here but when there is a ADC sampling a real-valued signal, this is simply equivalent to taking the real part of $x_{1c}(t)$, $x_1(t)$. When Re$\{x_{1c}(t)\}$ is taken, the conjugate frequency components of $x_{1c}(t)$ are introduced in $x_1(t)$, as described using single tone signal in (3), and the unfolding effect of the spectrum takes places. This is depicted in Figure 1.

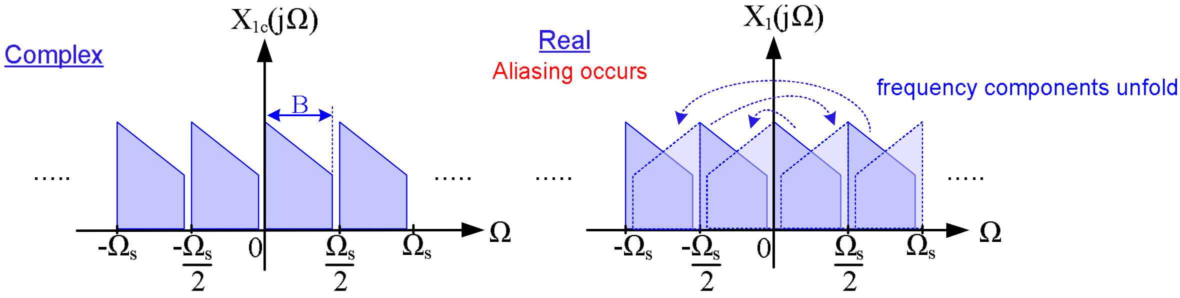

Figure 1. Unfolding of frequency components in the real spectrum.

It can be seen in Figure 1 that although in $x_{1c}(t)$ the spectral replicas do not overlap, since $0 \le B < \Omega_s/2$, taking only the real part $x_{1c}(t)$, introduces the conjugate frequencies in $x_1(t)$, causing aliasing to occur as the frequency components unfold. In order to avoid aliasing in $x_1(t)$, either the bandwidth B needs to be reduced by factor of 2 or the sampling rate needs to be doubled. If the sampling speed of a single ADC is limited, then 2 ADCs can be interleaved to double the sampling rate. The interleaving of ADCs was proposed in [1]. Figure 2 shows a 2-channel TI-ADC for the real-valued scenario for simplicity of illustration.

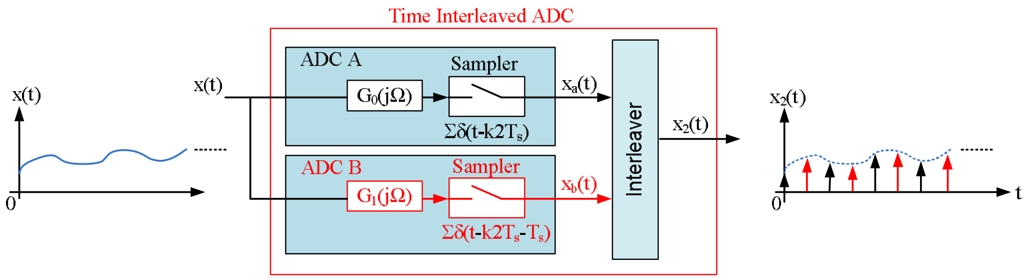

Figure 2. 2-channel Time-Interleaved ADC.

Each ADC samples at multiples of $2T_s$, however one ADC samples with a time offset T_s. Due to finite analog component matching in a 2-channel TI-ADC, each ADC will have a frequency dependent influence on the amplitude and phase of the signal being sampled. The same interleaved ADC model can be also extended to the complex TI-ADC case, where $G_0(j\Omega)$ and $G_1(j\Omega)$ are the frequency responses of the respective ADCs, as given by time-domain equation Eq. (4). Equation (5) shows the same operation in the frequency domain, where it can be seen that the spectral replica's cancellation now depend on $G_0(j\Omega)$ and $G_1(j\Omega)$.

$$x_{2c}(t) = g_0(t) * x_c(t) \sum_{k=-\infty}^{\infty} \delta(t - k2T_s) + g_1(t) * x_c(t) \sum_{k=-\infty}^{\infty} \delta(t - k2T_s - T_s) \tag{4}$$

$$X_{2c}(j\Omega) = \frac{1}{T_s}\left(\sum_{k=-\infty}^{\infty} \left(\frac{G_0(j\Omega) + G_1(j\Omega)(-1)^k}{2} \right) X_c\left(j\left\{ \Omega - k\frac{\Omega_s}{2} \right\} \right) \right) \tag{5}$$

If both ADC channels have the same frequency response, $G_0(j\Omega) = G_1(j\Omega)$, meaning no mismatches between the ADCs is present, then the spectral replica cancellation, for odd multiples of k, is complete and the distance between the replicas is doubled as shown in Figure 3.

