DSP algorithms for active power estimation – overview and comparison of algorithms by non-coherent sampling

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Abstract- This paper presents an overview of algorithms for one-phase active power estimation using digital signal processing in time domain and in frequency domain and compares properties of these algorithms. Besides comparing already published algorithms also a new algorithm and additional information concerning some known algorithms is included. Results of computer simulations in Matlab and results of measurement by means of computer plug-in boards, both multiplexed and using simultaneous signal sampling is presented. Using new cosine windows designed by iterative algorithm recently published by this paper authors is also included.

Key words: Power estimation, digital signal processing, non-coherent sampling

I. Introduction

Measurement of power is an important up-to-date part of electrical measurements and power quality assessment and many papers have been devoted to various aspects of this measurement (e.g. [1],[2]). This paper presents an overview and comparison of algorithms for one-phase active power estimation using digital signal processing. Besides known algorithms also a new algorithm is presented and additional information is provided to some of already published algorithms. Results of computer simulations in Matlab and of measurement by means of computer plug-in boards are presented. An example of using cosine windows designed by iterative algorithm implemented in a graphical user interface accessible on web and programmed and recently published by this paper authors [3], [11] are also included in this paper.
Power is estimated by processing two input signals, one corresponding to voltage on the load and the other one corresponding to current passing through the load. This paper analysis the bias caused by the used power estimation DSP algorithms, not including the influence of the load voltage and load current converters. The algorithms are analyzed for sinusoidal signals (representing basic components of any multifrequency signal) and for the general case of non-coherent sampling, i.e. also for non-integer number of signal periods sampled. Transfer from time domain to frequency domain is provided by Fourier transform, in case of time-discrete signals by Discrete-Time Fourier Transform (DTFT) and Discrete Fourier Transform (DFT, and its fast version - FFT).

II. Inspected methods of active power estimation

A. General information.

The active power is estimated as

$$P' = \frac{1}{T_M}\int_0^{T_M} v(t)i(t)\mathrm{d}t \cong \frac{1}{N}\sum_{n=0}^{N-1} v(n)i(n) \qquad (1)$$

where P' is active power estimate, T_M is time of measurement (in power definition one signal period) and $v(n)$ and $i(n)$ are sampled versions of continuous-time voltage $v(t)$ and current $i(t)$. N is number of acquired samples. The measurement (sampling) time $T_M = N\,T_S$ (T_S being sampling interval) can be expressed also as $T_M = (M + \lambda)\,T_{SIG}$, where integer M is number of sampled periods T_{SIG} and λ is the decimal part of the last period sampled ($0 \leq \lambda < 1$). There is $P' = P$ for $\lambda = 0$ (coherent sampling), and $P' \neq P$ for $\lambda \neq 0$ [4], [5]. Since exactly coherent sampling is usually not possible (e.g. because of time changes of power system frequency), the difference between P' and P (i.e. bias of the measured active power P caused by non-coherent sampling) has to be taken into account. It can be markedly reduced by instantaneous power windowing and additional signal processing. The relative bias of P estimation can be expressed as

$$\delta'_P = \frac{P' - P}{P} \tag{2}$$

Period of instantaneous power is half of period of sinusoidal voltage and current.
Additional bias can be generated by non-simultaneous voltage and current input signal sampling by using low-cost multiplexed PC DAQ boards. This bias can be suppressed by various signal interpolations. The effect of linear interpolation has been studied in [5], and a more-detailed experimental study of linear and spline interpolation will be presented at the symposium. Effect of non-simultaneous channel sampling can be eliminated by using DAQ plug-in boards with simultaneous channels sampling having a separate ADC (nowadays usually of sigma-delta type) in each channel (mentioned in this paper), or by using synchronized high-accuracy digital voltmeters for each signal channel sampling. The fundamental cause of active power uncertainty is noise in data acquisition channels, and its basic component is quantization noise. It can be expressed also by means of SNR of each of input channels [4].

