# EVALUATION OF UNCERTAINTY OF INDUCTANCE MEASURES ON STATE PRIMARY STANDARD OF THE UNIT OF INDUCTANCE

*O. Velychko, M. Surdu, S. Shevkun*

State Enterprise "All-Ukrainian state research and production center for standardization, metrology, certification and consumers` rights protection", Kyiv

**Abstract:** The materials on evaluation of the combined standard and expanded uncertainty of measurements at calibration of working standards of inductance on the State primary standard of the units of inductance are presented.

**Key words:** standard, inductance measure, combined standard uncertainty, expanded uncertainty.

## 1. INTRODUCTION

Development of the country in the field of science, technology and economics is inseparably connected with consistent improvement of the measuring system, control and testing under development, manufacturing and maintenance of the production. And it is impossible without ensuring of uniformity and the corresponding measurement accuracy within the country. That's why one pays reasonably great attention to the measurements. The requirements to their accuracy increases constantly and the limits of measurements of various physical quantities are expanded.

Uniformity of measurements at the national level is based on a system of state standards that ensure the reproduction of the physical unit and its transfer by means of secondary and/or working standards of different categories, from which the size of this unit is transferred to the measuring equipment.

Intensification of the process of international integration in the economics, science and industry requires the harmonization of domestic regulations with the international documents and unity in their approach to presentation of the measurement results. Thus, in many branches important for Ukraine the theory of errors is widely used for solving of practical metrological problems. But the current state of standard base of many developed countries to evaluate the metrological characteristics of standards provides for the application of the theory of uncertainties along with the theory of errors.

In Ukraine, in the national regulations the provisions of the Guide to the expression of uncertainty in measurement (GUM) [1, 2] are implemented. Thus, DSTU 3231:2007 [3] provides not only a definition of necessary errors for standards, but also the evaluation of the measurement uncertainties.

The purpose of the article is to provide the basic provisions of the standard methodology for evaluation of the combined and the expanded measurement uncertainty at the calibration of working standards of inductance on the State primary standard of the units of inductance and dissipation factor DETU 08-09-09 based on a modern approach to the processing of measurement results [4].

## 2. BASIC MATERIAL

### 2.1. Basic provisions

Calibration of working standards of inductance on the State primary standard DETU 08-09-09 is executed by means of direct measurements, i.e. directly, and therefore the evaluation of uncertainty of calibration results is performed without measurement equation (as influence coefficient of standard uncertainties is equal to 1).

For calibration of the working standards of inductance it is necessary to transfer the size of a unit from the standard measure of capacity with nominal value of 100 pF or 10 pF that has the appropriate calibration certificate, indicating the calibration uncertainty.

Such measures are included in the State standard units of electrical capacity and dissipation factor DETU 08-06-01 as a group of three standard measures type AH11A with nominal value of 100 pF and the three standard measures type AH11A with nominal value of 10 pF. Standard measures included in this group are being under investigation since 1998. At present, due to our in-house researches, as well as researches carried out by PTB and traceability to the standards of the leading National Metrological Institutes – NIST (USA) and NPL (Great Britain), the values of electrical capacity of standard measures AH11A, included in the group are known with the expanded uncertainty $U(C_{AH}) = 1,0 \cdot 10^{-6}$ with coverage factor *k=2*.

When calibrating working standards of inductance the set of transitional (intermediate) measures of capacity is used, which nominal parameters have decimal values, and for reproduction of the unit of inductance via the units of electrical capacity and frequency it is reasonable to have measures of capacity that are multiples of 2.5 (eg, 25 nF, 250 nF) at this very frequency.

