

## NEW METHOD FOR DETERMINATION OF AMPLITUDE AND MODULATION DEPTH OF FLUCTUATING HARMONICS

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**Abstract** - This paper describes an original method based on the Fourier transform, developed in order to determine the characteristics of a signal containing fluctuating harmonics. Problems arising from applying the Fourier transform to signals that contain an infinite number of components are mentioned, and the way to overcome them is explained. Some simulations have been performed to prove the efficiency of the method.

### I. Introduction

The testing of electrical appliances for harmonic emissions in relation to Electromagnetic Compatibility (EMC) regulations is required in many countries. One condition that has to be studied is when harmonic components are fluctuating. Some methods based on the wavelets [1] or on Kalman filters have already been tested [2]. An other method is described in [3]. The method proposed here is based on the Fourier analysis in addition with the estimation and the correction of the error made.

### II. Type of analyzed signals and parameters to determine

#### A. Type of analyzed signals

The simulated signals used to test the method contain a low frequency (typically  $f_0 = 50$  Hz) stationary fundamental component and two fluctuating harmonics modulated by square waves at frequencies ranging between 1 Hz and 10 Hz. These signals can be described by :

$$s(t) = U_1 \cdot \sin(2\pi \cdot f_0 \cdot t) + U_{n_1} \cdot [1 + k_{n_1} \cdot \text{Mod}_1(t)] \cdot \sin(2\pi \cdot n_1 \cdot f_0 \cdot t) + U_{n_2} \cdot [1 + k_{n_2} \cdot \text{Mod}_2(t)] \cdot \sin(2\pi \cdot n_2 \cdot f_0 \cdot t)$$

where  $\text{Mod}_1(t)$  and  $\text{Mod}_2(t)$  are two square waveforms taking values +1 and -1.

The maximum order of these harmonics is 30<sup>th</sup> ( $n_1 < n_2 < 30$ ). Their amplitudes  $U_{n_1}$  and  $U_{n_2}$  are such as  $0.05 \cdot U_1 < U_{n_1} < U_{n_2} < 0.25 \cdot U_1$  and their modulation depths  $k_{n_1}$  and  $k_{n_2}$  between 0.01 and 0.2.

#### B. Parameters to determine

The parameters to determine are the amplitudes  $U_1$  of the fundamental component and  $U_{n_1}$  and  $U_{n_2}$  of the harmonics, and the modulation depths  $k_{n_1}$  and  $k_{n_2}$  of the harmonics. The parameters that have to be known are the frequency  $f_0$  of the fundamental component, the orders  $n_1$  and  $n_2$  of the fluctuating harmonics and the frequencies of the modulating waveforms.

### III. Presentation of the method

#### A. Principle

A Fourier transform is applied to the simulated signal which is sampled over an integer number of periods of the fundamental component. The obtained spectrum shows directly the amplitudes of the fundamental component, of the harmonics, and of the modulation components around each fluctuating harmonic. An example of spectrum of such signals is given in figure 1. As the shape of the modulating waveforms is known, the modulation depth of each harmonic can be deduced from the amplitude of the considered harmonic and the first corresponding modulation component. Errors occur when some modulation components overlap other components used for the calculation of parameters of interest.

In addition, the modulation waveforms contain an infinite number of components that give rise to an infinite number of modulation components around each modulated harmonic. Then, the Shannon condition is no more fulfilled and folded components (which are not represented on figure 1) appear in the signal spectrum, that can lead to significant errors in case of overlapping with components used in the computation. Such overlapping occurs or does not occur dependently on frequencies of modulation waveforms, the frequency of the fundamental component of the signal and the sampling frequency.