A. Power estimation in time domain using windowing (P-WTD)

These algorithms use windowing of the instantaneous power sequence to reduce power estimation bias. All signal processing takes place in time domain, i.e. DFT is not used. Detailed information about these algorithms is presented in [5]. Windowing decreases markedly active power estimation bias with increasing signal sampling time and depending on the used window. This allows estimating power with low bias in a short time. Windowing is applied here to instantaneous power. It changes the active power value, so this power estimate has to be corrected by a multiplication factor depending on the used window.
Advantages of this method are low processing time and easy applicability for digital signal processors.

B. Power estimation using interpolation and re-sampling in time domain (P-I/RTD)

This method is not mentioned in [4] and [5]. Effect of non-coherent signal sampling can be reduced also by interpolating the signal samples sequence (i.e. using interpolation in time domain), truncating an integer number of signal periods and re-sampling the interpolated signal so that an integer number of signal samples are obtained. Any interpolation method can be used; the most often used are staircase, linear and spline interpolation.
Instead of interpolated signal re-sampling it is also possible just to truncate the original sample sequence to a length corresponding as close as possible to an integer number of the original analogue signal periods (in this case however FFT algorithms not requiring signal length 2^N samples have to be used). It allows shortening the required processing time, but a residual non-coherency of sampling results in non-zero power estimation bias. Application of this method for signal amplitude and phase estimation is described in [6].
Knowledge of signal period time with high accuracy is a basic condition for successful application of this method. Estimation of frequency has been studied by many authors (lately e.g. [7], [8]). Error in signal period time (or frequency) estimation is reflected in residual non-coherency of signal sampling and corresponding power estimation bias. This bias can be reduced by signal windowing and by increasing signal sampling time.
The P-I/RTD method provides high-accuracy active power estimation, but it is very sensitive to the presence of sub-harmonic or inter-harmonic components in a multi-frequency signal.

C. Power estimation in frequency domain using windowing (P-WFD)

Processing of signal in frequency domain, i.e. after transforming signal to frequency spectrum by a suitable type of Fourier transform, offers an illustrative reflection of time-domain operations into the frequency domain. Since active power is the DC component of instantaneous power, in can be found either as a mean value of instantaneous power by integration in time domain (1), or as a DC component of the normalized DFT spectrum of instantaneous power. DFT spectrum normalization means dividing the spectrum by N (by using two-sided spectrum), where N is the DFT length. If a non-integer number of signal periods are sampled, leakage of energy into surrounding frequency bins causes power estimation bias. The effect of sampling non-coherency can be substantially reduced by instantaneous power windowing as in P-WTD method. The normalized DC DFT component of the windowed instantaneous power has to be corrected by the same multiplication factor as the instantaneous power DC component in P-WTD method.
Since the P-WTD method and the P-WFD method differ only in the way of finding the instantaneous power DC

component, active power estimations by these two methods are practically identical.

D. Power estimation in frequency domain using windowing and interpolation in frequency domain (P-WIFD)

In this method, information about all components of windowed instantaneous power spectrum is at user's disposal for increasing active power estimation accuracy. In practice, number of DFT instantaneous power spectrum components larger than one (as in the previous method) in neighborhood of DC component is used for active power estimation. Knowledge of window spectrum and DTFT and DFT spectrum of windowed signal spectrum allows finding individual signal spectrum components in surroundings of local DFT spectrum maxima by various interpolated DFT methods (e.g. [9]). By active power estimation the DC component of instantaneous power spectrum has to be found. Its fractional frequency bin is zero and therefore simplified interpolation expressions can be used [4]. If a cosine window w(n) of the order L and of length N defined by (3) is used,

$$w(n) = \sum_{r=0}^{L} D_r \cos\left(2\pi r \frac{n}{N}\right), \qquad n = 0,\ 1, \cdots, N-1 \tag{3}$$

the general interpolation formula for active power estimate (not presented in [4]) is

