19th Symposium IMEKO TC 4 Symposium and 17th IWADC Workshop
Advances in Instrumentation and Sensors Interoperability
July 18-19, 2013, Barcelona, Spain

Calibration of magnetic field meters up to 50 kHz at CMI

Michal Ulvr

Czech Metrology Institute, Department of Electromagnetic Quantities, V Botanice 4, 150 72 Prague 5, phone +420257288340, mulvr@cmi.cz

Abstract- This paper presents a calibration method with a special search coil used at the Czech Metrology Institute (CMI) for calibration of AC magnetic field meters in the range of frequencies up to 50 kHz. Special attention is given a) to the construction of magnetic flux density standards, which are used for generating the AC magnetic flux density for frequencies up to 3 kHz and for the frequency range from 3 kHz up to 50 kHz, b) to a description of special search coils used for precise measurements of the generated AC magnetic flux density, and c) to the analysis of sources of uncertainty. Calibration expanded uncertainty of (0.4 up to 0.8) % for $k = 2$ can be achieved by this calibration method.

Keywords – AC magnetic field measurements, calibration, search coils, solenoid, uncertainty analysis.

I. Introduction

AC magnetic field analysers (e.g. ELT 400, EFA 300, C.A. 42), which work on the principle of integrating voltage measurement, are widely used by health and safety professionals, in manufacturing, and in service industries. The calibration of these instruments is therefore important. A widely-used method for calibration of instruments of this type involves inserting the probe of the instrument into Helmholtz coils and then comparing the magnetic flux density value measured by the instrument's probe with the magnetic flux density value generated by the Helmholtz coils, which are powered by an AC current [1]. The adjusted magnetic flux density value is then calculated from the current value and from the Helmholtz coils constant determined e.g. by the NMR method. However, the frequency dependence of the coils constant and their resonance frequency is needed in order to make a correct calculation of the adjusted value. For these calibrations, we use a method with a special search coil, which is described below.

II. Calibration method

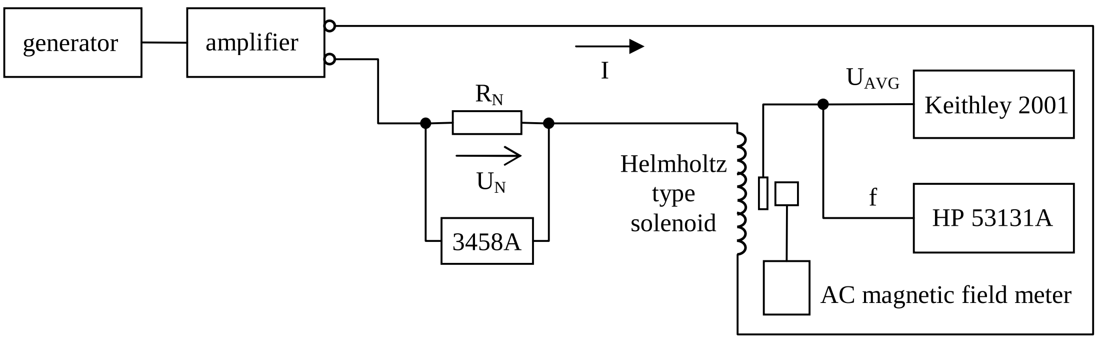

Figure 1. Schematic diagram of AC magnetic field meter calibration by comparing with the search coil.