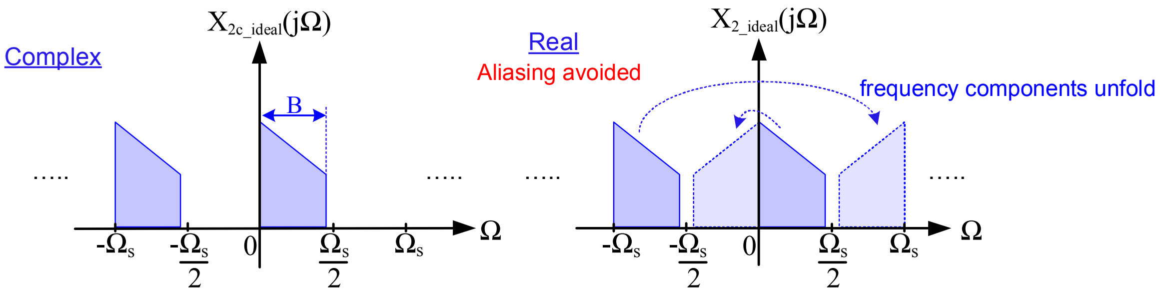

Figure 3. 2-channel TI-ADC spectrum without frequency response mismatch.

If each ADC channel has a different frequency response, $G_0(j\Omega) \neq G_1(j\Omega)$, then the spectral replica cancellation, for odd multiples of k, is incomplete and causes the replicas to remain in the spectrum as shown in Figure 4.

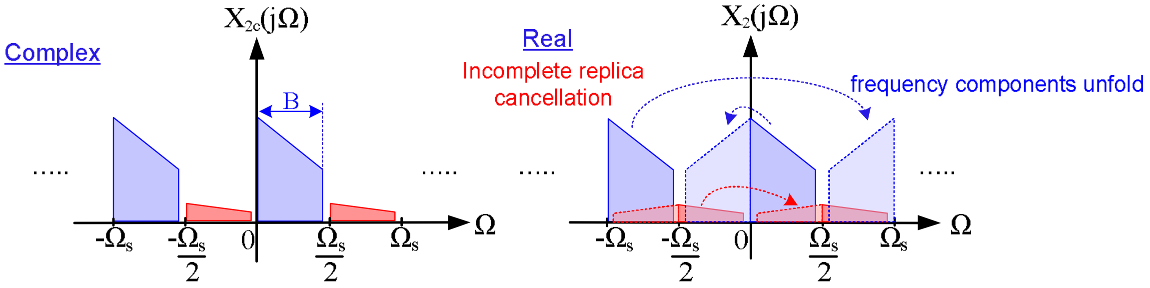

Figure 4. 2-channel TI-ADC spectrum with frequency response mismatch.

This results in spurious components in the spectrum and limits the Spurious-Free-Dynamic-Range (SFDR). Using $G_0(j\Omega)$ as reference, the mismatches are modeled only in $G_1(j\Omega)$. For one specific frequency position, let $G_1(j\Omega) = (1+A_\varepsilon)\exp(j\varphi_\varepsilon)$, where A_ε is the gain mismatch and φ_ε is the phase mismatch (rad). The replica cancellation upon interleaving depends on the values of A_ε and φ_ε.

$$X_{2c}(j\Omega) = \frac{1}{T_s}\left(\sum_{k=-\infty}^{\infty}\left(\frac{1+(1+A_\varepsilon)e^{j\varphi_\varepsilon}(-1)^k}{2}\right)X_c\left(j\left\{\Omega - k\frac{\Omega_s}{2}\right\}\right)\right) \quad (6)$$