The example of transfer of the size of physical quantity unit at the calibration of measure of inductance with nominal value of 1 H based on the standard measure of capacity 100 pF and application of the set of transitional (intermediate) measures of capacity is shown in fig.1.
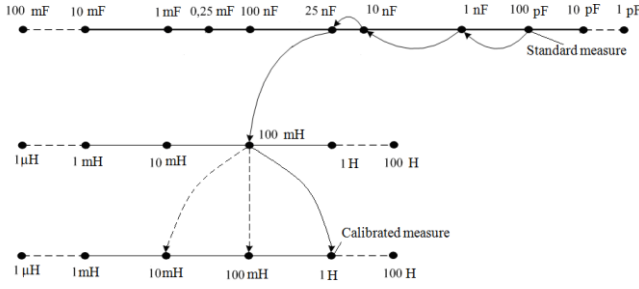
**Fig. 1. The example of transfer of the size of physical quantity unit at the calibration of measure of inductance**

$$u_C\left(C_{100\,\mathrm{pF}} \to L_X\right) =$$
$$= \sqrt{u^2\left(C_{AH,100\,\mathrm{pF}}\right) + \sum_{i=1}^{N} u_i^2\left(C_{c\Sigma}\right) + u^2\left(C_{c\Sigma} \to L_{c\Sigma}\right) + \sum_{j=1}^{M} u_j^2\left(L_{c\Sigma}\right) + u^2\left(L_M \to L_X\right)} \quad , \text{ (1)}$$

Transfer of the unit size from one measure to the measure of another category (working standard - WS) is made using comparator [5] which is a part of the standard DETU 08-09-09.

The comparator is created on the base of autotransformer bridges and performs:

- transfer of the size of unit of any impedance parameter (capacity, inductance, active resistance) over the entire range of values;

- reproduction of the unit of inductance, based on the units of capacity and frequency within the range from 10mH to 1 H.

### 2.2. Estimation of uncertainty at the calibration of working standard of inductance

At calibration of working standard of inductance by means of transfer of the size of physical quantity from standard measure of capacity with the nominal value of 100pF via the intermediate measures $1\,\mathrm{nF} \to 10\,\mathrm{nF} \to 25\,\mathrm{nF} \to 100\,\mathrm{mH}$ and further to 1 H (10 mH, 100 mH) the budget of uncertainty includes the following components: $u\left(C_{AH,100\,\mathrm{pF}}\right)$, $\sum u\left(C_{c\Sigma,100\,\mathrm{pF}} \to \ldots \to L_{c\Sigma,1H}\right)$ and $u\left(L_x\right)$.

Calculation of the combined standard ($u_C$) and expanded ($u_{NSE}$) uncertainty at transfer of the size of physical quantity from the standard measure of electric capacity type AN11A with nominal value of 100 pF to the calibrated measure of inductance $L_x$ with nominal value 1H is made according to the formulae (1, 2):

where

$$u\left(C_{AH,100\,\mathrm{pF}}\right) =$$
$$= U\left(C_{AH,100\,\mathrm{pF}}\right)/2 = 1,0 \cdot 10^{-6}/2 = 5,0 \cdot 10^{-7}$$

- expanded uncertainty with coverage factor k=2 for the standard measure of electrical capacity type AN11A with nominal value of 100 pF;

$u_i\left(C_{c\Sigma}\right)$ - combined standard uncertainty caused by an error of transfer of the size of physical quantity unit from the measure of capacity to the measure of capacity of another category;

$u\left(C_{c\Sigma} \to L_{c\Sigma}\right)$ - combined standard uncertainty, caused by an error of transfer of the size of physical quantity unit from the measure of capacity to the measure of inductance;

$u_j\left(L_{c\Sigma}\right)$ - combined standard uncertainty, caused by an error of transfer of the size of physical quantity unit from the measure of inductance to the measure of inductance;

$u\left(L_M \to L_X\right)$ - combined standard uncertainty, caused by an error of transfer of the size of physical quantity unit from the M-th measure of inductance to the calibrated measure of inductance $L_x$;

$$U\left(C_{100\,\mathrm{pF}} \to L_X\right) = k \cdot u_C\left(C_{100\,\mathrm{pF}} \to L_X\right). \quad (2)$$

Calculation of the combined standard ($u_{c\Sigma}$) and expanded ($U_{c\Sigma}$) uncertainty, caused by the error of transfer of the size of physical quantity unit from the standard measure of capacity with nominal value of 100 pF via the intermediate measures $1\,\mathrm{nF} \to 10\,\mathrm{nF} \to 25\,\mathrm{nF} \to 100\,\mathrm{mH}$ and further to 1 H is made as follows:

**1.** When transferring the unit size from the standard measure of capacity with nominal value of 100 pF to the intermediate measure of 1 nF:

**a)** combined standard uncertainty is:

$$u_{c\Sigma}\left(C_{100\,\mathrm{pF}-1nF}\right) = \sqrt{\left(\frac{u_{NSE}}{3}\right)^2 + u_{SRD}^2} = 1,202 \cdot 10^{-7},$$

where:

$u_{NSE}$ - standard uncertainty, caused by the non-excluded systematic error (NSE) of transfer of the size of physical quantity unit ($u_{NSE} = 2 \cdot 10^{-7}$);

$u_{SRD}$ - standard uncertainty, caused by the standard random deviation (SRD) of transfer of the size of physical quantity unit ($u_{SRD} = 1 \cdot 10^{-7}$).

Herewith one should take into account that values $u_{NSE}$ and $u_{SRD}$ are obtained on basis of calculated and experimental data, presented in the Table 1 from the nominal value to the nominal decimal values of the intermediate measures from the composition of the standard DETU 08-09-09.

**b)** expanded uncertainty (when coverage factor *k=2*) is:

$$U_{c\Sigma}\left(C_{100pF-1nF}\right) = k \cdot u_{c\Sigma}\left(C_{100pF-1nF}\right) =$$
$$= 2 \cdot u_{c\Sigma}\left(C_{100pF-1nF}\right) = 2,404 \cdot 10^{-7}.$$

**2.** When transferring the unit size from the intermediate measure of capacity of 1 nF to the intermediate measure of 10 nF:

**a)** combined standard uncertainty is:

$$u_{c\Sigma}\left(C_{1nF-10nF}\right) = \sqrt{\left(\frac{u_{NSE}}{3}\right)^2 + u_{SRD}^2} = 1,202 \cdot 10^{-7},$$

where $u_{NSE} = 2 \cdot 10^{-7}$;

$u_{SRD} = 1 \cdot 10^{-7}$.

**b)** expanded uncertainty (when coverage factor *k=2*) is:

$$U_{c\Sigma}\left(C_{1nF-10nF}\right) = k \cdot u_{c\Sigma}\left(C_{1nF-10nF}\right) =$$
$$= 2 \cdot u_{c\Sigma}\left(C_{1nF-10nF}\right) = 2,404 \cdot 10^{-7}.$$

**3.** When transferring the unit size from the intermediate measure of capacity of 10 nF to the intermediate measure of 25 nF:

**a)** combined standard uncertainty is:

$$u_{c\Sigma}\left(C_{10nF-25nF}\right) = \sqrt{\left(\frac{u_{NSE}}{3}\right)^2 + u_{SRD}^2} = 1,202 \cdot 10^{-7},$$

where $u_{NSE} = 2 \cdot 10^{-7}$;

$u_{SRD} = 1 \cdot 10^{-7}$.

**b)** expanded uncertainty (when coverage factor $k=2$) is:
$$U_{c\Sigma}\left(C_{10nF-25nF}\right) = k \cdot u_{c\Sigma}\left(C_{10nF-25nF}\right) =$$
$$= 2 \cdot u_{c\Sigma}\left(C_{10nF-25nF}\right) = 2,404 \cdot 10^{-7}.$$

**4.** When transferring the unit size from the intermediate measure of capacity of 25 nF to the standard temperature-controlled measure of inductance with nominal value of the 100 mH:

**a)** combined standard uncertainty is:

$$u_{c\Sigma}\left(C_{25nF} \to L_{100mH}\right) = \sqrt{\left(\frac{u_{NSE}}{3}\right)^2 + u_{SRD}^2} = 3,37 \cdot 10^{-6},$$

where $u_{NSE} = 1 \cdot 10^{-5}$;

$u_{SRD} = 5 \cdot 10^{-7}$.