The calibration method consists in comparing the measured values using a calibrated instrument with values measured by a special search coil. The special search coil is designed to achieve a good approximation of the magnetic dipole. The constant of the magnetic dipole for a coil is its area turns, and the constant is calibrated by a method with variable mutual inductance [2]. Figure 1 presents a schematic diagram of instrument calibration. First, the search coil is placed in the center of a Helmholtz type solenoid and the output voltage of the search coil is measured by a Keithley 2001 digital multimeter. The Helmholtz type solenoid is powered from a generator and amplifier, and the current I through the solenoid is measured as the voltage drop U_N on the standard resistor R_N, using an Agilent 3458A digital multimeter. Current I is measured for information only. The adjusted root mean square value of magnetic flux density B_{RMS} inside the solenoid is then calculated from a measured amplitude B_a

$$B_{RMS} = \frac{B_a}{\sqrt{2}} = \frac{U_{AVG}}{4K_S f \sqrt{2}}\,, \tag{1}$$

where U_{AVG} is the mean value of the output voltage of the search coil measured using a Keithley 2001 digital multimeter, K_S is the value of the constant of the search coil, and f is the frequency measured by an HP 53131A digital counter. Then the probe of the calibrating instrument is placed in the center of the Helmholtz type solenoid, and the measured value is recorded. The magnetic flux density sine wave also needs to be checked. The advantages of this method are that the adjusted value inside the solenoid is precisely measured, it is not dependent on the parameters of the Helmholtz type solenoid, and only constant K_S and the resonance frequency of the search coils need to be known. Only the influence of the magnetic field homogeneity inside the solenoid needs to be taken into account.

III. Construction and parameters of the solenoids and special search coils

For the calibration method described above, it was necessary to produce the proper magnetic flux density standards for generating AC magnetic flux density and special search coils for measuring the generated magnetic flux density For homogeneity and taking into account the size of the calibrating instrument probe (the probe is usually approximately 120 mm in diameter), two Helmholtz type solenoids were produced, each of them for a specific frequency range. Two special search coils were produced for the same frequency ranges.

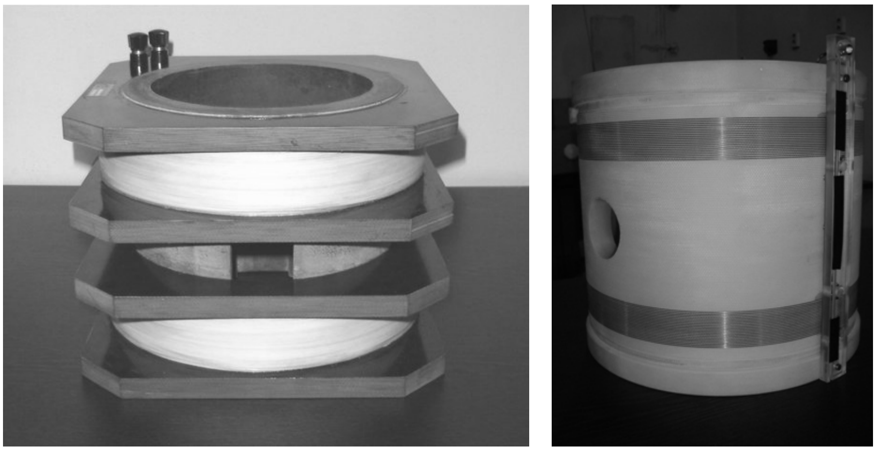

Figure 2. Multi-layer Helmholtz type solenoid No. 052 (left) and single-layer Helmholtz type solenoid No. 1201 (right).

A. Frequency range up to 3 kHz

A massive multi-layer Helmholtz type solenoid was designed and produced for the lower frequency range up to 3 kHz. The vector of magnetic field $\boldsymbol{B}$ inside the multi-layer solenoid can be described by components B_z and B_ρ as

$$B_z = \mu_0 J R_{in} k_j \left(1 + k_2 \frac{u_2}{R_{in}^2} + k_4 \frac{u_4}{R_{in}^4} + ... \right), \tag{2}$$

$$B_\rho = \mu_0 J R_{in} k_j \left(- k_2 \frac{v_2}{R_{in}^2} - k_4 \frac{v_4}{R_{in}^4} - ... \right), \tag{3}$$