Eq. (7) below reformulates Eq. (6) to show an ideal interleaved spectrum added with the undesired spurious replica, where $H_{2.0}(e^{j\omega}) = [1+G_1(e^{j\omega})]/2$ and $H_{2.1}(e^{j\omega}) = [1-G_1(e^{j\omega})]/2$. [Note: $H_{2.1}(j\Omega) = H_{2.1}(e^{j\omega})|_{\omega=\Omega T_s} = H_{2.1}(e^{j\Omega T_s})$]. For small values of A_ε and φ_ε, the approximation $H_{2.0}(e^{j\omega}) \approx 1$ and $H_{2.1}(e^{j\omega}) \approx (-A_\varepsilon - j\varphi_\varepsilon)/2$ can be made. Figure 5 shows the finite Replica Rejection Ratio (RRR) for a single tone case due to the interleaved ADC mismatches. The spur component's conjugate unfolds into the 1st Nyquist zone at $\Omega_s/2-\Omega_a$, which is scaled with $H^*_{2.1}(e^{j\omega})$, limiting the SFDR.

$$X_{2c}(e^{j\omega}) = \underbrace{H_{2.0}\left(e^{j\omega}\right)X_{2c\_ideal}\left(e^{j\omega}\right)}_{\text{ideal interleaved spectrum}} + \underbrace{H_{2.1}\left(e^{j[\omega-\pi]}\right)X_{2c\_ideal}\left(e^{j[\omega-\pi]}\right)}_{\text{undesired spurious replica}} \quad (7)$$

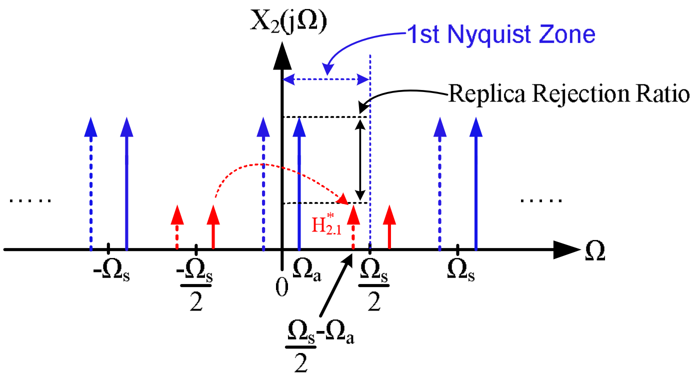

Figure 5. 2-channel TI-ADC RRR due to frequency response mismatch.

The resulting RRR can be expressed as:

$$RRR = 10\log\left(\frac{|H_{2.0}(j\Omega)|^2}{|H_{2.1}(j\Omega)|^2}\right) \approx -10\log\left(\left|\left(-\frac{A_\varepsilon}{2} - j\frac{\varphi_\varepsilon}{2}\right)\right|^2\right) \approx -10\log\left(\left(\frac{A_\varepsilon}{2}\right)^2 + \left(\frac{\varphi_\varepsilon}{2}\right)^2\right)(\text{dB}) \quad (8)$$

For realistic mismatch values, e.g. A_ε=1% and φ_ε=1° (to be inputted in rad), the results from the exact and approximated version are consistent to the expected value of 40 dB with negligible difference (Δ<0.05 dB). This can be easily verified via simulation using two sinusoids that are interleaved as shown in Figure 4, where e.g. $x_a(t)=\cos(\Omega_a t)$ and is $x_b(t)= (1+A_\varepsilon)\cos(\Omega_a t+\varphi_\varepsilon)$. The RRR across the whole bandwidth varies due to frequency response mismatch and Eq. (8) can be used to determine the RRR for the mismatch at each frequency point for its corresponding A_ε and φ_ε values.

III. Adaptive Mismatch Correction Algorithm of a 2-Channel TI-ADC

The mismatch correction process of the TI-ADC can be performed using the reverse of Eq. (7). The spur component resulting from the TI-ADC mismatch is first reconstructed and then subtracted from the TI-ADC's output signal, as shown in Figure 6 below. For this to be done, it is necessary to first identify the mismatch component present in the TI-ADC's output signal and then use this information to reconstruct the mismatch component.