**b)** expanded uncertainty (when coverage factor $k=2$) is:
$$U_{c\Sigma}\left(C_{25nF} \to L_{100mH}\right) = k \cdot u_{c\Sigma}\left(C_{25nF} \to L_{100mH}\right) =$$
$$= 2 \cdot u_{c\Sigma}\left(C_{25nF} \to L_{100mH}\right) = 6,74 \cdot 10^{-6}.$$

**5.** When transferring the unit size from the standard temperature-controlled measure of inductance with nominal value of the 100 mH to the intermediate measure of 1 H:

**a)** combined standard uncertainty is:

$$u_{c\Sigma}\left(L_{100mH-1H}\right) = \sqrt{\left(\frac{u_{NSE}}{3}\right)^2 + u_{SRD}^2} = 3,33 \cdot 10^{-6},$$

where $u_{NSE} = 1 \cdot 10^{-5}$;

$u_{SRD} = 1 \cdot 10^{-7}$;

**b)** expanded uncertainty (when coverage factor $k=2$) is:
$$U_{c\Sigma}\left(L_{100mH-1H}\right) = k \cdot u_{c\Sigma}\left(L_{100mH-1H}\right) =$$
$$= 2 \cdot u_{c\Sigma}\left(L_{100mH-1H}\right) = 6,67 \cdot 10^{-6}$$

**6.** When transferring the unit size from the standard temperature-controlled measure of inductance with nominal value of the 100 mH to the calibrated measure $L_x$ with nominal value of 1 H (10 mH, 100 mH):

**a)** combined standard uncertainty is:

$$u_{c\Sigma}\left(L_{100mH} \to L_X\right) = \sqrt{\left(\frac{u_{NSE}}{3}\right)^2 + u_{SRD}^2} = 3,33 \cdot 10^{-6},$$

where $u_{NSE} = 1 \cdot 10^{-5}$;

$u_{SRD} = 1 \cdot 10^{-7}$.

**b)** expanded uncertainty (when coverage factor $k=2$) is:
$$U_{c\Sigma}\left(L_{100mH} \to L_X\right) = k \cdot u_{c\Sigma}\left(L_{100mH} \to L_X\right) =$$
$$= 2 \cdot u_{c\Sigma}\left(L_{100mH} \to L_X\right) = 6,67 \cdot 10^{-6}.$$

The data presented in Table 1, should be used for estimation of the uncertainty of calibration results of working standards of inductance.

**Table 1. Generic results of the uncertainty of transfer of the size of physical quantity unit from the standard measure of capacity with nominal value of 100 pF, via the intermediate measures 1 nF → 10 nF → 25 nF → 100 mH, and then further to the calibrated measure $L_X$.**

| The direction of transfer of the size of physical quantity unit DETU 08-09-09 | The standard uncertainty caused by non-exclusion of systematic error of transfer of the size of physical quantity unit | The standard uncertainty caused by standard random deviation of transfer of the size of physical quantity unit | The overall standard uncertainty of transfer of the size of physical quantity unit | Expanded uncertainty of transfer of the size of physical quantity unit |
|---|---|---|---|---|
| Transfer of the size of physical quantity unit from the measure of capacity to the measure of capacity | | | | |
| 100 pF – 1 nF | $2 \cdot 10^{-7}$ | $1 \cdot 10^{-7}$ | $1,202 \cdot 10^{-7}$ | $2,404 \cdot 10^{-7}$ |
| 1 nF – 10 nF | $2 \cdot 10^{-7}$ | $1 \cdot 10^{-7}$ | $1,202 \cdot 10^{-7}$ | $2,404 \cdot 10^{-7}$ |
| 10 nF – 25 nF | $2 \cdot 10^{-7}$ | $1 \cdot 10^{-7}$ | $1,202 \cdot 10^{-7}$ | $2,404 \cdot 10^{-7}$ |
| Transfer of the size of physical quantity unit from the measure of capacity to the measure of inductance | | | | |
| 25 nF – 100 mH | $1 \cdot 10^{-5}$ | $5 \cdot 10^{-7}$ | $3,37 \cdot 10^{-6}$ | $6,74 \cdot 10^{-6}$ |
| Transfer of the size of physical quantity unit from the measure of inductance to the measure of inductance | | | | |
| 100 mH – 1 H | $1 \cdot 10^{-5}$ | $1 \cdot 10^{-7}$ | $3,37 \cdot 10^{-6}$ | $6,67 \cdot 10^{-6}$ |
| 1 H – 10 H | $1 \cdot 10^{-5}$ | $1 \cdot 10^{-7}$ | $3,37 \cdot 10^{-6}$ | $6,67 \cdot 10^{-6}$ |
| 100 mH – 10 mH | $1 \cdot 10^{-5}$ | $1 \cdot 10^{-7}$ | $3,37 \cdot 10^{-6}$ | $6,67 \cdot 10^{-6}$ |
| 10 mH – 1 mH | $1 \cdot 10^{-5}$ | $1 \cdot 10^{-7}$ | $3,37 \cdot 10^{-6}$ | $6,67 \cdot 10^{-6}$ |
| Transfer of the size of physical unit from the M-th measure of inductance to the calibrated measure of inductance | | | | |
| 100 mH – $L_X$ | $1 \cdot 10^{-5}$ | $1 \cdot 10^{-7}$ | $3,37 \cdot 10^{-6}$ | $6,67 \cdot 10^{-6}$ |