where μ_0 is a constant of magnetic vacuum permeability ($4\pi.10^{-7}$ H/m), J is current density (A/mm^{-2}), R_{in} is inner radius of the winding (mm), k_j, k_2 and k_4 are constants dependent on the geometry of the solenoid (they can be found in [3]), and u_2, u_4, v_2 and v_4 are functions of coordinates derived from the Legendre polynomials. All odd constants k_1, k_3, … are equal to zero as a result of symmetry. The solenoid was designed with $k_2 = 0$ for better homogeneity. After editing and substituting for u_4, v_4 into equations (2) and (3), we get

$$B_z = \mu_0 J R_{in} k_j \left(1 + k_4 \frac{8z^4 - 2z^2\rho^2 + 3\rho^4}{8R_{in}^4} + ... \right), \tag{4}$$

ISBN-10: 84-616-5438-2 | ISBN-13: 978-84-616-5438-3

$$B_\rho = \mu_0 J R_{\text{in}} k_{\text{j}} \left[-k_4 \frac{z\rho\left(4z^2 - 3\rho^2\right)}{2R_{\text{in}}^4} - ... \right], \qquad (5)$$

where z (mm) and ρ (mm) are cylindrical coordinates with their origin in the center of the solenoid and z is the longitudinal axis of the solenoid. We can approximately determine the magnetic flux density value and the homogeneity inside the solenoid by substituting for coordinates z and ρ into (4) and (5). The theoretical magnetic flux density value in the center of the solenoid ($z = 0$, $\rho = 0$) is approximately 2 mT.

A Helmholtz type solenoid No. 052 with a massive textit frame 230 mm in length, inner diameter 190 mm and winding inner diameter 238 mm was manufactured at CMI. Its winding has 12 layers, and each layer has 26 turns wound with enameled copper wire 2 mm in diameter. The value of the solenoid constant was determined by the NMR method with flowing water as (1.94409 ± 0.00097) mT/A. This solenoid can generate a maximum magnetic flux density value of 6 mT on 50 Hz for a short time. A resonance frequency value of 28 kHz was also determined. The real homogeneity values will be different from the theoretical values, because the dimensions and the homogeneity of the winding have a considerable effect on multi-layer solenoids. For this reason, it is necessary to measure the real solenoid homogeneity values. In this case, the homogeneity values were measured experimentally using special search coil No. EP 601 with suppressed octupole (Figure 3), and the results of the theoretical and measured homogeneity values are presented in Table 1. The search coil was moved from the center in the z and ρ axis, and the true homogeneity value was calculated from the change in the measured output voltage.

Length on the *z* axis (mm)	**Theoretical homogeneity values (%)**	**Measured homogeneity values (%)**	**Length on the ρ axis (mm)**	**Theoretical homogeneity values (%)**	**Measured homogeneity values (%)**
-40	0.87	0.82	-60	1.65	1.86
			-40	0.33	0.21
-20	0.05	0.26	-20	0.02	0.05
0	0.00	0.00	0	0.00	0.00
20	0.05	0.31	20	0.02	0.03
40	0.87	0.91	40	0.33	0.18
			60	1.65	1.79

Table 1. Theoretical and measured values of the homogeneity of solenoid No. 052.

B. Frequency range from 3 kHz up to 50 kHz

The multi-layer solenoid can be applicable only for generating AC magnetic flux density to a few kHz, due to the parasitic capacities of the winding. For this reason, a single-layer Helmholtz type solenoid was designed and produced for the high-frequency range from 3 kHz up to 50 kHz. The vector of magnetic field ***B*** inside the single-layer solenoid can be described by the equations

$$B_{\text{z}} = \mu_0 I \alpha k_\sigma \left(1 + k_2 \frac{u_2}{R^2} + k_4 \frac{u_4}{R^4} + k_4 \frac{u_6}{R^6} ... \right), \qquad (6)$$

$$B_\rho = \mu_0 I \alpha k_\sigma \left(-k_2 \frac{v_2}{R_{\text{i}}^2} - k_4 \frac{v_4}{R^4} - ... \right), \qquad (7)$$