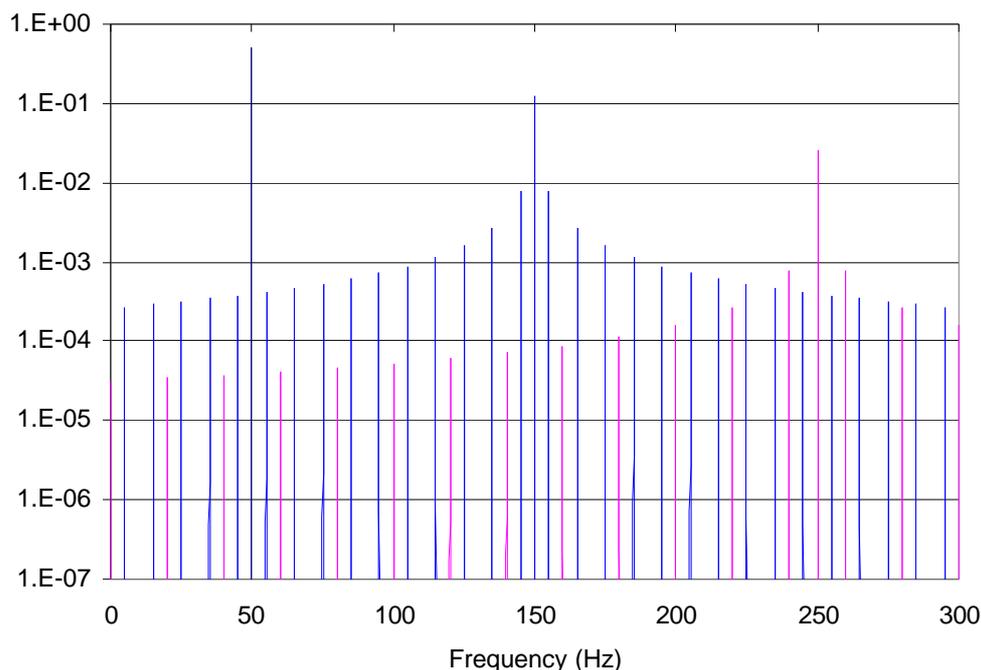


Fig. 1 : Spectrum of a signal consisting of a fundamental component at 50 Hz and two fluctuating harmonics of 3<sup>rd</sup> and 5<sup>th</sup> orders (observed over an infinite time interval).

Furthermore, the simulated signal is not generally sampled over an integer number of periods of modulation waves. This can also introduce some errors particularly on the value of the modulation depth that can be drastically reduced by the use of appropriate windows. In our simulations we have systematically used a Blackman-Harris 7 terms window before calculating the Fourier Transform.

Our first simulations showed that in the worst cases, the resulting errors generally do not exceed 1%. As this error remains too large for metrological applications, an iteration method has been developed in order to reduce these errors making them negligible.

### B. Iteration process

The principle is to estimate the error on the result in order to correct it. In a first step, the simulated signal is applied to the input of the Fourier transform algorithm which calculates the signal parameters. Let us call  $P$  the set of parameters (amplitude of the fundamental component, amplitudes and modulation depths of the harmonics) of the simulated signal and  $P_1$  the set of parameters calculated by the algorithm. Then a new signal simulated on the basis of these calculated parameters is applied to the same algorithm, and new values  $P'_1$  of the signal parameters are computed. The difference  $\varepsilon_1 = P'_1 - P_1$  between these new values and the previous values is an estimation of the error. This error is then subtracted from the parameters calculated in the first step giving a corrected result  $P_2 = P_1 - \varepsilon_1$ . The process consisting in simulating new signals on the basis of corrected values of the signal parameters to obtain a better estimation of the error, can be repeated as many times as needed. After each iteration, the resulting new error is reduced compared to the previous error. After  $n$  iterations, the  $n^{\text{th}}$  corrected result  $P_n$  is

given by  $P_n = P_{n-1} - \varepsilon_{n-1}$  where  $\varepsilon_{n-1} = P_{n-1}' - P_1$ . Our simulations showed that after 5 iterations, the errors on all calculated parameters become negligible.