$$P^{'} = \frac{|X(0)| \cdot |D_0| + |X(1)| \cdot |D_1| + |X(2)| \cdot |D_2| + \cdots}{D_0^2 + D_1^2/2 + D_2^2/2 + \cdots} = |X| \cdot K^T , \tag{4}$$

where $|X|$ is a row vector composed of values of right side of two-sided DFT magnitude spectrum of the windowed instantaneous power started with DC component $X(0)$, and K is row vector of the interpolation coefficients. Length of each of these vectors depends (but it is not equal) to the selected number of points of the used interpolation. Length of vectors $\boldsymbol{X}$ and $\boldsymbol{K}$ is usually lower or equal to the number of instantaneous power spectrum lines for nonnegative frequencies within the used window spectrum main-lobe centered on zero frequency. Method P-WFD corresponds to 1-point special case of P-WIFD method.
This method increases power estimation accuracy by windowing and interpolation in frequency domain. Its processing complexity is higher than those of time-domain methods, but thanks to fast FFT algorithms and available hardware means the computation time demands may not be markedly increased.

E. Power estimation in frequency domain using estimation of individual power components (P-WCFD)

Active power can be found alternatively in frequency domain by summing active powers of individual power spectrum components. These active power components represent powers of voltage and current harmonic components of the same order including the DC components

$$P^{'} = \sum_{k=0}^{M} \frac{U_k^{'} I_k^{'}}{2} \cos\left(\varphi_{u,k}^{'} - \varphi_{i,k}^{'}\right) \tag{5}$$

Here M is number of higher order harmonic components and U', I' are estimates of voltage and current components RMS values and φ' are estimates of voltage and current k-th order harmonic components phase shifts related to the sampling start. Windowing is applied here on voltage and current signals and interpolated DFT is used for estimation of the frequency and consequently RMS values and phases of individual harmonic components of voltage and current signals.
Finding active power by this procedure is rather complicated and time consuming. Power estimation uncertainty here is influenced basically by uncertainty of the largest active power value of all processed harmonic components. This active power estimation method however provides information about frequency structure of the estimated power and possibility to select frequency band in that the active power should be estimated.

F. Power estimation in frequency domain using windowing and repeated instantaneous spectrum components estimation (P-WSCFD)

This is a newly proposed method able to increase accuracy of active power estimate by processing signal in frequency domain. Its principle is as follows.

Normalized DTFT complex spectrum of a cosine window (3) is

$$W(\theta,\varphi)=\frac{e^{-j\left(\pi\theta\frac{N-1}{N}\right)}}{N}\sum_{r=0}^{L}\frac{D_r}{2}\left(e^{j\pi r\frac{N-1}{N}}\frac{\sin(\pi(\theta-r))}{\sin\left(\frac{\pi(\theta-r)}{N}\right)}+e^{-j\pi r\frac{N-1}{N}}\frac{\sin(\pi(\theta+r))}{\sin\left(\frac{\pi(\theta+r)}{N}\right)}\right) \qquad (6)$$

Here θ and r are frequencies in frequency bin (θ non-integer) and N is window length in samples.
Estimation bias of DC component and remaining components of instantaneous power spectrum is after applying correction multiplicative factor influenced only by long-range leakage. This leakage can be reduced numerically by repeated spectrum components estimation, in each step after subtracting the largest spectrum component from the instantaneous power spectrum. The largest component frequency, amplitude and phase can be found using WIFD method and subtracted (two mirrored components for positive and negative frequency with all their contributions to neighboring spectral components) from instantaneous power spectrum. Accuracy of estimation of remaining components is by this operation increased.
Starting from the highest component and repeating subtracting of actually largest component (if this happens to be the DC component, it will be stored and its possible multiple values are accumulated and the remaining number of estimates is realized) leads step-by-step to increased accuracy of remaining instantaneous power spectrum components estimation. The last estimate should always be the estimation of the DC component.
Three such estimations are sufficient for one-tone signal with non-zero DC component. For signals with unknown frequency structure, higher number of repeated estimations should be selected and these repetitions can be stopped if a spectral lines level's dispersion is not decreased for more than the selected value after the last estimation.
Even if the described principle looks very complicated, after a single windowed instantaneous power spectrum computation all following operations are performed on this spectrum. The most time-consuming of remaining signal processing is repeated window spectrum computation but if a low-order window (as Hann window) is used, this computation is simple.
This method allows reaching low active power estimation bias even in presence of sub-harmonic and inter-harmonic components.