where I is the current through the solenoid (A), α is a number of turns to one meter, R is the radius of the winding (mm), k_σ, k_2, k_4 and k_6 are constants dependent on the geometry of the solenoid (they can be found in [3]), and u_2, u_4, v_2 and v_4 are functions of coordinates derived from the Legendre polynomials, as previously. In addition, all odd constants k_1, k_3,… are equal to zero and the solenoid was designed with $k_2 = 0$, as previously. After substituting for u_4, u_6, v_4 into equations (6) and (7), we obtain

$$B_{\text{z}} = \mu_0 I \alpha k_\sigma \left(1 + k_4 \frac{8z^4 - 2z^2\rho^2 + 3\rho^4}{8R^4} + k_6 \frac{16z^2 - 120z^4\rho^2 + 90z^2\rho^4 - 5\rho^6}{16R^6} + ... \right), \qquad (8)$$

$$B_\rho = \mu_0 I \alpha k_\sigma \left[-k_4 \frac{z\rho\left(4z^2 - 3\rho^2\right)}{2R_{\text{in}}^4} - ... \right], \qquad (9)$$

where z and ρ has the same meaning as previously. When we substitute for $z = 0$, $\rho = 0$, the theoretical magnetic flux density value in the center of the solenoid is 0.105882 mT.

A single-layer Helmholtz type solenoid No. 1201 with a glass epoxy frame 300 mm in length, inner diameter 280 mm and inner winding diameter 337.5 mm was fabricated. The value of its constant (0.105497 ± 0.00005) mT/A was determined by direct comparison with the national magnetic flux density standard. A magnetic flux density value of 100 μT can be generated up to 50 kHz by means of this solenoid. A resonance frequency value of 535 kHz was determined. The real homogeneity of the solenoid was measured using a special search coil with suppressed octupole, as previously, and the results of the theoretical and measured homogeneity values are presented in Table 2.

Length on the *z* axis (mm)	**Theoretical homogeneity values (%)**	**Measured homogeneity values (%)**	**Length on the ρ axis (mm)**	**Theoretical homogeneity values (%)**	**Measured homogeneity values (%)**
-40	0.33	0.26	-60	0.73	0.66
			-40	0.14	0.13
-20	0.02	0.03	-20	0.01	0.03
0	0.00	0.00	0	0.00	0.00
20	0.02	0.02	20	0.01	0.02
40	0.33	0.23	40	0.14	0.11
			60	0.73	0.68

Table 2. Theoretical and measured values of the homogeneity of solenoid No. 1201.

C. Special search coils

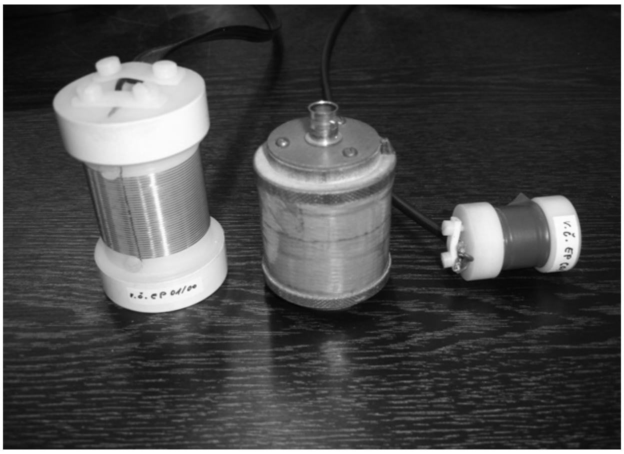

Figure 3. Special search coils (from left): EP 01/00, K_I and EP 601.