Numerical values of standard uncertainties caused by standard random deviation and non-excluded systematic error of transfer of the size of physical quantity unit were obtained during the State metrological certification of the National standard DETU 08-09-09.

In this case, a complete measurement result will be the following:

$$L_X \pm U\left(C_{100\,pF} \to L_X\right). \qquad (3)$$

In the combined standard uncertainty of the calibration result it is necessary to take into account the components provided by the influence of such factors as temperature coefficient of capacitance of measures and the frequency dependence of transfer of units during transfer of the size from the measures of capacity to the measures of inductance.

Measures of capacity have temperature coefficient of capacitance equal to $3 \cdot 10^{-5} /^{\circ} C$, while instability of thermostating of measures is $\pm 0,002^{\circ} C$. Meanwhile the uncertainty introduced by the instability of thermostating is $6 \cdot 10^{-8}$.

Frequency instability influences greatly the measuring results when transferring the unit size from the measures of capacity to the measures of inductance. However, during the measurements the frequency drift is negligible and the uncertainty of measurements is about $1 \cdot 10^{-10}$.

Thus, the components of the uncertainty introduced by the temperature coefficient of capacitance of measures and the frequency dependence can be neglected.

## 3. CONCLUSION

The researches that were carried out allowed us to develop a methodology of evaluation of the combined standard and expanded uncertainty of measurements at calibration of working standards of inductance on the State primary standard of the units of inductance and dissipation factor DETU 08-09-09 relying on the standard measures of capacity type AH11A with nominal value of 100 pF or 10 pF. Use of the developed methodology allows prepare the National standard DETU 08-09-09 to the regular KOOMET audit in 2011.

## REFERENCES

[1] ISO/IEC Guide 98-3:2008.Uncertainty of measurement. – Part 3: Guide to the expression of uncertainty in measurement.

[2] DSTU N RMG 43-2001.Recommendations on the interstate standardization. The application of "Guide to the Expression of Uncertainty in Measurement" (in Russian).

[3] DSTU 3231:2007. Metrology. National measurement standards, primary standards and secondary standards. Basic principles, procedures of development, approval, registration, storage and application (in Ukrainian).

[4] I. Zakharov, M. Sergienko, O. Velychko, V. Chepela. Methods of estimation of measurement uncertainty in the performance of metrology works / / Information processing systems. - Kharkov. - 2006. - Issue 6 (56). - P. 32-36 (in Russian).

[5] M. Surdu, A. Ahmadov, S. Ahmadov, S. Kursin, A. Lameko, M. Muharovsky. Precision comparator for reproduction of the unit of inductance and the transfer of its size within the range of values / / Ukrainian Journal of Metrology. - 2008. - Issue 4. - P.14-23 (in Russian).