### III. Results of simulations

#### A. Simulation conditions and limitation of the method

In our simulations, the sampling frequency is 3 kHz and the number of samples is 30000 which is compatible with instrument like the Agilent 3458A (with extended memory) for future real measurement applications. The sampling frequency allows to make signal analysis taking into account harmonics up to the 30<sup>th</sup> order for signal with fundamental frequency at 50 Hz and the resulting observation time interval (10 s) leads to a frequency resolution of 0.1 Hz. As a consequence, the tested modulation frequencies must be an integer multiple of 0.1 Hz.

#### B. Simulation results

Results presented here have been obtained for a signal containing a 5<sup>th</sup> and a 7<sup>th</sup> order fluctuating harmonics with amplitudes equal to 25% and 10% of the fundamental component and modulation depths of 0.1 and 0.05. The modulation frequency of the 7<sup>th</sup> order harmonic is 5 Hz whereas it varies between 1 Hz and 10 Hz for the other harmonic. Fig. 2 shows the variation of the error on the calculated amplitude of the 7<sup>th</sup> order harmonic as a function of the modulation frequency of the other harmonic. The error has been computed by comparing the result of the simulation ( $P_{n+1}$ , for  $n = 0, 2$  and  $5$ ) with the original parameters ( $P$ ) of the simulated signal. In most cases, without any iteration, this error is lower than 1 part in  $10^{13}$ . But for some values of the modulation frequency of the 5<sup>th</sup> order harmonic, the error can reach 0.1%. After 2 iterations the error does not exceed 1 part in  $10^7$  and after 5 iterations it is always smaller than a few parts in  $10^{15}$ , except for one point where it is approximately 1 part in  $10^{13}$ . As a conclusion, after 5 iterations, the error becomes negligible. A similar behavior is observed for the other signal parameters we have determined.

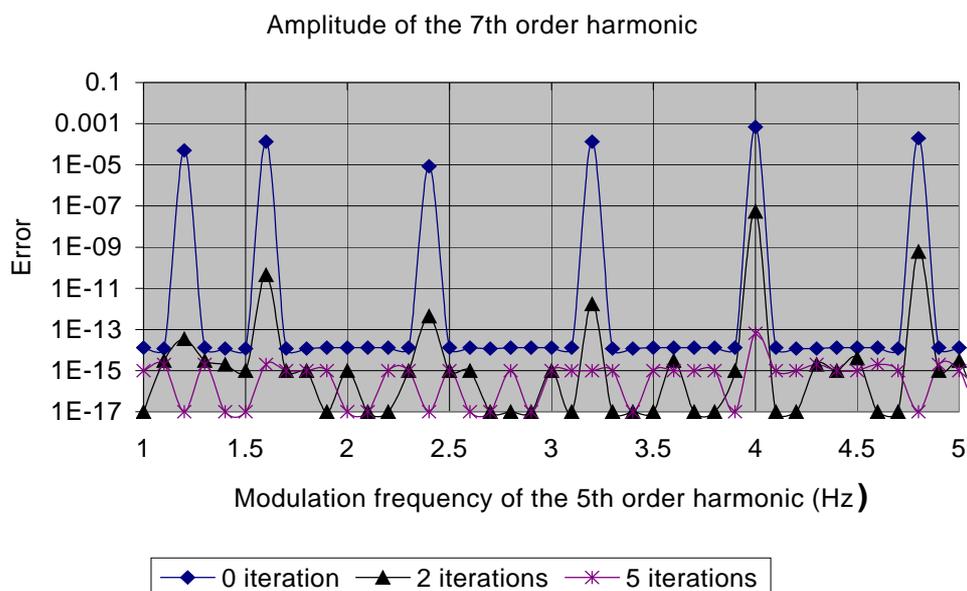


Fig. 2 : Error on the amplitude of the 7<sup>th</sup> order harmonic as a function of the modulation frequency of the 5<sup>th</sup> order harmonic for various numbers of iterations.

#### **IV. Conclusion and future work**

An original method has been developed to determine parameters of signals containing a low frequency stationary fundamental component and two fluctuating harmonics modulated by square waveforms. Simulations showed that the error resulting from the calculation process is negligible.

Future work will be the application of the method to other modulation waveforms, to signals containing a larger number of harmonics and finally to real measurements.

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