The system of axially symmetric conductive loops with area turns *NA* passed by current *I* can be described by the magnetic moment $m = I{\cdot}NA$, which is a vector quantity describing the magnetic dipole of the search coil. In the real case of a system of loops, an accurate description is a series of multipoles with multipole constants p_0, p_1, p_2, p_3,..., p_n. A multipole with constant p_0 equals zero because of the absence of a magnetic monopole. A multipole with constant p_1 is a magnetic dipole; p_1 is the area turns for a coil. A multipole with constant p_2 is a quadrupole that equals zero and all even multipole constants are equal to zero, due to the symmetry of the system. The third multipole, with a constant of p_3, is an octupole, etc. The influence of the higher multipoles (their participation in the magnetic field generated by the coil) declines with distance. The influence can be significant in real distances (e.g. 5 - 10 multiple of the dimensions of the coil). For this reason, we attempt to suppress the higher multipoles, especially p_3, when designing the search coil. The search coils used in the calibration method described above are designed as symmetrical cylindrical windings – the constants of the even multipoles are zero. For a single-layer search coil, the constant of an octupole is zero, when we choose the ratio of the coil's length and diameter $\sqrt{3}/2$. This ratio can also be used for the multi-layer search coil, but the thickness of the windings must also be taken into account.

Two special search coils for the calibration method described here have a cylindrical frame and a suppressed octupole. A special multi-layer search coil No. K_I with a textit frame and with calibrated constant K_S = (1.3312 ±

0.0011) m^2 was used for measuring the adjustable magnetic field value inside solenoid No. 052 in the frequency range up to 3 kHz. Search coil No. K_I has a coaxial connector, and its resonance frequency value is 56 kHz. A single-layer search coil No. EP 01/00 with a frame made of material close to teflon and with calibrated constant K_S = (0.045394 ± 0.000036) m^2 was used for measuring the magnetic field value inside solenoid No. 1201 in the frequency range from 3 kHz up to 50 kHz. The connector of this search coil is at a distance of 1 meter from the coil. The resonance frequency of EP 01/00 is 3.76 MHz.

IV. Uncertainty analysis

The type B uncertainty of the calibration method has several components, as follows

$$u_{Bc} = \sqrt{u_f^2 + u_{SC}^2 + u_{AVG}^2 + u_d^2 + u_h^2}\ , \tag{10}$$

where u_f is the standard uncertainty of the frequency measurement, u_{SC} is the uncertainty of the search coil constant, u_{AVG} is the uncertainty of the search coil output voltage measurement, u_d is the uncertainty of the directional dependence measurement of the search coil, and u_h is the uncertainty of the influence of homogeneity inside the solenoid in the volume of the probe/search coil. The maximum uncertainty value of the frequency measurement usually varies in the tens of ppm, so it can be neglected, because the other uncertainties are essentially higher. The uncertainty of the search coil constant determined by calibration with variable mutual inductance (for search coil No. K_I and EP 01/00) is 0.04 %. The uncertainty of the search coil output voltage measurement is dependent on the specification of the Keithley 2001 digital multimeter, because its accuracy of voltage range depends on the frequency of the measured voltage value. The value of u_{AVG} can lie in the order of hundredths to tenths of one percent. The value of the measured output voltage from the search coil also depends on the direction in which this coil is inserted into the magnetic field. In principle, the value depends on cos φ, where φ is the angle between the axis of the search coil and the direction of the magnetic flux density vector. The search coil must be set to the position where cos φ = 1, which is the maximum value from the output of the search coil. For example, there is a difference of about 0.02 % from the true value of measured voltage for φ = 1°, there is a difference of about 0.06 % for φ = 2°, and so on. This means that the value of u_d can lie in the order of hundredths to tenths of one percent. The value of u_h can be estimated in one of two ways. First, we want to measure $B_z p_1$, but we measure it in the volume of the probe/search coil. If we know the theoretical value of B_z from (4) or (8) and we also know a number of multipole constants p_0, p_1, p_2, p_3,.., which characterize the search coil, then the magnetic flux (the induced voltage) is the product of these variables. We can estimate the value of u_H from the constants characterize homogeneity of the solenoid and from the multipole constants p_0, p_1, p_2, p_3, and so on. The second way is by mechanical integration of the magnetic flux density in the volume of the probe/search coil. This involves finding the average value of the magnetic flux density in the volume of the probe/search coil. The value of u_h can lie in the order of tenths of one percent. A total calibration expanded uncertainty value of (0.4 up to 0.8) % for k = 2 can be achieved using this method.