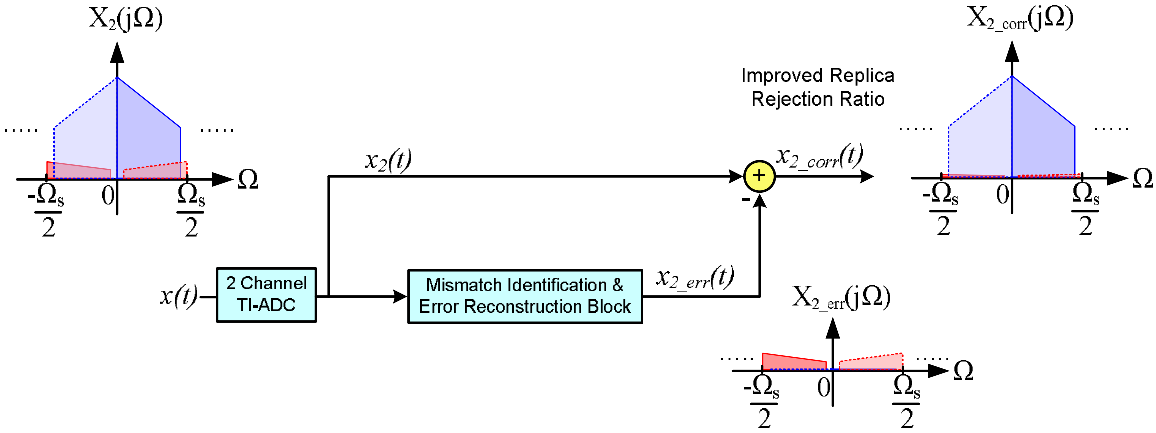

Figure 6. 2-channel TI-ADC mismatch correction principle.

Since the TI-ADC overcomes the sampling rate limitation of a single core ADC, much research effort has been invested for some decades to develop correction methods to mitigate its performance degradation, i.e. loss in the TI-ADC's effective resolution due to the interleaved ADC's mismatches. It is preferred to have correction methods that operate in the background as such methods do not interrupt the operation of the ADC. Extensive analysis on the types of mismatches has been done in [2], where it is also emphasized that the influence of each interleaved ADC mismatches on the amplitude and phase of the signal being sampled is frequency dependent. The popular solutions for correcting the TI-ADC mismatches can be categorized into mixed signal domain solutions and digital domain solutions. In [3],[4] good example of recent work in the mixed-signal domain can be found and in [5]-[7] are good examples of recent work done in the digital domain. The methods proposed in [5],[7] require oversampling in order to allocate a region where only the TI-ADC mismatch spurs are present and [6] also uses oversampling to detect the timing offset error. These methods [5],[7] suffer the shortcoming that they are not able to detect the frequency-dependent mismatch throughout the band, except in the region allocated via oversampling. It has been stated recently in [8],[9] that online adaptive mismatch identification methods for detecting the frequency dependent mismatch are of interest and an open challenge. Therefore this contribution proposes a new mismatch identification and correction method for a 2-channel TI-ADC relying on complex I/Q signal processing techniques [10]. Figure 7 shows the novel proposed correction method.