III. Examples of DSP of simulated and measured signals

Figures 1 and 2 below compare relative power estimation bias for different input signals and for the inspected power estimation methods.

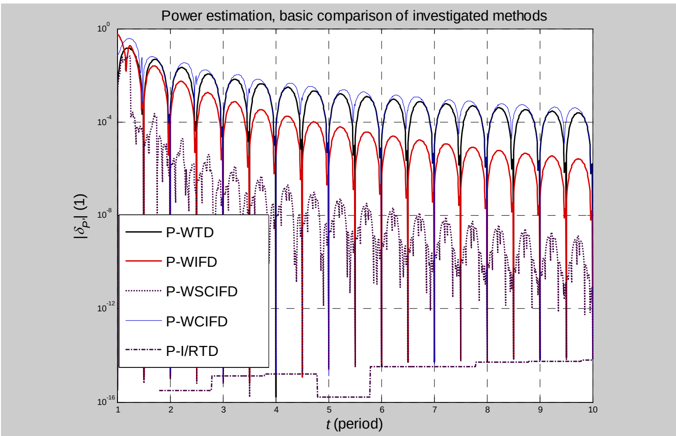

Fig. 1. Dependence of relative bias of active power estimation by the inspected methods on sampling time expressed in input signal periods

Fig. 1 compares the investigated methods of active power estimation for the simplest input signals. Sinusoidal waveforms of voltage and current, without DC components, and quantization and external noise were used for simulation. Load impedance phase is 80° (bias increases by load impedance phase approaching 90°). Window Hann and 3-point interpolation is used. As shown in Fig.1, under these conditions the most accurate is method

P-I/RTD using 3-point Hann window interpolation in signal period estimation and re-sampling for leakage reduction. The second best is the P-WSCIFD method using repeated instantaneous power spectrum correction and the third is method P-WIFD using again 3-point Hann window interpolation in frequency domain.

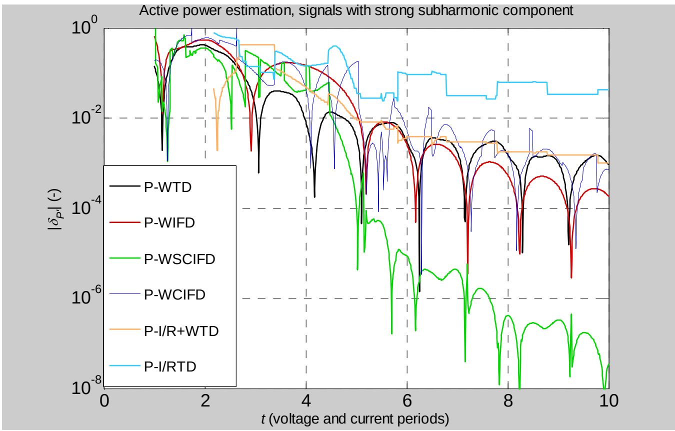

Fig. 2. .Dependence of relative bias of active power estimation by inspected methods on sampling time expressed in input signal periods