V. Measurement results

The method described above was tested on a C.A. 42 type analyser in the range of frequencies 50 Hz up to 50 kHz. The results are presented in Table 3, where the magnetic field value generated by the solenoid is calculated from the search coil constant and the measured average value on the search coil output.

Frequency (Hz)	**Magnetic field value generated by the solenoid (μT)**	**Magnetic field value measured by C.A. 42 (μT)**	**Relative error of measured value (%)**	**Expanded calibration uncertainty (k = 2) (%)**
50	500.00	502.70	0.54	0.8
150	500.00	502.70	0.54	0.6
500	800.00	807.20	0.90	0.5
1000	400.00	403.22	0.81	0.5
5000	100.00	100.11	0.11	0.4
10000	100.00	100.22	-0.22	0.4
20000	70.00	69.21	-1.13	0.6
30000	80.00	79.49	-0.64	0.6
40000	60.00	59.43	-0.95	0.7
50000	50.00	49.66	-0.68	0.8

Table 3. C.A. 42 calibration measurement results in the range of frequencies 50 Hz up to 50 kHz.

VI. Conclusions

A method for calibration of magnetic field meters in the frequency range up to 50 kHz has been proposed. The construction and the parameters of the solenoids and special search coils are described and the uncertainty analysis is also presented. When a method with a special search coil is used, the calibration uncertainty depends mainly on the uncertainty of the search coil constant, on the uncertainty of the measured output voltage, and on the uncertainty of search coil placement in the center of the solenoid. Homogeneity inside the solenoid must also be taken into account. The measurement uncertainty is not dependent on solenoid parameters such as the constant and the frequency dependence of its constant. Because the search coils described above were used far below their resonance frequency, we can state that the search coils are frequency independent in the described frequency range. The method was successfully tested on a C.A. 42 type analyser in the frequency range from 50 Hz up to 50 kHz. The expanded uncertainty value (k = 2) of the calibration method described here varies in the range of (0.4 up to 0.8) %. This method could be used with some changes and improvements in the range of frequencies up to 100 kHz.

References

[1] Celso Fabricio de Melo at al, “Calibration of magnetic field meters at 60 Hz using a Helmholtz coil: Constructive aspects and calculation of associated uncertainties”, XVIII IMEKO World Congress, Rio de Janeiro, 2006.

[2] M. Ulvr, “A system for calibration of search coils with area turns up to 100 m^2”, Proc. of CPEM 2012, July 2012.

[3] J. V. Afanasjev et al: “Sredstva izmerenij parametrov magnitnogo polja”, Leningrad, Energia 1979.

[4] ICNIRP, “Guidelines for limiting exposure to time varying electric and magnetic fields (1 Hz – 100 kHz),” *Health Phys.*, vol. 99, no. 6, pp. 818-836, 2010.

[5] Stefan Cirulo, Andrea Mariscotti and Angelo Vicava, “A Helmholtz coil for high frequency high field intensity applications”, *Metrol. Meas. Syst.*, vol. XVI, no. 1, pp. 117-127, 2009.

[6] D. A. Rollett and A. E. Drake, “Traceable magnetic field strength measurements to 120 kHz,” *Proc. Inst. Electr. Eng.—Sci. Meas. Technol.*, vol. 143, no. 4, pp. 255–258, Jul. 1996.

[7] C. Carobbi, S. Lazzerini, L. Millanta: High frequency operation of the coils for standard magnetic field generation using the bulk-current technique. *IEEE Transactions on Instrumentation and Measurements*, vol. 54, no. 4, pp. 1427-1432, 2005.