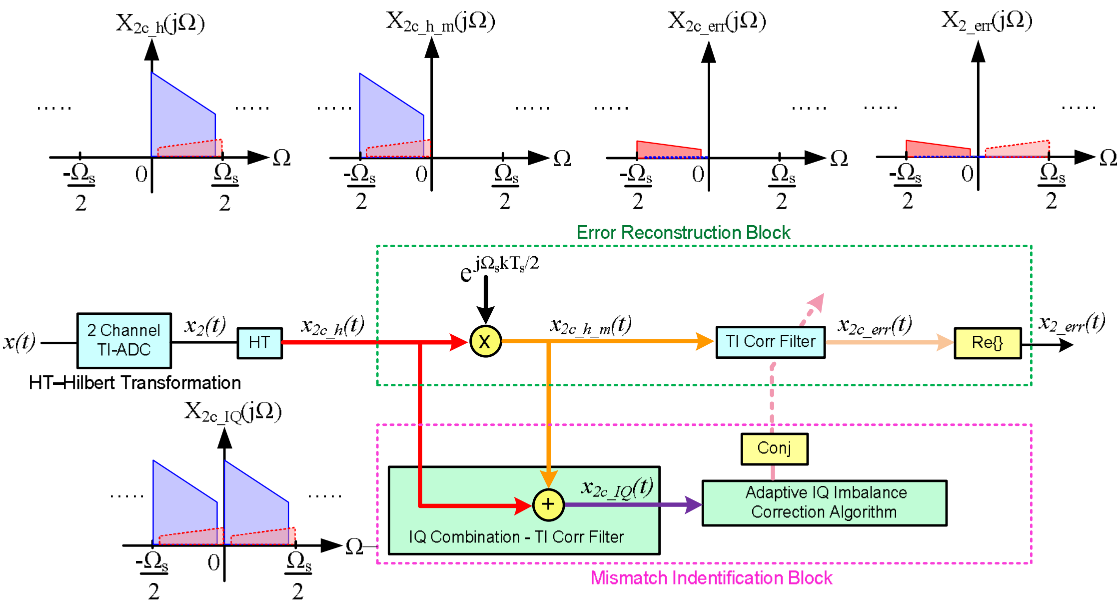

Figure 7. 2-channel TI-ADC mismatch identification and error reconstruction block.

First, a complex signal $x_{2c\_h}(t)$ is generated using a Hilbert filter from $x_2(t)$, the real-valued output of 2-channel TI-ADC. Then $x_{2c\_h}(t)$ is frequency shifted by $\Omega_s/2$ to the spur location to produce $x_{2c\_h\_m}(t)$, which is to be filtered to reconstruct the TI-ADC's error component, $x_{2c\_err}(t)$. Then by simply taking $\mathrm{Re}\{x_{2c\_err}(t)\}$, a real-valued error component $x_{2\_err}(t)$ is reconstructed and then subtracted from $x_2(t)$ to produce $x_{2\_corr}(t)$ which has an improved RRR. The key requirement in correcting the TI-ADC mismatches adaptively lies in the identification of the interleaved ADC mismatches. After performing a Hilbert transform, one side of the redundant spectral information is eliminated, e.g. the negative frequencies, and this vacant space can be used to create an interleaving mismatch identification signal. Adding together $x_{2c\_h}(t)$ and $x_{2c\_h\_m}(t)$ produces $x_{2c\_IQ}(t)$, where the interleaving mismatch spurs are placed at the conjugate frequency location of the desired signal tones, creating an I/Q imbalance pairs. For a single tone case, e.g. $x_{2c\_h}(t) = \exp(j\Omega_a t) + h^*_{2.1}\exp(j[\Omega_s/2-\Omega_a]t)$ and $x_{2c\_h\_m}(t) = \exp(-j[\Omega_s/2-\Omega_a]t) + h^*_{2.1}\exp(-j\Omega_a t)$. Therefore $x_{2c\_IQ}(t)$ can be expressed as:

$$x_{2c\_IQ}(t) = \underbrace{e^{j\Omega_a t} + h^*_{2.1}e^{-j\Omega_a t}}_{\text{I/Q Pair 1}} + \underbrace{e^{j(-\Omega_s/2+\Omega_a)t} + h^*_{2.1}e^{j(\Omega_s/2-\Omega_a)t}}_{\text{I/Q Pair 2}} \tag{9}$$

Here adaptive I/Q mismatch correction algorithms can be utilized for TI-ADC mismatch identification, e.g. circularity based algorithms [11]-[14]. A multi-tap version of the circularity based algorithm, i.e. $\boldsymbol{w}_{opt}$ with N taps interpreted as the weight vector of a digital correction filter, can follow frequency-dependent mismatches for a multi-tone scenario [11] and therefore extract the frequency-dependent TI-ADC's mismatches, enabling the identification of $H_{2.1}(j\Omega)$ for almost the complete bandwidth. The optimum filter's block estimate coefficients are:

$$\boldsymbol{w}_{opt} = \boldsymbol{c}_{x_{2c\_IQ}} \Big/ \left(\boldsymbol{\Gamma}_{x_{2c\_IQ}} + \bar{\boldsymbol{\Gamma}}_{x_{2c\_IQ}}\right) \tag{10}$$