Fig.2 is depicted for signal containing fundamental harmonic component and a subharmonic component (with frequency ratio $f_{sub}/f_1 = 0.63$ and magnitude ratio $A_{sub}/A_1=0.5$), zero DC component and phase of load 80°. Three point Hann window interpolation is used (with the exception of P-I/RTD method).
As can be seen from Fig.1 and Fig.2, a strong sub-harmonic component influences bias by all power estimation methods, but the most influenced method is the method P-I/RTD, moving from the lowest bias for conditions of Fig.1 to the highest bias in strong sub-harmonic component presence.
Fig.3 compares results of processing simulated signals and signals generated by a two-channel signal generator HP3245A (allowing synchronization and defined phase difference of the two signal outputs) and measured by two different DAQ plug-in boards of the National Instruments (NI) company. A low cost multiplexed board PCI 6023A and a simultaneously sampling board PCI 4472 have been used. Input ranges of the two boards have been 10 V, sampling frequency was 100 kSa/s (lower sampling frequencies were realized by measured samples sequence decimation). Input signals were sinusoidal 50 Hz with phase differences in the range -80 deg to 80 deg. Signal magnitude was 5 V, corresponding to a half of plug-in boards range. Values gained by NI 4472 board overlap with values gained by simulation in Fig.3.

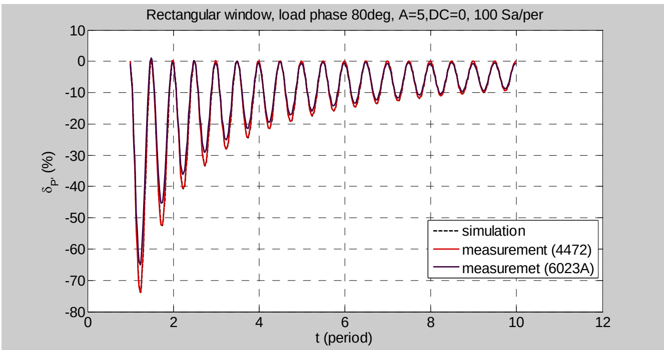

Fig. 3. Comparison of relative bias of active power estimation gained by simulation and by measurement.

Fig 4 shows the influence of the used window on power estimation bias. Bias dependence on signal length using a classic window of the first order (Han window) and four 3rd order windows is shown. Two of the 3rd order windows are classic Rife-Vincent windows. The remaining two windows are examples of the newly defined cosine window classes (defined in [3] and [11]) and their coefficients have been found by iterative window spectrum zeros placement method described in [3] and [11]. The results in Fig.4 have been gained for the load phase 60 degrees and by signal processing in frequency domain. Logarithmic vertical axis scale was selected to increase differences between displayed curves.

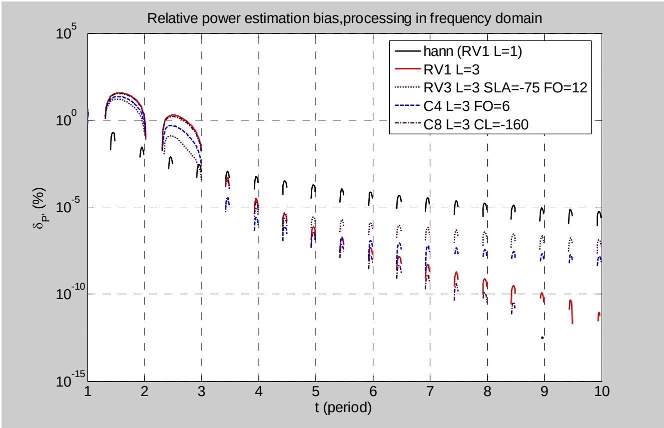

Fig. 4. Influence of the used window on power estimation bias for various windows.

IV. Conclusions

Six active power estimation algorithms were described and compared by computer simulation and measurement. Sensitivity of the described methods to subharmonic and interharmonic components and to the used window were shown for selected windows. It was also shown that using a PC plug-in board able to sample both input channel simultaneously and with high resolution leads to results very close to results gained by computer simulation, whereas using a multiplexed plug-in board leads to higher bias even if linear interpolation of the second channel signal is used.

Acknowledgement

The research of Milos Sedlacek was supported by the research program No. MSM6840770015 "Research of Methods and Systems for Measurement of Physical Quantities and Measured Data Processing " of the CTU in Prague sponsored by the Ministry of Education, Youth and Sports of the Czech Republic.

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