where $\boldsymbol{\Gamma}_{x_{2c\_IQ}}$ is the NxN matrix valued autocorrelation function (ACF) and $\boldsymbol{c}_{x_{2c\_IQ}}$ is an N-element complementary autocorrelation function (CACF) vector [11]. The conjugate of $\boldsymbol{w}_{opt}$ is then used to filter $x_{2c\_h\_m}(t)$ to produce $x_{2c\_err}(t)$ and then by subtracting $\mathrm{Re}\{x_{2c\_err}(t)\}$, $x_{2\_err}(t)$, from $x_2(t)$ to produce $x_{2\_corr}(t)$. In the next step, the proposed TI-ADC's correction method is then put to the test on a state-of-the-art TI-ADC, ADC12D1800RF, which consists of 2 ADCs, each with a sampling frequency of 1.8 GHz, thus having an interleaved sampling frequency of 3.6 GHz. It is therefore able to sample the signal directly at RF [15]. Initially, the test was performed using a single tone and with a one-tap correction. Then 3 signal tones were simultaneously inputted and the correction was performed using an 8-tap correction filter according to Eq. (10). The interleaving spur level after correction is reduced to the noise floor level for both cases as shown in Figure 8 and Figure 9.

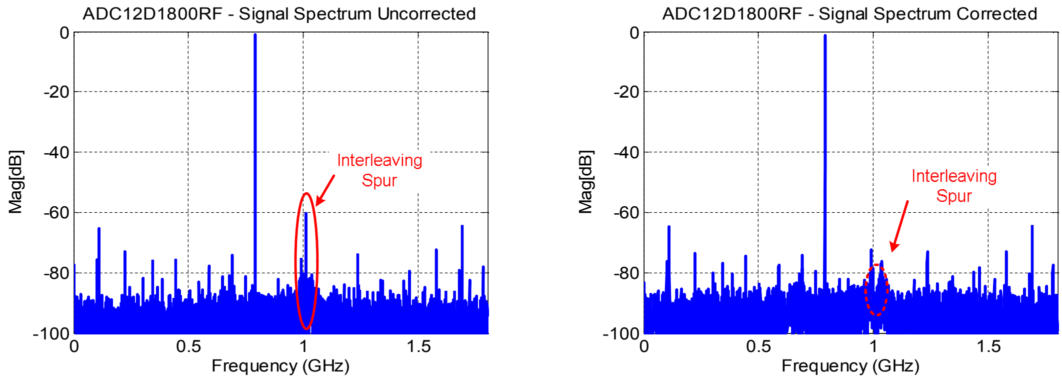

Figure 8. 2-channel TI-ADC single-tone spectrum before and after the proposed correction algorithm.

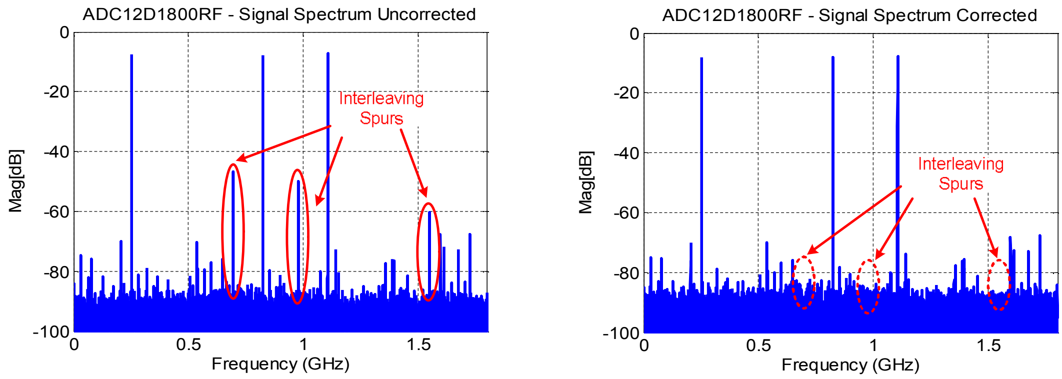

Figure 9. 2-channel TI-ADC multi-tone spectrum before and after the proposed correction algorithm.

III. Conclusions

The proposed adaptive correction algorithm recovers the TI-ADC's lost effective resolution due to the interleaved ADCs' frequency response mismatch using complex I/Q signal processing. It requires neither oversampling [5],[6] nor a tone injection into an allocated band for mismatch identification [7], thus circumvents further reduction of TI-ADC's usable analog bandwidth apart from the anti-aliasing filter. A patent application has been filed to protect the subject matter of this work [16]. In general, a link was established in this contribution between the TI-ADC mismatch correction and I/Q mismatch correction field of work which opens an interesting new track for future research.

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